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A NEW DUALITY FOR SEMIDEFINITE MULTIPLICATIVE PROGRAMMING

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Abstract: Multiplicative programs are a difficult class of nonconvex programs that have received increasing attention because of their various applications. However, due to the nonconvex nature, few theoretical results are available. In this paper, we show that the semidefinte multiplicative programming is a special geometric programming. Using the results from the classical geometric programming, a new duality is obtained. The duality programming is convex programming. Meantime, the optimal conditions are also obtained.

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1. Introduction

Multiplicative optimization problems have an objective function which involves

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the product of several functions. These arise in many application areas such as Cobb Douglas type production functions in economics and in engineering (see [14]) and modular design (see [3]). In their most general form, such programs are nonconvex and hence difficult to analyze.

The semidefinite programming, which is defined in the cone consisting of all positive semidefinite matrices over the field of all real numbers, is an extension of the linear programming. M.L. Overton said, "Semidefinite programming is an exciting subject that remains very active, and will no doubt be so for some time to come. Here is an area. Where nonlinear convex programming has shown its strength, beauty, and versatility. One finds elegant theoretical results, probably efficient algorithms, and a wide variety of applications." However, few researchers concerned the non-linear semidefinite programming due to its difficulties.

In this paper, we will concern a special nonlinear semidefinte programming: semidefinte multiplicative programming. We will show that semidefinte multiplicative programming is a special geometric programming. And using the results from classical geometric programming, we give the new duality of semidefinte multiplicative programming,which is a convex programming. Meantime, the optimal condition will be also obtained.

2. Basis on the Geometric Duality

Let $x^I = (x^i)_{i\in I}$, $x^J = (x^j)_{j\in J}$, and $x = (x^0, x^I, x^J)$, where I and J are index sets, $x^i \in \mathcal{R}^{n_i}$, $i \in I$ $x^j \in \mathcal{R}^{n_j}$, $j \in J$. Let χ be a cone of $\mathcal{R}^n$, whose conjugate cone is $\chi^* = \{x^* | \langle x, x^*\rangle \geq 0, x \in \chi\}$, $n = n_0 + \sum\limits_{i\in I} n_i + \sum\limits_{J\in J} n_j$. $g_k : \mathcal{R}^{n_k} \to \mathcal{R}$ is a closed proper convex function (see [13]), its domain is C_k, and its conjugate function is $g_k^*(x^*)$, the positive homogenous extension of $g_k^+(x^k, \mu_k)(k \in \{0\} \bigcup I \bigcup J)$, where

$$g_k^*(x^{k*}) = \sup_{x^k\in\mathcal{R}_k^n} \left\{\langle x^k, x^{k*}\rangle - g_k(x^k)\right\}, \tag{1}$$

$$g_k^+(x^k, \mu_k) = \begin{cases} \sup\limits_{c^k\in C_k} \left\{\langle x^k, c^k\rangle\right\}, & \text{if } \mu_k = 0, \text{ and } \sup\limits_{c^k\in C_k} \left\{\langle x^k, c^k\rangle\right\} < +\infty, \\ \mu_k g_k(\frac{x^k}{\mu_k}), & \text{if } \mu_k > 0, \text{ and } x^k \in \mu_k C_k, \\ +\infty, & \text{else.} \end{cases} \tag{2}$$

We define the geometric duality as follows:

$$\min\left\{ G(x,\mu)=g_0(x^0)+\sum_{j\in J} g_j^{+}(x^j,\mu_j) \;\middle|\; \begin{array}{ll} g_i(x^i)\le 0, & i\in I \\ \mu_j\ge 0, & j\in J \\ x\in\chi & \end{array} \right\}, \tag{GP}$$

$$\min\left\{ G^*(x^*,\lambda)=g_0^*(x^{0*})+\sum_{i\in I} g_i^{*+}(x^{i*},\lambda_i) \;\middle|\; \begin{array}{ll} g_j^*(x^{j*})\le 0, & j\in J \\ \lambda_i\ge 0, & i\in I \\ x^*\in\chi^* & \end{array} \right\}. \tag{DGP}$$

The following implications (3a)-(3g), are said to be the optimal condition of (GP) and (DGP)

$$x\in\chi,\quad x^*\in\chi^*, \tag{3a}$$

$$g_i(x^i)\le 0,\quad i\in I;\qquad g_j^*(x^{j*})\le 0,\quad j\in J, \tag{3b}$$

$$\langle x,x^*\rangle = 0, \tag{3c}$$

$$x^{0*}\in\partial g_0(x^0), \tag{3d}$$

$$\lambda_i=0, \langle x^i,x^{i*}\rangle=\sup_{c^i\in C_i}\langle c^i,x^{i*}\rangle,\quad \lambda_i>0, x^{i*}\in\lambda_i\partial g_i(x^i), i\in I, \tag{3e}$$

$$\mu_j=0, \langle x^j,x^{j*}\rangle=\sup_{c^j\in C_j}\langle c^j,x^{j*}\rangle,\quad \mu_j>0, x^{j*}\in\mu_j\partial g_j(x^j/\mu_j), j\in J, \tag{3f}$$

$$\lambda_i g_i(x^i)=0, i\in I;\ \mu_j g_j^*(x^{j*})=0, j\in J, \tag{3g}$$

where, $\partial g_0(x)\triangleq\{x^*|\forall z\in\mathcal{R}^n, g_0(z)\ge g_0(x)+\langle x^*,z-x\rangle\}$ means that the gradient set of $g_0(x)$ at x. If $C_i=\mathcal{R}^{n_i}(i\in I), C_j=\mathcal{R}^{n_j}(j\in J)$, then (3e) and (3f)can be replaced with (3e)$'$ and (3f)$'$.

$$\lambda_i\ge 0,\quad x^{i*}\in\lambda_i\partial g_i(x^i),\quad i\in I, \tag{$3e)'$}$$

$$\mu_i\ge 0,\quad x^{j*}\in\mu_j\partial g_j(x^j/\mu_j),\quad j\in J, \tag{$3f)'$}$$

Proposition 1. (see [13]) *$g(x)$ is a closed proper convex function, $g^*(x^*)$ is the conjugate function of $g(x)$, then*

$$\langle x,x^*\rangle\le g^*(x^*)+g(x),$$

and furthermore "=" holds if and only if $x^*\in\partial g(x)$.

Proposition 2. *If (x,μ) and (x^*,λ) are the feasible solution of (GP) and (DGP), respectively, then*

(i) $\langle x^i,x^{i*}\rangle\le\lambda_i g_i(x^i)+g_i^{*+}(x_i^*,\lambda_i),\forall i\in I$. And furthermore, "=" holds if and only if (3e) *holds;*

(ii) $\langle x^j,x^{j*}\rangle\le\mu_j g_j^*(x^{j*})+g_j^{+}(x_j,\mu_j),\forall j\in J$. And furthermore, "=" holds if and only if (3f) *holds.*

Proof. We only need to prove case (i), case (ii) is same to case (i).

When $\lambda_i = 0$: $\langle x^i, x^{i*}\rangle \leq \sup_{z\in\mathcal{R}^{n_i}} \langle z, x^{i*}\rangle = \lambda_i g_i(x^i) + g_i^{*+}(x_i^*, \lambda_i)$. And furthermore, "=" holds if and only if (3e) holds.

When $\lambda_i > 0$:

$$\lambda_i g_i(x^i) + g_i^{*+}(x^{i*}, \lambda_i) = \lambda_i g_i(x^i) + \lambda_i g_i^*(\frac{x^{i*}}{\lambda_i})$$
$$\geq \lambda_i g_i(x^i) + \lambda_i(\langle x^i, \frac{x^{i*}}{\lambda_i}\rangle - g_i(x^i)) = \langle x^i, x^{i*}\rangle.$$

According to Proposition 1, "=" holds if and only if (3e) holds. □

Theorem 3. *If* (x, μ) *and* (x^*, λ) *are the feasible solution of* (GP) *and* (DGP)*, respectively, then*

$$0 \leq G(x, \mu) + G^*(x^*, \lambda).$$

And furthermore, "=" holds if and only if (3c)-(3g) *holds.*

Proof.

$$G(x, \mu) + G^*(x^*, \lambda)$$
$$= g_0(x^0) + \sum_{j\in J} g_j^+(x^j, \mu_j) + g_0^*(x^{0*}) + \sum_{i\in I} g_i^{*+}(x^{i*}, \lambda_i)$$
$$\geq g_0(x^0) + g_0^*(x^{0*}) + \sum_{j\in J}(g_j^+(x^j, \mu_j) + \mu_j g_j^*(x^{j*})) + \sum_{i\in I}(g_i^{*+}(x^i*, \lambda_i) + \lambda_i g_i(x^i))$$
$$\geq \langle x^0, x^{0*}\rangle + \sum_{j\in J}\langle x^j, x^{j*}\rangle + \sum_{i\in I}\langle x^i, x^{i*}\rangle = \langle x, x^*\rangle \geq 0.$$

According to Proposition 1, "=" holds if and only if (3c)-(3g) is satisfied. □

For the detail of geometric duality also see [5].

3. Geometric Duality for Semidefinite Multiplicative Programming

An optimization problem is said to be semidefinite multiplicative programming if it has the following form:

$$\max\left\{\prod_{i=1}^{p} f_i(y)^{a_i} \,\middle|\, \mathcal{A}^T y \preceq C\right\}, \tag{SDMP1}$$

where, $\mathcal{A}^T y = y_1 A_1 + \cdots + y_m A_m, y \in \mathcal{R}^m$, $A_i (i = 1, \cdots, m) \in R^{nn}$, C is n a $n \otimes n$ symmetric, $\mathcal{A}^T y \preceq C$ denotes that $C - \mathcal{A}^T y$ is a semidefinite matrix. Suppose that every subfunction in object function is a positive value function over feasible region, and $a_i \geq 0$.

In a narrow sense, the following optimization problem is said to be geometric programming (see [1]), p. 531):

$$\min\{f(x)|g_i(x) \leq 1, i = 1, 2, \cdots, m, x > 0\},$$

where $f(x) = \sum\limits_{k \in K_0} T_k, g_i(x) = \sum\limits_{k \in K_i} T_k$, $T_k = \alpha_k \prod\limits_{j=1}^{n} x_j^{\alpha_{kj}}$, $K_i (i = 0, 1, \cdots, m)$ is index set, $\alpha_k > 0$, α_{kj} is rational number. And furthermore, if $f(x)$ is a product of linear functions, $g_i(x)$ is a linear function and $f(x)$ is positive value function over feasible region, then it will be said to be a linear multiplicative programming. It is obvious that linear multiplicative programming is a special multiplicative programming.

Multiplicative programming is a quite important geometric programming. Most of algorithms are base on arithmetic-geometric mean inequality. So, Peterson (see [5]) call it as "Geometric Programming". The aim of this paper is to give the geometry duality of semidefinite multiplicative programming.

$(SDMP1)$ can be equivalently transformed into the following programming:

$$\begin{array}{ll} \min & -\sum\limits_{i=1}^{p} a_i \log\ s_i + \delta(\alpha|\{C\}), \\ \text{s.t.} & -f_i(y^i) + s_i' \leq 0,\ i = 1, \cdots, p, \\ & s_i - s_i' = 0,\ i = 1, \cdots, p, \\ & y - y^i = 0,\ i = 1, \cdots, p, \\ & \mathcal{A}^T y - \alpha \leq 0. \end{array} \tag{SDMP2}$$

In order to show that $(SDMP1)$ is a special geometric programming, we give the following corresponding relations:

$$I = \{1, 2, \cdots, p\}, \quad J = \phi, \tag{4a}$$

$$x^0 \leftrightarrow (y, s_1, \cdots, s_p, \alpha), \tag{4b}$$

$$x^I \leftrightarrow (y^1, \cdots, y^p, s_1', \cdots, s_p'), \tag{4c}$$

$$\chi \leftrightarrow \left\{ (y, y^1, \cdots, y^p, s_1, \cdots, s_p, s_1', \cdots, s_p', \alpha) \left| \begin{array}{l} \mathcal{A}^T y - \alpha \leq 0, \\ y - y^i = 0, \\ s_i - s_i' = 0, \\ i = 1, \cdots, p \end{array} \right. \right\}, \tag{4d}$$

$$g_0(x^0) \leftrightarrow -\sum_{i=1}^{p} a_i \log\ s_i + \delta(\alpha|\{C\}), \tag{4e}$$

$$g_i(x^i) \leftrightarrow -f_i(y^i) + s'_i \leq 0, \quad i = 1, \cdots, p. \tag{4f}$$

Now, we begin to finish the geometric duality of $(SDMP2)$.

(1) Object function.

Let $h_i(s_i) = -a_i \log s_i$ $(h_i)^*(s_i^*) = \sup\limits_{s_i \in \mathcal{R}} \{s_i * s_i^* + a_i \log s_i\} \triangleq \sup\limits_{s_i \in \mathcal{R}} p_i(s_i)$,

— when $s_i^* \geq 0$, $(h_i)^*(s_i^*) = +\infty$;

— when $s_i^* < 0$, $p'_i(s_i) = s_i^* + \frac{a_i}{s_i}$, $p''_i(s_i) = -\frac{a_i}{s_i^2} < 0$.

So, $p_i(s_i)$ is a strictly concave function over $\mathcal{R}^+$, and $s_i = -\frac{a_i}{s_i^*}$ is a unique maximal point. Therefore,

$$(h_i)^*(s_i^*) = -a_i + a_i \log\left(-\frac{a_i}{s_i^*}\right) + \delta(s_i^*|\{s_i^* < 0\}).$$

And it is obvious that

$$(\delta(\ |\{C\}))^*(\alpha^*) = \delta^*(\alpha^*|\{C\}) = \langle \alpha^*, C\rangle.$$

Let $g_i(y^i, s'_i) = -f_i(y^i) + s'_i$, then

$$g_i^*(y^{i*}, s_i'^*) = (-f_i)^*(y^{i*}) + \delta(s_i'^*|\{1\}).$$

So,

$$(g_i)^{*+}(y^{i*}, s_i'^*, \lambda_i) = \lambda_i(-f_i)^*(\frac{y^{i*}}{\lambda_i}) + \lambda_i\delta(s_i'^*|\{\lambda_i\}).$$

Additionally, we impose $0 = \langle 0, y\rangle$ on the object function, its conjugate function is $\sup\limits_{y\in\mathcal{R}^m} \langle y, y^*\rangle$. So, the object function is

$$\sum_{i=1}^{p}(-a_i + a_i \log(-\frac{a_i}{s_i^*})) + \sum_{i=1}^{p} \lambda_i(-f_i)^*(\frac{y_i^*}{\lambda_i}) + \langle \alpha^*, C\rangle,$$

its domain is

$$\left\{ (y^*, y^{*1}, \cdots, y^{*p}, s_1^*, \cdots, s_p^*, s_1'^*, \cdots, s_p'^*, \alpha^*) \;\middle|\; \begin{array}{l} s_i'^* = \lambda_i > 0, \\ s_i^* < 0, \\ i = 1, \cdots, p, \\ y^* = 0 \end{array} \right\}.$$

(2) χ^*.

Proposition 4.

$\{(y, C) \in \mathcal{R}^m \times \mathcal{S}^{n\times n}|\mathcal{A}^T y \leq C\} = \{(-\mathcal{A}X, X) \in \mathcal{R}^m \times \mathcal{S}^{n\times n}|X \geq 0\}^*$;

$\{(y,C)\in\mathcal{R}^m\times\mathcal{S}^{n\times n}|\mathcal{A}^Ty\leq C\}^*=\{(-\mathcal{A}X,X)\in\mathcal{R}^m\times\mathcal{S}^{n\times n}|X\geq 0\}.$

Proof. $\{(-\mathcal{A}X,X)\in\mathcal{R}^m\times\mathcal{S}^{n\times n}|X\geq 0\}$ is a closed convex cone. So,

$$\{(-\mathcal{A}X,X)\in\mathcal{R}^m\times\mathcal{S}^{n\times n}|X\geq 0\}^{**}=\{(-\mathcal{A}X,X)\in\mathcal{R}^m\times\mathcal{S}^{n\times n}|X\geq 0\}.$$

Therefore, we only need to prove the first formula.

It is obvious that

$\{(y,C)\in\mathcal{R}^m\times\mathcal{S}^{n\times n}|\mathcal{A}^Ty\leq C\}\subseteq\{(-\mathcal{A}X,X)\in\mathcal{R}^m\times\mathcal{S}^{n\times n}|X\geq 0\}^*.$

For $\forall a\in\mathcal{R}^n$, we have

$$(-\mathcal{A}aa^T,aa^T)\in\{(-\mathcal{A}X,X)\in\mathcal{R}^m\times\mathcal{S}^{n\times n}|X\geq 0\}.$$

For $\forall(y,C)\in\{(-\mathcal{A}X,X)\in\mathcal{R}^m\times\mathcal{S}^{n\times n}|X\geq 0\}^*$, we have

$$\langle(y,C),(-\mathcal{A}aa^T,aa^T)\rangle=-\langle\mathcal{A}y,aa^T\rangle+\langle C,aa^T\rangle=-a^T(\mathcal{A}^Ty-C)a\geq 0.$$

It means that $\mathcal{A}^Ty-C\leq 0$ and $(y,C)\in\{(y,C)\in\mathcal{R}^m\times\mathcal{S}^{n\times n}|\mathcal{A}^Ty\leq C\}$. So,

$$\{(y,C)\in\mathcal{R}^m\times\mathcal{S}^{n\times n}|\mathcal{A}^Ty\leq C\}\supset\{(-\mathcal{A}X,X)\in\mathcal{R}^m\times\mathcal{S}^{n\times n}|X\geq 0\}^*.\quad\square$$

Now, we turn to solve $\chi*$.

Given

$$(y^*,y^{1*},\cdots,y^{p*},s_1^*,\cdots,s_p^*,s_1'^*,\cdots,s_p'^*,\alpha^*)\in\chi^*.$$

For

$$\forall(y,y^1,\cdots,y^p,s_1,\cdots,s_p,s_1',\cdots,s_p',\alpha)\in\chi,$$

we have

$$\langle y,y^*\rangle+\sum_{i=1}^p\langle y^i,y^{i*}\rangle+\langle\alpha,\alpha^*\rangle+\sum_{i=1}^p(s_is_i^*+s_i's_i'^*)\geq 0.$$

It is equivalent to:

— when $\mathcal{A}^Ty\leq\alpha$, $\langle y,y^*+\sum_{i=1}^p y^{i*}\rangle+\langle\alpha,\alpha^*\rangle\geq 0$ holds.

— when $s_i-s_i^*=0,\ s_is_i^*+s_i's_i'^*\geq 0,\ i=1,\cdots,p$ holds.

Obviously, $s_i^*=-s_i'^*,\ i=1,\cdots,p$. According to Proposition 4, $(y^*+\sum_{i=1}^p y^{i*},\alpha^*)$ has the form of $(-\mathcal{A}X,X)$, where $X\geq 0.v^i=-y^{i*}$. Then, $y^*=$

$-\mathcal{A}X+\sum\limits_{i=1}^{p}v^i$, i.e.

$$\chi^*\leftrightarrow\left\{(y^*,y^{*1},\cdots,y^{*p},s_1^*,\cdots,s_p^*,s_1'^*,\cdots,s_p'^*,\alpha^*)\;\middle|\;\begin{array}{l}y^{i*}=-v^{i*},\\ \alpha^*=X,\\ s_i^*=w_i,\\ v^i\in\mathcal{R}^m, w_i\in\mathcal{R},\\ i=1,\cdots,p,\\ y^*=-\mathcal{A}X+\sum\limits_{i=1}^{p}v^i,\\ X\geq 0\end{array}\right\}.$$

From (1) and (2), the geometric duality of $(SDMP2)$ has the following form:

$$\begin{array}{ll}\min & \sum\limits_{i=1}^{p}(-a_i+a_i\log(\frac{a_i}{\lambda_i}))+\sum\limits_{i=1}^{p}\lambda_i(-f_i)^*(\frac{-v^i}{\lambda_i})+\langle X,C\rangle,\\ \text{s.t} & -\mathcal{A}X+\sum\limits_{i=1}^{p}v^i=0,\\ & \lambda_i>0,\quad v^i\in\mathcal{R}^m,\quad i=1,\cdots,p,\\ & X\geq 0.\end{array}\qquad\text{(SDMP3)}$$

(3) Optimal conditions.

$J=\phi$, So(3f) vanishes. According to the optimal condition of geometric programming (3c), we have

$$\begin{aligned}&\sum_{i=1}^{P}(\langle y^i,y^{i*}\rangle+s_is_i^*+s_i's_i'^*)+\langle y,y^*\rangle+\langle C,X\rangle\\ &=\langle y,\sum_{i=1}^{P}y^{i*}+y^*\rangle+\sum_{i=1}^{P}(s_is_i^*+s_i's_i'^*)+\langle C,X\rangle\\ &=\langle y,\sum_{i=1}^{P}-v^i\rangle+\langle C,X\rangle=\langle y,-\mathcal{A}X\rangle+\langle C,X\rangle=\langle C-\mathcal{A}^Ty,X\rangle=0.\end{aligned}$$

According to the optimal conditions of geometric programming (3d), we have

$$(y^*,s_1^*,\cdots,s_p^*,X)\in\partial(-\sum_{i=1}^{p}a_i\log\ s_i+\delta(\alpha|\{C\})),$$

which is equivalent to the following formulas:

$$y^*\in\partial\langle 0,y\rangle(y)=\{0\},$$

$$X\in\partial\delta(\alpha|\{C\})(\alpha),$$

$$s_i^* \in \partial(-a_i \log s_i)(s_i) = \{-\frac{a_i}{s_i}\} = \{-\lambda_i\}, \quad i = 1, \cdots, p.$$

According to the optimal condition of geometric programming (3e), we have

$$y^{i*} \in \lambda_i \partial(-f_i)(y^i), \quad s_i'^* = \lambda_i,$$

i.e.

$$v^i \in \lambda_i \partial f_i(y^i), \quad s_i'^* = \lambda_i.$$

According to the optimal conditions of geometric programming (3g), we have

$$f_i(y) = f_i(y^i) = s_i' = s_i = \frac{a_i}{\lambda_i}.$$

If the feasible solution of $(SDMP2)$ (respectively, $(SDMP3)$) satisfies the following conditions, then it is an optimal solution of $(SDMP2)$ (respectively, $(SDMP3)$).

$$\langle C - \mathcal{A}^T y, X\rangle = 0,$$

$$f_i(y) = \frac{a_i}{\lambda_i}, \quad i = 1, \cdots, p,$$

$$v^i \in \lambda_i \partial f(y^i), \quad i = 1, \cdots, p.$$

When $f_i(y^i) = c^{iT} y + d_i$,

$$(-f_i)^* \left(\frac{-v^i}{\lambda_i}\right) = \sup_{y^i \in \mathcal{R}^m} \left\{ \left\langle y^i, -\frac{v^i}{\lambda_i} \right\rangle + \langle c^i, y^i\rangle + d_i \right\} = \left\{ \begin{array}{ll} +\infty, & v^i \neq \lambda_i c^i, \\ d_i, & v^i = \lambda_i c^i. \end{array}\right.$$

And hence, $(SDMP3)$ is equivalent to the following convex programming:

$$\begin{array}{ll} \min & \sum_{i=1}^p (d_i \lambda_i - a_i \log \lambda_i) + \sum_{i=1}^p (a_i \log a_i - a_i) + \langle X, C\rangle, \\ \text{s.t} & -\mathcal{A}X + \sum_{i=1}^p \lambda_i c^i = 0, \\ & \lambda_i > 0, \cdots, \quad i = 1, p, \\ & X \geq 0. \end{array} \qquad \text{(SDMP4)}$$

Its optimal conditions are

$$\langle C - \mathcal{A}^T y, X\rangle = 0, \qquad c^{iT} y + d_i = \frac{a_i}{\lambda_i}, \qquad i = 1, \cdots, p.$$

This finishes the geometric duality and gives optimal conditions of semidefinite multiplicative programming.

4. Conclusions

In this paper, we have showed that semidefinite multiplicative programming is a special geometric programming. By classical geometric duality, a new duality of semidefinite multiplicative programming, which is a convex programming, is obtained. And furthermore, we also give the optimal conditions of the semidefinite multiplicative programming.

References

[1] M.S. Bazaraa, C.M. Shetty, *Nonlinear Programming Theory and Algorithms*, John Wiley and Sons, New York (1993).

[2] H.P. Benson, G.M. Boger, Out-space cutting-plane algorithm for linear multiplicative programming, *Journal of Optimization Theory and Applications*, **104**, No. 2 (2000), 301-322.

[3] A. Charnes, M. Kirby, Modular design, generalized inverses, and convex programming, *Operations Research*, **13** (1965), 863-867.

[4] M.C. Dorneich, N.V. Sahinidis, Global optimization algorithms for chip design and compaction, *Engineering Optimization*, **25**, No. 2 (1995), 131-154.

[5] L.P. Elmor, Geometric programming, *SIAM Review*, **18**, No. 1 (1976), 1-51.

[6] C. Helmberg, *Semidefinite Programming for Combinatorial Optimization*, http://www.zib.de/helmberg (2000).

[7] R. Horst, N.V. Thoai, J. De Vries, On finding new vertices and redundant constrains in cutting plane algorithms for global optimization, *Operations Research Letters*, **7** (1988), 85-90.

[8] H. Konno, H. Shirakawa, H. Yamazaki, A mean-absolute deviation-skewness portfolio optimization model, *Annals of Operations Research*, **45** (1993), 205-220.

[9] H. Konno, T. Kuno, Linear multiplicative programming, *Mathematical Programming*, **56** (1992), 51-64.

[10] T. Kuno, Y. Yajima, H. Konno, An outer approximation method for minimizing the product of several convex functions on a convex set, *Journal of Global Optimization*, **3**, No. 3 (1993), 325-335.

[11] J.M. Mulvey, R.J. Vanderbei, S.A. Zenios, Robust optimization of large-scale systems, *Operations Research*, **43** (1995), 264-281.

[12] P.M. Pardalos, Polynomial time algorithms for some classes of constrained quadratic problems, *Optimization*, **21**, No. 6 (1990), 843-853.

[13] R.T. Rockafellar, *Convex Analysis*, Princeton Unversity Press, Princeton (1970).

[14] E. Rosenberg, Optimal module sizing in Vlsi floor planning, *Zeitschrift fur Operations Research*, **33** (1989), 134-143.

[15] S. Schiable, C. Sodini, Finite algorithm for generalized linear multiplicative programming, *Journal of Optimization Theory and Applications*, **87**, No. 2 (1995), 441-455.

[16] C.H. Scott, T.R. Jefferson, On duality for a class of quasiconcave multiplicative programs, *Journal of Optimization Theory and Applications*, **117**, No. 3 (2003), 575-583.

[17] M. Sniedovich, S. Findlay, Solving a class of multiplicative programming problems via *C*-programming, *Journal of Global of Optimization*, **6** (1995), 313-319.