International Journal of Pure and Applied Mathematics
Volume 55 No. 1 2009, 121-123

ON THE POSTULATION OF A GENERAL UNION OF DOUBLE POINTS AND TRIPLE POINTS IN $\mathbb{P}^3$

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Abstract: Assume char $\neq 2, 3$. Fix integers d, x, y such that $d \geq 15$, $x \geq 0$ and $y \geq 0$. Let $Z \subset \mathbb{P}^3$ be a general union of x triple points and y double points. Then either $h^0(\mathbb{P}^3, \mathcal{I}_Z(d)) = 0$ (case $10x+4y \geq \binom{d+3}{3}$) or $h^1(\mathbb{P}^3, \mathcal{I}_Z(d)) = 0$ (case $10x + 4y \leq \binom{d+3}{3}$).

AMS Subject Classification: 14N05, 15A72, 65D05
Key Words: polynomial interpolation, triple point, double point, zero-dimensional scheme

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Here we work over an algebraically closed field $\mathbb{K}$ such that $\mathrm{char}(\mathbb{K}) \neq 2, 3$ and discuss the following result.

Theorem 1. *Fix integers d, x, y such that $d \geq 15$, $x \geq 0$ and $y \geq 0$. Let $Z \subset \mathbb{P}^3$ be a general union of x triple points and y double points. Then either $h^0(\mathbb{P}^3, \mathcal{I}_Z(d)) = 0$ (case $10x + 4y \geq \binom{d+3}{3}$) or $h^1(\mathbb{P}^3, \mathcal{I}_Z(d)) = 0$ (case $10x + 4y \leq \binom{d+3}{3}$).*

(Explanation: a triple (resp. double) point of $\mathbb{P}3$ has length 1) (resp. 4).

In a previous preprint we claimed the result for $d \geq 7$, but for low d we used a *Macaulay 2* check of the initial cases and hence we could not claim

Received: July 5, 2009

the result in positive characteristic. In the meantime much more was prove (even for quartuple points) (see [2] and [3]) and it seems worthwile to remark that an obvious adaptation of [2] gives Theorem 1. Since both papers depends heavily from a deep lemma in [1], there is no hope to extend the methods to characteristic 2 and 3.

With respect to [2] we have no quartuple point. Instead of [2], Lemma 8, we have the following numerical lemma.

Lemma 1. *Fix non negative integers x, y, e, f, t such that $t \geq 9$, $0 \leq e \leq 1$, $0 \leq f \leq 2$, and*

$$6x + 3y + 3e + f \leq (t+2)(t+1)/2. \tag{1}$$

Then

$$3x + y + 6e + 6f \leq (t+1)t/2. \tag{2}$$

If $e = 0$ and $y > 0$, then (2) holds for all $t \geq 7$.

Proof Assume that (2) fails, i.e. assume

$$3x + y + 6e + 6f \geq 1 + (t+1)t/2. \tag{3}$$

Subtracting the left hand side of (3) from (1), we get

$$3x + 2y - 3e - 5f \geq t. \tag{4}$$

Subtracting the double of (4) from (1), we get

$$9e + 11f - y \geq (t+2)(t+1)/2 - 2t. \tag{5}$$

Since $e \leq 1$, $f \leq 2$, and $y \geq 0$, we get $t \leq 8$ (and $t \leq 6$ if $e = 0$ and $y > 0$). □

(Explanation: a triple point (resp. double point, resp. simple point) of $\mathbb{P}^2$ has length 6 (resp. 3, resp. 1).

Then as in the proof of assertion H_d of [2] we only need to check the inequality

$$10(d-9) + \binom{9+3}{3} \leq \binom{d+3}{3}/10 - 2, \tag{6}$$

which is true for all $d \geq 15$. For the low cases in characteristic 0 or in characteristic 31991, adapt (deleting the quartuple points) the *Macaulay 2* program ([4]) listed in [2], §4.

Acknowledgements

The author was partially supported by MIUR and GNSAGA of INdAM (Italy).

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