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THE FUZZY PRICE OF DEMAND, ELASTICITY OF DEMAND AND SOME APPLICATIONS

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Abstract: In this paper, the optimal fuzzy-number-valued profit function for the price and the fuzzy-number-valued elasticity function are given and discussed using the fuzzy price function. Mean while the fuzzy-number-valued elasticity function is studied.

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1. Introduction and Concepts

The concept of fuzzy set was introduced by Zadeh [5] in 1965. After that, many applications of fuzzy sets have been developed. One of them is the article uses fuzzy set theory to study economic problems. In [1], a new solution procedure was considered to solve fuzzy equations to problems in economics and finance. In the perfectly competitive markets, the demand fluctuates slightly, thus it is real important to consider the changes of demand function with respect to that of price. In this paper, we discuss the fuzzy-number-valued elasticity functions and its applications.

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2. Preliminaries

In this section, we describe some basic concepts of fuzzy numbers [2], [3], [4]. A triangular fuzzy number $\widetilde{u} : R \to [0,1]$ is defined by

$$\widetilde{u}(x) = \begin{cases} 0, & \text{if } x < u - \delta_1; \\ \dfrac{x - u + \delta_1}{\delta_1}, & \text{if } u - \delta_1 \le x \le u; \\ \dfrac{u + \delta_2 - x}{\delta_2}, & \text{if } u \le x \le u + \delta_2; \\ 0, & \text{if } u > \delta_2, \end{cases}$$

and denoted by $(u - \delta_1, u, u + \delta_2)$. We call u is around δ_1 left and δ_2 right unsymmetrically.

If $\delta_1 = \delta_2 = \delta$, then it is called a symmetric triangular fuzzy number and is denoted by $(u - \delta, u, u + \delta)$ $(\delta > 0)$.

By the results of [2] and [4], for a fuzzy set, if we define $u_\alpha = \{x : \widetilde{u}(x) \ge \alpha\}$ for $0 < \alpha \le 1$, and $u_0 = \text{cl}\ \{\text{support}\ \widetilde{u}\}$ for $\alpha = 0$, then $\widetilde{u}$ is completely determined by the intervals $u_\alpha = [u_\alpha^-, u_\alpha^+]$. Suppose now that $\widetilde{u} \in F(R)$ are fuzzy numbers represented by $\{[u_\alpha^-, u_\alpha^+] : 0 < \alpha \le 1\}$. If $\widetilde{u}, \widetilde{v}$ are two triangular fuzzy numbers, then

(1) $u_\alpha = [(u_2 - u_1)\alpha + u_1, (u_2 - u_3)\alpha + u_3]$;

(2) $\widetilde{u} + \widetilde{v} = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$;

(3) $k\widetilde{u} = \begin{cases} (ku_1, ku_2, ku_3), & \text{if } k \ge 0; \\ (ku_3, ku_2, ku_1), & \text{if } k < 0. \end{cases}$

3. Fuzzy Price for Demand Functions

Let $p(x)$ be a crisp price function and x the demand quantity. In general, $p(x)$ is a decreasing function on $[0, \alpha]$. Then, the crisp revenue function is $R(x) = p(x)x, \quad x \in [0, \alpha]$.

Let the crisp cost function be $C(x) = C_0 + C_1(x)$, where C_0 is the fixed cost and $C_1(x)$ is the changeable cost. It is well known that the crisp profit function

$$L(x) = R(x) - C(x) = p(x)x - (C_0 + C_1(x))$$

is maximum satisfyies $R'(x_0) = C'(x_0)$. However, in the perfectly competitive markets, the price fluctuates slightly, thus considering the price as a fuzzy-number-valued function denoted by $\widetilde{p}(x), x \in [0, \alpha]$. In this section, we discuss the cases when $\widetilde{p}(x)$ is a symmetrical and unsymmetrical triangular fuzzy-number-valued function.

3.1. Symmetrical Triangular Fuzzy-Number-Valued Price Function

Let $\widetilde{p}(x)$ be a symmetrical triangular fuzzy-number-valued price function denoted by $\widetilde{p}(x) = (p(x) - \delta(x), p(x), p(x) + \delta(x))$. Since x is position, the fuzzy profit function is

$$\begin{aligned}\widetilde{L}(x) &= \widetilde{R}(x) - C(x) = \widetilde{p}(x)x - (C_0 + C_1(x)) \\ &= (p(x)x - \delta(x)x - C_0 - C_1(x), p(x)x - C_0 - C_1(x), \\ &\quad p(x)x + \delta(x)x - C_0 - C_1(x)).\end{aligned}$$

Thus, in the fuzzy sense, if $p(x)$ moves around $\delta(x)$ symmetrically, the profit $L(x) = R(x) - C(x) = p(x)x - (C_0 + C_1(x))$ is moves around $\delta(x)x$ symmetrically. Furthermore, $\widetilde{L}(x)$ get maximum at x_0 satisfying $R'(x_0) = C'(x_0)$ and the maximum $L(x_0) = p(x_0)x_0 - (C_0 + C_1(x_0))$ moves around $\delta(x_0)x_0$ symmetrically.

3.2. Unsymmetrical Triangular Fuzzy-Number-Valued Price Function

Let $\widetilde{p}(x)$ be a unsymmetrical triangular fuzzy-number-valued price function denoted by $\widetilde{p}(x) = (p(x) - \delta_1(x), p(x), p(x) + \delta_2(x))$. Since x is position, the fuzzy profit function is

$$\begin{aligned}\widetilde{L}(x) &= \widetilde{R}(x) - C(x) = \widetilde{p}(x)x - (C_0 + C_1(x)) \\ &= (p(x)x - \delta_1(x)x - C_0 - C_1(x), p(x)x - C_0 - C_1(x), \\ &\quad p(x)x + \delta_2(x)x - C_0 - C_1(x)).\end{aligned}$$

Thus, in the fuzzy sense, if $p(x)$ moves around $\delta_1(x)$ left and $\delta_2(x)$ right unsymmetrically, the profit $L(x) = R(x) - C(x) = p(x)x - (C_0 + C_1(x))$ moves around $\delta_1(x)$ left and $\delta_2(x)$ right unsymmetrically. Furthermore, $\widetilde{L}(x)$ gets maximum x_0 satisfying $R'(x_0) = C'(x_0)$ and the maximum $L(x_0) = p(x_0)x_0 - (C_0 + C_1(x_0))$ is around $\delta_1(x_0)x_0$ left and $\delta_2(x_0)x_0$ right unsymmetrically.

4. The Fuzzy-Number-Valued Elasticity Functions

In the perfectly competitive markets, the demand fluctuates slightly, thus it is real important to consider the changes of demand function with respect to that of the price. In this section, we discuss the fuzzy-number-valued elasticity functions and its applications.

Definition 4.1. (see [3]) Let $\widetilde{X} : [a,b] \to F(R)$ be a fuzzy-number-valued function on $[a,b]$. We say that it satisfies the condition (H) on $[a,b]$, if for any $x_1, x_2 \in [a,b]$ satisfying $x_1 < x_2$, there exists a fuzzy number $\widetilde{Y} \in F(R)$ such that $\widetilde{X}(x_2) = \widetilde{X}(x_1) + \widetilde{Y}$.

Definition 4.2. (see [3]) A fuzzy-number-valued function $\widetilde{X} : [a,b] \to F(R)$ is said to be differentiable at $x_0 \in [a,b]$, if there exists a $\widetilde{X}'(x_0) \in F(R)$ such that both limits

$$\lim_{h\to 0^+} \frac{\widetilde{X}(x_0+h) - \widetilde{X}(x_0)}{h}, \lim_{h\to 0^+} \frac{\widetilde{X}(x_0) - \widetilde{X}(x_0-h)}{h}$$

exist and are equal to $\widetilde{X}'(x_0)$.

Lemma 4.1. (see [3]) *Let $\widetilde{X} : [a,b] \to F(R)$ be differentiable at $x_0 \in [a,b]$. Then for any $\alpha \in [0,1]$, $X_\alpha^-(x), X_\alpha^+(x)$ are differentiable at $x_0 \in [a,b]$. Furthermore, $[X'(x_0)]_\alpha = [X_\alpha^{-'}(x), X_\alpha^{+'}(x)]$.*

Let $\widetilde{X} : [a,b] \to F(R)$ be a fuzzy-number-valued function on $[a,b]$ assuming values on the set of triangular fuzzy numbers. We write as $\widetilde{X}(w) = (X(w) - \delta_1(w), X(w), X(w) + \delta_2(w))$. If $\widetilde{X}(w)$ is differentiable on $[a,b]$, then $X_\alpha^-(w) = \delta_1(w)(\alpha-1) + X(w)$, $X_\alpha^+(w) = (1-\alpha)\delta_2(w) + X(w)$ are differentiable on $[a,b]$ for all $\alpha \in [a,b]$. From Lemma 4.1, we have

$$(X_\alpha^-(w))' = \delta_1'(w)(\alpha-1) + X'(w), \; (X_\alpha^+(w))' = (1-\alpha)\delta_2'(w) + X'(w).$$

That is $\widetilde{X}'(w) = (X'(w) - \delta_1'(w), X'(w), X'(w) + \delta_2'(w))$.

Let $\widetilde{Q} = \widetilde{Q}(p)$ be a triangular fuzzy-number-valued demand function on $[0,\alpha]$, i.e., $\widetilde{Q}(p) = (Q(p) - \delta_1(p), Q(p), Q(p) + \delta_2(p))$. If $\widetilde{Q}(p)$ is differentiable at $p \in [0,\alpha]$, then $\widetilde{\varepsilon}_{Q_p} = -\widetilde{Q}'(p)\frac{p}{Q(p)}$ is called the *fuzzy price-demand elasticity* at $p \in [0,\alpha]$. Obviously,

$$\widetilde{\varepsilon}_{Q_p} \approx -\frac{\frac{\widetilde{Q}(p+\Delta p) - \widetilde{Q}(p)}{Q(p)}}{\frac{\Delta p}{p}}.$$

Thus, $\widetilde{\varepsilon}_{Q_p} = -\widetilde{Q}'(p)\frac{p}{Q(p)}$ means that when the change of price is 1 (i.e., $\frac{\Delta p}{p} = 1$), the change of demand is $\widetilde{\varepsilon}_{Q_p}$.

Example 4.1. In the perfectly competitive markets, the price fluctuates slightly, thus considering the price as a fuzzy-number-valued function denoted by $\widetilde{p}(x) = \widetilde{a} - \widetilde{b}x$, $x \in [0, \frac{a}{b}]$.

If $\widetilde{a}, \widetilde{b}$ are fuzzy numbers around δ_{11} left and δ_{12} right, δ_{21} left and δ_{22} right, respectively, i.e., $\widetilde{a} = (a - \delta_{11}, a, a + \delta_{12})$, $\widetilde{b} = (b - \delta_{21}, b, b + \delta_{22})$, $\widetilde{p}(x) = (a - \delta_{11} - (b + \delta_{22})x, a - bx, a + \delta_{12} - (b - \delta_{21})x)$. Since x is position, the fuzzy profit function is

$$\begin{aligned}\widetilde{L}(x) &= \widetilde{R}(x) - C(x) = \widetilde{p}(x)x - (C_0 + C_1(x))\\ &= ((a - \delta_{11})x - (b + \delta_{22})x^2 - C_0 - C_1 x, ax - bx^2 - C_0 - C_1 x,\\ &\quad (a + \delta_{12})x - (b - \delta_{21})x^2 - C_0 - C_1 x).\end{aligned}$$

Thus, in the fuzzy sense, if a moves around δ_{11} left and δ_{12} right, and b around δ_{21} left and δ_{22}, then the profit $L(x) = R(x) - C(x) = p(x)x - (C_0 + C_1 x)$ moves around $(\delta_{11} + \delta_{22}x)x$ left and $(\delta_{12} + \delta_{21}x)x$ right. Furthermore, $\widetilde{L}(x)$ get maximum at x_0 satisfying $R'(x_0) = C'(x_0)$ and the maximum $L(x_0) = R(x_0) - C(x_0) = p(x_0)x_0 - (C_0 + C_1 x_0$ is around $(\delta_{11} + \delta_{22}x_0)x_0$ left and $(\delta_{12} + \delta_{21}x_0)x_0$ right. Notice that $\widetilde{Q}(x) = (-(b + \delta_{22}), -b, -(b - \delta_{21}))$. The fuzzy price-demand elasticity at $p \in [0, \alpha]$ is

$$\widetilde{\varepsilon}_{Q_p} = \left((b - \delta_{21})\frac{p}{a - bp}, b\frac{p}{a - bp}, (b + \delta_{22})\frac{p}{a - bp}\right).$$

The above representation shows that the fuzzy price-demand elasticity at $p \in [0, \alpha]$ is $b\dfrac{p}{a - bp}$ rounding $(b - \delta_{21})\dfrac{p}{a - bp}$ left and $(b + \delta_{22})\dfrac{p}{a - bp}$ right.

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