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ESSENTIAL COMPONENTS OF THE SET OF EQUILIBRIUM POINTS FOR GENERALIZED GAMES IN THE UNIFORM TOPOLOGICAL SPACE OF BEST REPLY CORRESPONDENCES

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Abstract: In this article, we discuss the stability of the set of equilibrium points for generalized games. We first give a existence theorem of generalized games. Then, we show that, for every generalized game in the uniform topological space of best reply correspondences, there exists at least one essential component of the set of equilibrium points for generalized games.

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1. Introduction and Preliminaries

Recently, essential components of solution set have been an important aspect of stability for nonlinear problems. In game theory, the stability and perfection of Nash equilibrium points have become important topics, see [4] and references therein. In 1986, Kohlberg and Mertens [11] proposed a list of requirements which satisfactory solution concepts for finite games should satisfy and they introduced hyper-stable set, fully stable set, and stable set of Nash equilibrium points, each of which satisfies most of the requirements. Hillas [7] also pro-

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posed other versions of strategic stability, i.e., he considered the subset of Nash equilibrium points robust against perturbations of the best response correspondences. On the other hand, in 1963, Jiang [8] has proved that every finite game has at least one essential component of its Nash equilibrium points. Jiang's work was motivated by [9] in which Kinoshita introduced the notion of essential components of the set of fixed point and proved that for any continuous map of the Hilbert cube into itself, there exists at least one essential component of the set of its fixed points. In recent years, some existence results for essential components of the solution set of nonlinear problems have been obtained, see [12]-[16].

In this article, we discuss the stability of the set of equilibrium points for generalized games. We first give an existence theorem of generalized games. Then, we show that, for every generalized game in the uniform topological space of best reply correspondences, there exists at least one essential component of the set of equilibrium points for generalized games.

Now we introduce some notations and definitions. We shall consider a finite-players games in its strategic form $\Gamma := (X_i, f_i, G_i)_{i\in N}$, where $N := \{1, 2, \dots, n\}$. For each $i \in N, X_i$ is the set of strategies for the player i, each f_i is a mapping from $X := \prod_{i\in N} X_i$ into R, which is called the payoff function of the i-th player. For each $i \in N$, denote

$$\hat{i} = N\backslash\{i\}, X_{\hat{i}} = \prod_{j\in N\backslash\{i\}} X_j, x_{\hat{i}} = (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n) \in X_{\hat{i}}$$

and $x := (x_i, x_{\hat{i}}) \in X$. The feasible strategy correspondence of player i is $G_i : X_{\hat{i}} \to 2^{X_i}$. Then we call $\Gamma := (X_i, f_i, G_i)_{i\in N}$ is an generalized game. The equilibria of generalized game is: Find x^* such that for all $i \in N$, $x^*_i \in G_i(x^*_{\hat{i}})$ and $\max\{f_i(z, x^*_{\hat{i}}) : z \in G_i(x^*_{\hat{i}})\} = f_i(x^*)$.

Definition 1. For each fixed $x_{\hat{i}} \in X_{\hat{i}}$, the best strategy set for i-th player is such that:

$$B_i(x_{\hat{i}}) = \{x_i \in G_i(x_{\hat{i}}) : f_i(x_i, x_{\hat{i}}) = \max f_i(x'_i, x_{\hat{i}}), \forall x'_i \in G_i(x_{\hat{i}})\}.$$

$B_i : X_{\hat{i}} \to 2^{X_i}$ is a set-value map. B_i is said to be the best reply correspondence of i's player. Let $B : X \to 2^X$ such that $B(x_i, x_{\hat{i}}) = \prod_{i\in N} B_i(x_{\hat{i},}), \forall x = (x_i, x_{\hat{i}}) \in X$, then B is said to be the best reply correspondence of a generalized game $\Gamma := (X_i, f_i, G_i)_{i\in N}$.

Definition 2. Let $X$and Y are the Hausdorff spaces, $F : Y \to 2^X$ is a set-valued mapping, then:

(1) F is said to be upper semicontinuous (u.s.c.) at $y \in Y$, if for each open set $G \supset F(y)$, there exists an open neighborhood $O(y)$ of y such that $G \supset F(y')$

for any $y' \in O(y)$.

(2) F is said to be lower semicontinuous (l.s.c.) at $y \in Y$, if for each open set $G \cap F(y) \neq \emptyset$, there exists an open neighborhood $O(y)$ of y such that $G \cap F(y') \neq \emptyset$ for any $y' \in O(y)$.

(3) F is an usco mapping, if F is upper semicontinuous on Y, and for each $y \in Y, F(y)$ is compact.

Lemma 1. (see [3], [5]) *Let X be a nonempty compact and convex subset of a Hausdorff topological vector space E. Suppose a set-valued mapping $T : X \to 2^X$ has the following properties:*

(1) $T(X)$ is nonempty and convex for each $x \in X$;

(2) T has open inverse valued, i.e., $T^{-1}(y) = \{x \in X : y \in T(x)\}$ for each $y \in X$ is open in X.

Then T has at least one fixed point.

Lemma 2. (see Theorem 2 in [6]) *Let X be a metric space, Y be a Baire space, $F : Y \to 2^X$ is an usco mapping. Then there is a dense G_δ subset Q' of Y such that F is l. s. c on Q'.*

2. The Existence and Generic Stability of the Set of Equilibrium Points for Generalized Game

In this section, we first give an existence theorem for generalized game. Then, we study the generic stability of the set of equilibrium points for generalized game.

Theorem 1. *Let $\Gamma = (X_i, f_i, G_i)_{i\in N}$ be a given generalized game. Suppose that Γ satisfies the following condtions:*

(1) for each $i \in N, X_i$ is a nonempty convex compact set of a normed space E_i;

(2) for each $i \in N, f_i$ is continuous on $X = \prod_{i\in N} X_i$, $G_i : X_{\hat{i}} \to 2^{X_i}$ is continuous and with nonempty convex compact values;

(3) for each $x \in X$, and each $i \in N, f_i(\cdot, x_{\hat{i}})$ is quasi-concave on X_i;

(4) for each $y \in X$, the set

$$\{x \in X : \max\{f_i(z, x_{\hat{i}}), z \in G_i(x_{\hat{i}})\} = f_i(y_i, x_{\hat{i}}), i \in N\}$$

is open set.

Then the generalized game $\Gamma = (X_i, f_i, G_i)_{i\in N}$ has an equilibrium.

Proof. For condtion (2), We can obtain that $f_i(z, x_{\hat{i}})$ attained the maximum on $G_i(x_{\hat{i}})$. Let function $V_i : X_{\hat{i}} \to R$ as following:for each $i \in N$

$$V_i(x_{\hat{i}}) = \max\{f_i(z, x_{\hat{i}}) : z \in G_i(x_{\hat{i}})\}.$$

Let set-valued mapping $M_i : X_{\hat{i}} \to 2^{X_i}$ as following: for each $i \in N$

$$M_i(x_{\hat{i}}) = \{z \in G_i(x_{\hat{i}}) : V_i(x_{\hat{i}}) = f_i(z, x_{\hat{i}})\}.$$

By condition (3) and the definition of V_i and M_i, we can obtain that $M_i(x_{\hat{i}})$ is nonempty and convex.

Define a set-valued mapping $F : X \to 2^X$ by $F(x) = \{y \in X : y_i \in M_i(x_{\hat{i}})\}$ for each $i \in N$, then for each $x \in X, F(x)$ is a nonempty convex subset in X. Since for each $y \in X$,

$$\begin{aligned} F^{-1}(y) &= \{x \in X : y \in F(x)\} = \{x \in X : y_i \in M_i(x_{\hat{i}}), i \in N\} \\ &= \{x \in X : \max\{f_i(z, x_{\hat{i}}), z \in G_i(x_{\hat{i}})\} = f_i(y_i, x_{\hat{i}}), i \in N\}. \end{aligned}$$

By condition (4), we obtain that $F^{-1}(y)$ is open. Now, by Lemma 1, there exists $x^* \in X$ such that $x^* \in F(x^*)$, then $x_i^* \in M_i(x_{\hat{i}})$ for each $i \in N$. That means the generalized game $\Gamma = (X_i, f_i, G_i)_{i \in N}$ has a equilibrium. The proof is complete.

Let $M = \{(f, G) : f, G$ staisfying the conditions of Theorem 1 and

$$\sup_{x \in X} \sum_{i=1}^{n} \|f_i(x)\| < +\infty\}.$$

Let $u = (f, G) \in M$, here $f = (f_1, f_2, \ldots, f_n)$, $G = (G_1, G_2, \ldots, G_n)$, for each $u_1 = (f^1, G^1), u_2 = (f^2, G^2) \in M$. Define

$$\rho(u_1, u_2) = \sum_{i=1}^{n} \sup_{x \in X} \|f_i^1(x) - f_i^2(x)\| + \sup_{x \in X} \sum_{i=1}^{n} h_i(G_i^1(x), G_i^2(x))$$

for each $i \in N, x \in X$, where $h_i(G_i^1(x), G_i^2(x))$ denotes the Hausdorff distance between $G_i^1(x)$ and $G_i^2(x)$ on E_i.

Clearly (M, ρ) is a mertric space. For any $u \in M$, by Theorem 1, there exists $x^* \in X$ be a equilibrium of the genernalized game $\Gamma = (X_i, f_i, G_i)_{i \in N}$. Let $E(u)$ be the set of all equilibrium points of u, then $E(u) \neq \emptyset$ and thus defines a set-valued mapping from M into X.

Lemma 3. *Let X, Y be two Hausdorff topological space, $\{A_\alpha\}_{\alpha \in \Lambda}$ be a net on $K(X)$, where $K(X)$ means the collection of all nonempty compact subsets of X, $\{y^\alpha\}_{\alpha \in \Lambda}$ be a net on Y, $\{f^\alpha(x, y)\}_{\alpha \in \Lambda}$ be a net of continuous real function on $X \times Y$. If $A_\alpha \to A \in K(X), y^\alpha \to y \in Y$ and $\sup_{(x,y) \in X \times Y} \|f^\alpha(x, y) -$*

$f(x,y)\| \to 0, f$ *is a continuous real function on* $X \times Y$*, then*

$$\max_{v\in A_\alpha} f^\alpha(v, y^\alpha) \to \max_{v\in A} f(v, y).$$

Proof. For the conclusion of Aubin (Theorem 3 in [1]), we have

$$\max_{v\in A_\alpha} f(v, y^\alpha) \to \max_{v\in A} f(v, y).$$

Then we only need to prove

$$\max_{v\in A_\alpha} f^\alpha(v, y^\alpha) \to \max_{v\in A_\alpha} f(v, y^\alpha).$$

For each $\alpha \in \Lambda$, $\exists v_\alpha \in A_\alpha$, $v'_\alpha \in A_\alpha$ such that

$$f^\alpha(v_\alpha, y^\alpha) = \max_{v\in A_\alpha} f^\alpha(v, y^\alpha), \qquad f(v'_\alpha, y^\alpha) = \max_{v\in A_\alpha} f(v, y^\alpha).$$

Since

$$f^\alpha(v'_\alpha, y^\alpha) - f(v'_\alpha, y^\alpha) \le \max_{v\in A_\alpha} f^\alpha(v, y^\alpha) - \max_{v\in A_\alpha} f(v, y^\alpha) \le f^\alpha(v_\alpha, y^\alpha) - f(v_\alpha, y^\alpha).$$

Then

$$\begin{aligned}\|\max_{v\in A_\alpha} f^\alpha(v, y^\alpha) - \max_{v\in A_\alpha} f(v, y^\alpha)\| &\le \max(\|f^\alpha(v'_\alpha, y^\alpha) - f(v'_\alpha, y^\alpha)\|, \|f^\alpha(v_\alpha, y^\alpha) \\ &- f(v_\alpha, y^\alpha)\|) \le \sup_{(x,y)\in X\times Y} \|f^\alpha(x,y) - f(x,y)\| \to 0.\end{aligned}$$

Thus $\max_{v\in A_\alpha} f^\alpha(v, y^\alpha) \to \max_{v\in A} f(v, y)$. The proof is complete. □

Lemma 4. $S : M \to 2^X$ *is an usco mapping.*

Proof. Since X is a compact, by Theorem 7.1.16 of [10], it suffices to show that S is a closed mapping, i.e., the graph Graph(S) of S is closed in $M \times X$, where Graph(S)$= \{(u, x) \in M \times X : x \in S(u)\}$.

Let $\{(u^\alpha, x^\alpha)\}_{\alpha\in\Lambda}$ be a net in Graph(S) with $\{(u^\alpha, x^\alpha)\}_{\alpha\in\Lambda} \to (u, x) \in M \times X$. Then $u^\alpha \to u, x^\alpha \to x$ and $x^\alpha \in S(u^\alpha)$. Then for all $\alpha \in \Lambda$, there exists $x_i^\alpha \in G_i^\alpha(x_{\hat{i}}^\alpha), f_i^\alpha(x_i^\alpha, x_{\hat{i}}^\alpha) = \max_{y_i\in G_i(x_{\hat{i}}^\alpha)} f_i^\alpha(y_i, x_{\hat{i}}^\alpha)$ for all $i \in N$. Then for all $i \in N$, $d_i(x_i, G_i(x_{\hat{i}})) \le d_i(x_i, x_i^\alpha) + d_i(x_i^\alpha, G_i^\alpha(x_{\hat{i}}^\alpha)) + h_i(G_i^\alpha(x_{\hat{i}}^\alpha), G_i(x_{\hat{i}}^\alpha)) + h_i(G_i(x_{\hat{i}}^\alpha), G_i(x_{\hat{i}})) \le d_i(x_i, x_i^\alpha) + \rho(u^\alpha, u) + h_i(G_i(x_{\hat{i}}^\alpha), G_i(x_{\hat{i}})) \to 0$. Hence, we obtain $x_i \in G_i(x_{\hat{i}})$.

Since

$$\begin{aligned}\|f_i^\alpha(x_i^\alpha, x_{\hat{i}}^\alpha) - f_i(x_i, x_{\hat{i}})\| &\le \|f_i^\alpha(x_i^\alpha, x_{\hat{i}}^\alpha) - f_i(x_i^\alpha, x_{\hat{i}}^\alpha)\| + \|f_i(x_i^\alpha, x_{\hat{i}}^\alpha) - f_i(x_i, x_{\hat{i}})\| \\ &\le \rho(u^\alpha, u) + \|f_i(x_i^\alpha, x_{\hat{i}}^\alpha) - f_i(x_i, x_{\hat{i}})\| \to 0.\end{aligned}$$

There exists $f_i^\alpha(x_i^\alpha, x_{\hat{i}}^\alpha) \to f_i(x_i, x_{\hat{i}})$.

Moreover, since

$$h_i(G_i^\alpha(x_{\hat{i}}^\alpha), G_i(x_{\hat{i}})) = h_i(G_i^\alpha(x_{\hat{i}}^\alpha), G_i(x_{\hat{i}}^\alpha)) + h_i(G_i(x_{\hat{i}}^\alpha), G_i(x_{\hat{i}}))$$
$$\leq \rho(u^\alpha, u) + h_i(G_i(x_{\hat{i}}^\alpha), G_i(x_{\hat{i}})) \to 0.$$

Then, we obtain $G_i^\alpha(x_{\hat{i}}^\alpha) \to G_i(x_{\hat{i}})$. For Lemma 3, we have

$$\max_{y_i \in G_i^\alpha(x_{\hat{i}}^\alpha)} f_i^\alpha(y_i, x_{\hat{i}}^\alpha) \to \max_{y_i \in G_i(x_{\hat{i}})} f_i(y_i, x_{\hat{i}}).$$

Then $f_i(x_i, x_{\hat{i}}) = \max_{y_i \in G_i(x_{\hat{i}})} f_i(y_i, x_{\hat{i}})$. Therefore $x \in S(u)$, and thus Graph(S) is closed. The proof is complete. □

Definition 3. For each $u \in M, x \in S(u)$ is said to be an essential equilibrium of u provided that for any neighborhood O of x in X, there exists an open neighborhood V of u in M such that $S(u') \cap O \neq \emptyset$ for all $u' \in V$. Further, u is said to be essential if all its equilibrium are essential; u is said to be weakly essential if there exists one equilibrium of u is essential.

By Defintion 2 and Defintion 3, it is easy to obtain the following results:

Lemma 5. *u is essential if and only if the set-valued mapping S is l.s.c. on u.*

Theorem 2. *There exists a dense G_δ subset Q of M such that each $u \in Q$, u is essential.*

Proof. According to Lemma 2, Lemma 4, Lemma 5, the results can easily be obtain. □

3. Essential Components in the Uniform Topological Space of Best Reply Correspondence

In this section, we consider the essential components for the generalized game in the uniform topological space of best reply correspondences. First, two lemmas is attained. Then, the main result of this article is attained.

Lemma 6. *$B : X \to 2^X$ is upper semicontinuous with convex and compact values.*

Proof. By Definition 1, we only need to prove that B_i is upper semicontinuous with convex and compact values.

Since for each $x_{\hat{i}} \in X_{\hat{i}}, u_i \to f_i(u_i, x_{\hat{i}})$ is continuous and $G_i(x_{\hat{i}})$ is compact, then $B_i(x_{\hat{i}}) \neq \emptyset$.

For any $x_i^1, x_i^2 \in B_i(x_{\hat{i}}), \forall \lambda \in (0,1)$, then $x_i^1, x_i^2 \in G_i(x_{\hat{i}})$. Since $G_i(x_{\hat{i}})$ is convex, then $\lambda x_i^1 + (1-\lambda) x_i^2 \in G_i(x_{\hat{i}})$. For $x_i^1, x_i^2 \in B_i(x_{\hat{i}})$, we denote $\max_{u_i \in G_i(x_{\hat{i}})} f_i(u_i, x_{\hat{i}}) = a$, then $f_i(x_i^1, x_{\hat{i}}) = a, f_i(x_i^2, x_{\hat{i}}) = a$. For each $x_{\hat{i}} \in X_{\hat{i}}$, function $f_i(\cdot, x_{\hat{i}})$ is quasi-concave on X_i. Then

$$f_i(\lambda x_i^1 + (1-\lambda)x_i^2, x_{\hat{i}}) \geq \min(f_i(x_i^1, x_{\hat{i}}), f_i(x_i^2, x_{\hat{i}})) = a.$$

Since $\lambda x_i^1 + (1-\lambda)x_i^2 \in G_i(x_{\hat{i}})$, then $f_i(\lambda x_i^1 + (1-\lambda)x_i^2, x_{\hat{i}}) \leq a$. Therefore $f_i(\lambda x_i^1 + (1-\lambda)x_i^2, x_{\hat{i}}) = a$, thus $\lambda x_i^1 + (1-\lambda)x_i^2 \in B_i(x_{\hat{i}})$, hence $B_i(x_{\hat{i}})$ is convex.

By the condition (2) of Theorem 1, it is easily obtained that f_i is continuous on X. Then for each fixed $x_{\hat{i}} \in X_{\hat{i}}, f_i(\cdot, x_{\hat{i}})$ is continuous on X_i, Hence the set $B_i(x_{\hat{i}})$ is closed. At the same time $B_i(x_{\hat{i}}) \subseteq X_i$, and X_i is compact. Therefore $B_i(x_{\hat{i}})$ is compact. Thus for each $i \in N, B_i$ is convex and compact values.

Now, we proved that for each $i \in N, B_i$ is u.s.c. For each $x_{\hat{i}} \in X_{\hat{i}}, B_i(x_{\hat{i}}) = \{x_i \in G_i(x_{\hat{i}}) : f_i(x_i, x_{\hat{i}}) = \max_{y_i \in G_i(x_{\hat{i}})} f_i(y_i, x_{\hat{i}})\}$. Since (1) $f_i : X = X_i \times X_{\hat{i}} \to R$ is continuous on $X_i \times X_{\hat{i}}$; (2) $G_i : X_{\hat{i}} \to 2^X \backslash \{\emptyset\}$ is continuous and with compact values. Then, by Lemma 9.4.3 of [2], we obtain that $B_i : X_{\hat{i}} \to 2^X$ is u.s.c. thus the proof is complete. □

By Lemma 6 and Fan-Glickberg Fixed Point Theorem, there exists $x^* \in X$ such that $x^* \in B(x^*)$. Denote $Fix(B)$ be the collection of all fixed points of B.

Lemma 7. *For each $u \in M$, $Fix(B) = E(u)$, where B is the best reply correspondences of the generalized game u.*

Proof. If $x^* \in B(x^*)$, then for all $i \in N, x_i^* \in B_i(x_{\hat{i}}^*)$. i.e., $x_i^* \in G_i(x_{\hat{i}}^*)$ and $f_i(x_i^*, x_{\hat{i}}^*) = \max_{u_i \in G_i(x_{\hat{i}}^*)} f_i(u_i, x_{\hat{i}}^*)$. Thus, $x^* \in E(u)$.

Also, if $x^* \in E(u)$, it can be easily obtained that $x^* \in B(x^*)$. The proof is complete. □

Let $C = \{B : X \to 2^X$, B is u.s.c. with nonempty convex and compact values$\}$. For any $B, B' \in C$, define $\rho_1(B, B') = \sup_{x \in X} h(B(x), B'(x))$, where h is the Hausdorff distance defined on X. By Theorem 3.4 of [15], we can obtain that for each $B \in C, Fix(B)$ possesses at least one essential compact.

Definition 4. Let $u \in M$, $e(u)$ be a compact of $E(u)$. $e(u)$ is said to be essential component of $E(u)$ if for each open set $O \supset e(u), \exists \delta > 0$, then for all B' satisfying $\rho_1(B, B') < \delta$ such that $Fix(B) \cap O \neq \emptyset$. Here B is the best reply correspondences of the generalized game u.

Theorem 3. *For each $u \in M$, there exists at least one essential component of the generalized game u.*

Proof. For each $u \in M$, denote $B : X \to 2^X$ to be the best reply correspondences of u. Then by Lemma 7, $Fix(B) = E(u)$. Then, if $e(u)$ is a component of $E(u)$, it must be a component of $Fix(B)$. Since for the components of $Fix(B)$, there exists at least one is essential. Supposed it is $e(u)$, i.e., for each open set $O \supset e(u), \exists \delta > 0$, then for all B' satisfying $\rho_1(B, B') < \delta$ such that $Fix(B) \cap O \neq \emptyset$. Therefore, $e(u)$ is the essential component of $E(u)$, thus the proof is complete. □

Remark. Theorem 3 is derived from the uniform topological space of best reply correspondences, but the existence result of [12], [14], [16] are derived from the uniform topological space of payoff functions and feasible strategy correspondences.

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