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OSCILLATION CRITERIA FOR SECOND ORDER NONLINEAR NEUTRAL DIFFERENTIAL EQUATION ON TIME SCALES

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Abstract: We consider the oscillation of second order nonlinear neutral differential equations on time scales. We give some examples to illustrate the main results.

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1. Introduction

Consider the second order nonlinear neutral differential equation

$$(c(t)(x(t)-P(t)x[g(t)])^{\triangle})^{\triangle}+H(t,x[h(t)])+G(t,x[r(t)])=0, \qquad (1)$$

where the independent variable t is in a time scale T. A time scale T is a closed subset of the set R of real numbers with the topology and ordering inherited from R along with the forward jump operator $\sigma(t):=\inf\{s\in T: s>t\}$ and the backward jump operator $\rho(t):=\sup\{s\in T: s<t\}$. The graininess function μ is defend by $\mu(t):=\sigma(t)-t$.

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For a function $f : T \to R$ (the range R of f may actually be replaced by any Banach space), the (delta) derivative is defined by

$$f^{\triangle}(t) = \lim_{s \to t} \frac{f(\sigma(t)) - f(s)}{\sigma(t) - s}, \quad \text{where} \quad s \in T - \{\sigma(t)\}.$$

In this paper, we assume that:

(H_1) $\sup T = \infty$ and that $g(t)$, $h(t)$ and $r(t)$ all $\to \infty$ as $t \to \infty$,
$g, h, r \in C_{rd}(T, T)$ and $g(t) < t$, $h(t) < t$, $r(t) < t$, and $P(t) \in$
$C_{rd}(T, R)$, $c(t) \in C_{rd}(T, R^+)$ and $\int_{t_0}^{\infty} \frac{1}{c(t)} \triangle t = \infty$ for $t_0 \in T$,
$H(t, x), G(t, x) : T \times R \to R$ are continuous in x for each $t \in T$,

where $C_{rd}(T, S)$ denotes the set of all function $f : T \to S \subseteq R$ which are right-dense continuous on T. We say that f is right-dense continuous on T if f is continuous at all right-dense points, points where $\sigma(t) = t$, and the left-sided limit of f exists at all left-dense, right-scattered points, that is, points where $\sigma(t) > t$ and $\rho(t) = t$.

For $a, b \in T$ and a differentiable function f, the Cauchy integral of $f^{\triangle}$ is defined by

$$\int_a^b f^{\triangle}(t) \triangle t = f(b) - f(a),$$

and infinite integrals are defined by

$$\int_a^{\infty} f(t) \triangle t = \lim_{b \to \infty} \int_a^b f(t) \triangle t.$$

By a solution of equation (1) we mean a real-valued function x, defined on an interval $[b, \infty)$, that satisfies equation (1) on $[c, \infty)$ where $c > b$ is chosen so that $g(t) \geq b, h(t) \geq b, r(t) \geq b$ for $t \geq c$. A solution x of equation (1) is called oscillatory if it is neither eventually positive nor eventually negative; otherwise it is called nonoscillatory. Equation (1) is called oscillatory if all solutions are oscillatory.

In this paper, we are interested in oscillation of solutions of equation (1). It is organized as follows. The next section is devoted to the proof of the sufficient conditions for oscillation of all solutions of equation (1). In Section 3, we present some examples to illustrate our main results.

2. Main Results

Suppose that there exist two nonnegative function $\varphi, \phi \in C_{rd}(T, R^+)$ and there is a $t_0 \in T$ such that for all $x \in R$ and $t \geq t_0$:

$$(H_2) \quad xH(t,x) > 0 \text{ for } x \neq 0 \text{ and } \mid H(t,x) \mid \geq \varphi(t) \mid x \mid^{\lambda},$$
$$xG(t,x) > 0 \text{ for } x \neq 0 \text{ and } \mid G(t,x) \mid \geq \phi(t) \mid x \mid^{\lambda},$$

where $0 < \lambda = p/q < 1, p, q$ are odd integers and neither $\varphi(t)$ nor $\phi(t)$ is eventually identical to zero.

In order to analyze the effect of the neutral delay term g on the behavior of solutions of equation (1), the assumption we adopt in this paper is the following interval condition on g:

$$(H_3) \quad \text{there are sequences } \{a_n\} \text{ and } \{b_n\} \text{ from } T \text{ so that } \lim_{n\to\infty} a_n = \infty,$$
$$a_n < b_n \leq a_{n+1} \text{ and } g : [a_{n+1}, b_{n+1}] \to [a_n, b_n], \ \forall n \geq 1.$$

This condition allows $g(t)$ to vary between $g(a_k)$ and $g(b_k)$ for $t \in [a_k, b_k]$ in any fashion consistent with $g \in C_{rd}(T, T)$ but requires that $g(t) < g(s)$ if $t \in [a_k, b_k]$ and $s \in [a_{k+1}, b_{k+1}]$.

Lemma 2.1. *Let $x(t)$ be an eventually positive solution of equation (1). Assume $(H_1), (H_2), (H_3), P(t) \in C_{rd}(T, R^+)$ and that there exists a sequence $\{c_n\}$ from T such that*

$$a_n < c_n < b_n \text{ and } g(c_{n+1}) = c_n \text{ for all } n \geq 1, \tag{2}$$

and

$$\sum_{i=1}^{\infty} \prod_{j=1}^{i} (P(c_j))^{-1} = \infty. \tag{3}$$

Then $y(t) := x(t) - P(t)x[g(t)]$ satisfies $(c(t)y^{\triangle}(t))^{\triangle} < 0, y^{\triangle} > 0, y > 0$ eventually.

Proof. Assume that $x(t)$ is an eventually positive solution of equation (1), such that $x(t) > 0$ for all $t \geq T_0$. In view of equation (1) and from (H_2), there exists integer T_1 such that

$$(c(t)y^{\triangle}(t))^{\triangle} = -H(t, x[h(t)]) - G(t, x[r(t)]) < 0,$$

and that $h(t) \geq T_0$ and $r(t) \geq T_0$ for $t \geq T_1$. Hence $c(t)y^{\triangle}(t)$ is strictly decreasing on $[T_1, \infty)$. Let $l = \inf\{y(t) : t \geq T_1\}$. If $l < 0$, then there exists $T^* \geq T_1$ and $\alpha < 0$ such that $y(t) < \alpha$ for $t \geq T^*$. Next pick a positive integer k_0 such that $s_0 := c_{k_0} > T^*$, and define $s_k = c_{k_0+k}$. Then $x(s_{k+1})$

$< P(s_{k+1})x(s_k)+\alpha$ for $k \geq 0$. Iterating this inequality for $k = 0, 1, \cdots, n$ gives

$$x(s_n) < \alpha\big(1 + \sum_{i=2}^{n}\prod_{j=i}^{n} P(s_j)\big) + x(s_0)\prod_{i=1}^{n} P(s_i)$$

$$= \big(\alpha\frac{1+\sum\limits_{i=2}^{n}\prod\limits_{j=i}^{n} P(s_j)}{\prod\limits_{i=1}^{n} P(s_i)} + x(s_0)\big)\prod_{i=1}^{n} P(s_i)$$

$$= \big[\alpha(\sum_{i=1}^{n}\prod_{j=1}^{i}(P(s_j))^{-1}) + x(s_0)\big]\prod_{i=1}^{n} P(s_i).$$

Since $s_j = c_{k_0+j}$, the series involved here is that of (3) with the first k_0 terms of i omitted and then multiplied by the positive constant $\prod\limits_{i=1}^{k_0} P(c_i)$. Thus, by (3), the term on the right side of this inequality is negative for large n making $x(s_n) < 0$ and giving a contradiction to our assumption that $x(t) > 0$ for $t \geq T_0$. Thus $l \geq 0$, so $y(t) \geq 0$.

We claim that $c(t)y^{\triangle}(t) \geq 0$ eventually. Assume not, then there is a $T_2 \in [T_1, \infty)$ such that

$$c(t)y^{\triangle}(t) < 0, \quad t \in [T_2, \infty).$$

Then we can choose a negative constant C and $T_3 \in [T_2, \infty)$ such that

$$c(t)y^{\triangle}(t) \leq C < 0,$$

for $t \in [T_3, \infty)$. Intergrating from T_3 to t, we obtain

$$y(t) \leq y(T_3) + \int_{T_3}^{t} \frac{C}{c(s)} \triangle s.$$

Letting $t \to \infty$, then $y(t) \to -\infty$, this is contrary to $y(t) \geq 0$, hence we have $c(t)y^{\triangle}(t) \geq 0$. Since $\varphi(t), \phi(t)$ are not eventually identical to zero and $c(t) > 0$, it is easy to see $y^{\triangle}(t) > 0, y(t) > 0$ eventually. This completes the proof. □

We can prove the following lemma in the same way as Lemma 2.1.

Lemma 2.2. *Let $x(t)$ be an eventually negative solution of equation (1). Assume (H_1), (H_2), (H_3), $P(t) \in C_{rd}(T, R^+)$ and (2), (3)hold. Then $y(t) :$ $= x(t) - P(t)x[g(t)]$ satisfies $(c(t)y^{\triangle}(t))^{\triangle} > 0, y^{\triangle} < 0, y < 0$ eventually.*

Lemma 2.3. *Let $x(t)$ be an eventually positive solution of equation (1). Assume (H_1), (H_2), $P(t) \in C_{rd}(T, R^*)$, in which that $R^* = [-1, 0]$. Then $y(t) := x(t) - P(t)x[g(t)]$ satisfies $(c(t)y^{\triangle}(t))^{\triangle} < 0, y^{\triangle} > 0, y > 0$ eventually.*

Proof. Assume that $x(t)$ is an eventually positive solution of equation (1), such that $x(t) > 0$ for all $t \geq T_0$. Then

$$y(t) = x(t) - P(t)x[g(t)] > 0,$$

for $t \in [T_0, \infty)$. We can obtain that $(c(t)y^{\triangle}(t))^{\triangle} < 0, y^{\triangle} > 0$. The proof is similar to that of Lemma 2.1 and therefore it is omitted. This completes the proof. □

Using the same way as Lemma 2.3, we can obtain Lemma 2.4.

Lemma 2.4. *Let $x(t)$ be an eventually negative solution of equation (1). Assume $(H_1), (H_2), P(t) \in C_{rd}(T, R_0)$ in which that $R_0 = [-1, 0]$. Then $y(t) := x(t) - P(t)x[g(t)]$ satisfies $(c(t)y^{\triangle}(t))^{\triangle} > 0, y^{\triangle} < 0,\ y < 0$ eventually.*

Lemma 2.5. (see [2]) *Let $f \in C_{rd}(T, R)$ and $\lambda \in R$. Assume f is strictly increasing and $f^{\triangle} \in C_{rd}(T, R)$. Then*

$$\int_a^b \frac{f^{\triangle}(t)}{f^{\lambda}(t)} \triangle t = \int_{f(a)}^{f(b)} \frac{1}{t^{\lambda}} \triangle t,$$

where $a, b \in T$ with $a \leq b$.

Lemma 2.6. *If equation (1) satisfies $(c(t)y^{\triangle}(t))^{\triangle} > 0, y^{\triangle} < 0, y < 0$ eventually. Then*

$$y(t) < c(t)\alpha(t, t_1)y^{\triangle}(t), \quad t_1 \in [T_0, \infty),$$

where $\alpha(t, t_1) = \int_{t_1}^{t} \frac{1}{c(s)} \triangle s$.

The proof is easy, so it is omitted.

Theorem 1. *Assume that $H(t, x)$ and $G(t, x)$ are nondecreasing in x and satisfy $(H_1), (H_2), (H_3), (2), (3)$. Also supposed $P(t) \geq 0$ for all $t \geq t_0$. If*

$$\int_{t_0}^{\infty} \big(\varphi(t)\alpha^{\lambda}(g(h(t)), g(h(t_0))) + \phi(t)\alpha^{\lambda}(g(r(t)), g(r(t_0)))\big) \triangle t = \infty, \qquad (4)$$

then equation (1) it is oscillatory.

Proof. Suppose to the contrary that $x(t)$ is a nonoscillatory solution of equation (1). Without loss of generality, we may assume that $x(t)$ is an eventually negative solution of equation (1). We shall consider only this case, since by Lemma 2.1, in a similar fashion, we can prove that equation (1) has no eventually positive solution. So there exists $t_1 \in T$ such that $x(t) < 0, x[g(t)] < 0, x[h(t)] < 0, x[r(t)] < 0$, when $t \geq t_1$. By Lemma 2.2, we have

$$\lim_{t \to \infty} c(t)y^{\triangle}(t) = A \leq 0.$$

From $y(t) = x(t) - P(t)x[g(t)]$, so $x(t) = y(t) + P(t)x[g(t)]$. Iterating this equality, we have

$$x(t) = y(t) + P(t)\big(y[g(t)] + P[g(t)]x[g(g(t))]\big) \leq y(t) + P(t)y[g(t)]$$

$$\leq y[g(t)](1+P(t)) \leq y[g(t)]. \quad (5)$$

Since $(c(t)y^{\triangle}(t))^{\triangle} > 0$, select $t_2 \geq t_1$ such that

$$\begin{aligned} c(t)y^{\triangle}(t) &> c[g(h(t))]y^{\triangle}[g(h(t))], \\ c(t)y^{\triangle}(t) &> c[g(r(t))]y^{\triangle}[g(r(t))], \end{aligned} \quad (6)$$

for $t \geq t_2$. In view of $0 < \lambda = p/q < 1$ with odd integers p, q, also (H_2), (5), (6) and Lemma 2.6 yield

$$\begin{aligned} &(c(t)y^{\triangle}(t))^{\triangle} + \varphi(t)\Big[c(t)y^{\triangle}(t)\alpha\big(g(h(t)), g(h(t_2))\big)\Big]^{\lambda} \\ &+ \phi(t)\Big[c(t)y^{\triangle}(t)\alpha\big(g(r(t)), g(r(t_2))\big)\Big]^{\lambda} \\ &\geq (c(t)y^{\triangle}(t))^{\triangle} + H\Big(t, c(t)y^{\triangle}(t)\alpha\big(g(h(t)), g(h(t_2))\big)\Big) \\ &+ G\Big(t, c(t)y^{\triangle}(t)\alpha\big(g(r(t)), g(r(t_2))\big)\Big) \\ &> (c(t)y^{\triangle}(t))^{\triangle} + H\Big(t, c[g(h(t))]y^{\triangle}[g(h(t))]\alpha\big(g(h(t)), g(h(t_2))\big)\Big) \\ &+ G\Big(t, c[g(r(t))]y^{\triangle}[g(r(t))]\alpha\big(g(r(t)), g(r(t_2))\big)\Big) \\ &> (c(t)y^{\triangle}(t))^{\triangle} + H(t, y[g(h(t))]) + G(t, y[g(r(t))]) \\ &> (c(t)y^{\triangle}(t))^{\triangle} + H(t, x[h(t)]) + G(t, x[r(t)]) = 0. \end{aligned}$$

Since $c(t)y^{\triangle}(t) < 0$ for $t \geq t_2$, then

$$\varphi(t)\alpha^{\lambda}(g(h(t)), g(h(t_2))) + \phi(t)\alpha^{\lambda}(g(r(t)), g(r(t_2))) < -\frac{(c(t)y^{\triangle}(t))^{\triangle}}{(c(t)y^{\triangle}(t))^{\lambda}}.$$

Integrate this inequality from t_2 to ∞. Since $\lambda < 1$, Lemma 2.5 implies

$$\begin{aligned} &\int_{t_2}^{\infty} (\varphi(t)\alpha^{\lambda}(g(h(t)), g(h(t_2))) + \phi(t)\alpha^{\lambda}(g(r(t)), g(r(t_2))))\triangle t \\ &< \int_{t_2}^{\infty} -\frac{(c(t)y^{\triangle}(t))^{\triangle}}{(c(t)y^{\triangle}(t))^{\lambda}}\triangle t = \lim_{t\to\infty}\int_{c(t_2)y^{\triangle}(t_2)}^{c(t)y^{\triangle}(t)} -\frac{1}{s^{\lambda}}\triangle s = \int_{c(t_2)y^{\triangle}(t_2)}^{A} -\frac{1}{s^{\lambda}}\triangle s < \infty. \end{aligned}$$

Which is a contradiction with (4), so equation (1) has no eventually negative solution. Thus equation (1) is oscillatory. This completes the proof. □

By Lemma 2.3 and Lemma 2.4, we can prove the following theorem in the same way as Theorem 1, therefore it is omitted.

Theorem 2. *Assume that $H(t, x)$ and $G(t, x)$ are nondecreasing in x and* *satisfy* $(H_1), (H_2), (H_3)$. *Also supposed* $P(t) \in [-1, 0]$ *for all* $t \geq t_0$. *If*

$$\int_{t_0}^{\infty} \Big(\varphi(t)\big((1 + P[h(t)])\alpha(g(h(t)), g(h(t_0)))\big)^{\lambda} + \phi(t)\big((1 + P[r(t)])\alpha(g(r(t)), g(r(t_0)))\big)^{\lambda}\Big) \triangle t = \infty,$$

then equation (1) is oscillatory.

When $c(t) \equiv 1$, by Lemma 2.1 – Lemma 2.5 and Lemma 2.6, using the same way as Theorem 1, we have the following Theorem 3 and Theorem 4. Their proofs are omitted.

Theorem 3. *Assume that* $H(t, x)$ *and* $G(t, x)$ *are nondecreasing in* x *and satisfy* (H_1), (H_2), (H_3), (2), (3). *Also supposed* $P(t) \geq 0$ *for all* $t \geq t_0$. *If*

$$\int_{t_0}^{\infty} (\varphi(t) + \phi(t)) \triangle t = \infty,$$

then equation (1) is oscillatory.

Theorem 4. *Assume that* $H(t, x)$ *and* $G(t, x)$ *are nondecreasing in* x *and satisfy* (H_1), (H_2), (H_3). *Also supposed* $P(t) \in [-1, 0]$ *for all* $t \geq t_0$. *If*

$$\int_{t_0}^{\infty} (\varphi(t)(1 + P[h(t)])^{\lambda} + \phi(t)(1 + P[r(t)])^{\lambda}) \triangle t = \infty,$$

then equation (1) is oscillatory.

3. Applications

In this section, we give some examples to illustrate our main results.

Example 3.1. Consider the dynamic equation

$$\left[\frac{1}{t}\big(x(t) - \big(1 + \frac{\gamma}{t}\big)x(t - l)\big)^{\triangle}\right]^{\triangle} + t(x(t - \gamma_1))^{\frac{1}{3}} + t(x(t - \gamma_2))^{\frac{1}{3}} = 0 \tag{7}$$

on the time scale T, where $\gamma, l, \gamma_1, \gamma_2 > 0$, and $T = [m, \infty)$ in which that $m > \max\{\gamma_1 + l, \gamma_2 + l\}$.

Then for any $t_0 \in T$

$$\sum_{i=1}^{\infty} \prod_{j=1}^{i} (P(c_j))^{-1} = \sum_{i=1}^{\infty} \prod_{j=1}^{i} \frac{c_0 + jl}{c_0 + jl + \gamma} = \frac{c_0 + l}{c_0 + l + \gamma} + \sum_{i=2}^{\infty} [\prod_{j=1}^{i-1} \frac{c_0 + jl}{c_0 + jl + \gamma} \frac{c_0 + il}{c_0 + il + \gamma}]$$

$$\geq (c_0+l)\Big(\frac{1}{c_0+l+\gamma}+\sum_{i=2}^{\infty}\Big(\prod_{j=1}^{i-1}\frac{c_0+jl}{c_0+jl+\gamma}\Big)\Big(\frac{1}{c_0+il+\gamma}\Big)\Big)=\infty,$$

and

$$\int_{t_0}^{\infty}\big(\varphi(t)\alpha^{\lambda}(g(h(t)),g(h(t_0)))+\phi(t)\alpha^{\lambda}(g(r(t)),g(r(t_0)))\big)\triangle t$$
$$=\int_{t_0}^{\infty}\Big[t\Big(\int_{t_0-\gamma_1-l}^{t-\gamma_1-l}s\triangle s\Big)^{\frac{1}{3}}+t\Big(\int_{t_0-\gamma_2-l}^{t-\gamma_2-l}s\triangle s\Big)^{\frac{1}{3}}\Big]\triangle t$$
$$=\Big(\frac{1}{2}\Big)^{\frac{1}{3}}\int_{t_0}^{\infty}\Big[t\big((t+t_0-2\gamma_1-2l)(t-t_0)\big)^{\frac{1}{3}}$$
$$+t\big((t+t_0-2\gamma_2-2l)(t-t_0)\big)^{\frac{1}{3}}\Big]\triangle t\geq\Big(\frac{1}{2}\Big)^{\frac{1}{3}}\int_{t_0}^{\infty}2(t-t_0)^{\frac{5}{3}}\triangle t=\infty.$$

Hence, by Theorem 1, equation (7) is oscillatory.

Example 3.2. Consider the dynamic equation

$$\Big[\frac{1}{t}\big(x(t)-\frac{1}{t+\gamma_2}x(t-l)\big)^{\triangle}\Big]^{\triangle}+2t^2(x(t-\gamma_1))^{\frac{1}{5}}+2t^2(x(t-\gamma_2))^{\frac{1}{5}}=0 \quad (8)$$

on the time scale T, where $\gamma, l, \gamma_1, \gamma_2 > 0$, and $T=[m,\infty)$ in which that $m>\max\{\gamma_1+l,\gamma_2+l\}$.

Then for any $t_0\in T$

$$\int_{t_0}^{\infty}\Big(\varphi(t)\big((1+P[h(t)])\alpha(g(h(t)),g(h(t_0)))\big)^{\lambda}$$
$$+\phi(t)\big((1+P[r(t)])\alpha(g(r(t)),g(r(t_0)))\big)^{\lambda}\Big)\triangle t$$
$$=\int_{t_0}^{\infty}\Big[2t^2\big((1+\frac{1}{t-\gamma_1+\gamma_2})\int_{t_0-\gamma_1-l}^{t-\gamma_1-l}s\triangle s\big)^{\frac{1}{5}}+2t^2\big((1+\frac{1}{t})\int_{t_0-\gamma_2-l}^{t-\gamma_2-l}s\triangle s\big)^{\frac{1}{5}}\Big]\triangle t$$
$$>\int_{t_0}^{\infty}\Big[2(t-t_0)^{\frac{14}{5}}\big((1+\frac{1}{t-\gamma_1+\gamma_2})^{\frac{1}{5}}+(1+\frac{1}{t})^{\frac{1}{5}}\big)\Big]\triangle t>\int_{t_0}^{\infty}2(t-t_0)^{\frac{14}{5}}\triangle t=\infty.$$

Using Theorem 2: Equation (8) is oscillatory.

Example 3.3. Consider the dynamic equation

$$\big[x(t)-\big(3+\frac{\gamma}{2t}\big)x(t-l)\big]^{\triangle\triangle}+\frac{1}{2t}(x(t-\gamma_1))^{\frac{1}{5}}+\frac{1}{2t}(x(t-\gamma_2))^{\frac{1}{5}}=0 \quad (9)$$

on the time scale T, where $\gamma, l, \gamma_1, \gamma_2 > 0$ for $T=R$ or $T=Z$ in which case l, γ_1, γ_2 are in Z^+.

Then for any $t_0\in T$

$$\sum_{i=1}^{\infty}\prod_{j=1}^{i}(P(c_j))^{-1}=\sum_{i=1}^{\infty}\prod_{j=1}^{i}\frac{2c_0+2jl}{6c_0+6jl+\gamma}$$
$$=\frac{2c_0+2l}{6c_0+6l+\gamma}+\sum_{i=2}^{\infty}[\prod_{j=1}^{i-1}\frac{2c_0+2jl}{6c_0+6jl+\gamma}\frac{2c_0+2il}{6c_0+6il+\gamma}]$$
$$\geq 2(c_0+l)(\frac{1}{6c_0+6l+\gamma}+\sum_{i=2}^{\infty}(\prod_{j=1}^{i-1}\frac{2c_0+2jl}{6c_0+6jl+\gamma})(\frac{1}{6c_0+6il+\gamma}))=\infty,$$

and

$$\int_{t_0}^{\infty}(\varphi(t)+\phi(t))\triangle t=\int_{t_0}^{\infty}\frac{1}{2t}+\frac{1}{2t}\triangle t=\int_{t_0}^{\infty}\frac{1}{t}\triangle t$$
$$=\begin{cases}\infty, & T=R\ ,\\ \lim\limits_{t\to\infty}\sum\limits_{k=t_0}^{t-1}\frac{1}{t}=\infty, & T=Z\ .\end{cases}$$

We can conclude from Theorem 3 that equation (9) is oscillatory.

Example 3.4. Consider the dynamic equation

$$\big[x(t)-\big(-1+\frac{\gamma}{t}\big)x(t-l)\big]^{\triangle\triangle}+t^{\frac{11}{5}}(x(t-\gamma_1))^{\frac{1}{5}}+t^{\frac{11}{5}}(x(t-\gamma_2))^{\frac{1}{5}}=0 \quad (10)$$

on the time scale T. Note $K=\max\{\gamma_1,\gamma_2\}$ and $\gamma\leq K+1$, where $\gamma,l,\gamma_1,\gamma_2>0$ for $T=[K+1,\infty)=R^*$ or $T=\{K+1,K+2,\cdots,K+N,\cdots\}=Z_0^+$ in which case l,γ_1,γ_2 are in Z^+.

Then for any $t_0\in T$

$$\int_{t_0}^{\infty}(\varphi(t)(1+P[h(t)])^{\lambda}+\phi(t)(1+P[r(t)])^{\lambda})\triangle t$$
$$=\int_{t_0}^{\infty}(t^{\frac{11}{5}}(\frac{\gamma}{t-\gamma_1})^{\frac{1}{5}}+t^{\frac{11}{5}}(\frac{\gamma}{t-\gamma_2})^{\frac{1}{5}})\triangle t$$
$$>\int_{t_0}^{\infty}((t-\gamma_1)^{\frac{11}{5}}(\frac{\gamma}{t-\gamma_1})^{\frac{1}{5}}+(t-\gamma_2)^{\frac{11}{5}}(\frac{\gamma}{t-\gamma_2})^{\frac{1}{5}})\triangle t$$
$$=\int_{t_0}^{\infty}\gamma^{\frac{1}{5}}((t-\gamma_1)^2+(t-\gamma_2)^2)\triangle t$$
$$=\begin{cases}\infty, & T=R^*\ ,\\ \lim\limits_{t\to\infty}\sum\limits_{k=t_0}^{t-1}\frac{\gamma^{\frac{1}{5}}}{3}[(k-\gamma_1)^3+(k-\gamma_2)^3]=\infty, & T=Z_0^+.\end{cases}$$

Using Theorem 4: Equation (10) is oscillatory.

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