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EVOLUTION OF NUCLEONS AND HYDROGEN

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Abstract: In this article I present computations and projective geometrical structures for the evolution of nucleons after high speed collisions or a black hole explosion. An observation in physics is made that first a gluon-quark plasma is generated from annihilated or emitted matter. I set the discrete dynamical generation of nucleons after that on a unified treatment of all four basic interactions and extend Einstein's view on affine or Schwarzschild scaled metrics to a projective view where bags for nucleons and particles are generated.

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1. Electromagnetism, EMI and WI

The geometrical bag-structure for electromagnetism EM, the electromagnetic (EMI) and weak (WI) interactions are set in this article on the 3-dimensional sphere $S^3 \subset \mathbb{R}^4$ Hopf geometry of its symmetry group SU(2) for WI and on a U(1) circle $\mathbb{P} = \mathbb{R} \cup \{\infty\}$ for EMI. The circle, as well as a 2-dimensional sphere S^2 and S^3 can be projected through stereographic projections $st_n,\ n = 1, 2, 3,$ onto a real line, complex plane, $\mathbb{R}^3$ or S^3 through the Hopf map $h : S^3 \to S^2$ onto a 2-dimensional sphere in $\mathbb{R}^3$.

The last map has the Minkowski metric spacetime coordinates $(z_1, z_2) =$

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$(x+iy, z+ict)$, t time scaled by the speed of light c, (x, y, z) as Cartesian space coordinates. The map h has been described in [7] through the 3 generating Pauli matrices σ_j, $j = 1, 2, 3$, of SU(2) by

$$h(z_1, z_2) = ((\overline{z_1}, \overline{z_2})\sigma_1(z_1, z_2)^{tr}, (\overline{z_1}, \overline{z_2})(-\sigma_2)(z_1, z_2)^{tr}, (\overline{z_1}, \overline{z_2})\sigma_3(z_1, z_2)^{tr}).$$

For the composed maps holds projectively

$$st \circ h(z_1, z_2) = \frac{z_1}{z_2} = \frac{z_1/z_2}{1} = z \in \mathbb{C}, \ z_2 \neq 0,$$

and $st \circ h(z_1, 0) = (1, 0) = \infty \in \mathbb{P} \subset S^2$. The inverse Hopf map, composed with st_3 presents the EM structure of electrically charged parts of particles or nucleons, called systems. It is a field description in spacetime. Eigen-time is a periodic, rolled coordinate with a z-axis as rotation axis. The above sphere S^2 is a bag boundary inside which the quantum mechanical QM main probability distribution $\psi\psi^*$ of its system as a ψ wave can be found.

EM has 6 possible charges, 4 for quarks and 2 for charged leptons. WI-decays with $W^{\pm}, Z^0$ bosons show how these preserved quantities relate to one another. For instance, in the quarks d, u decay $d \to u + W^- \to u + e^- + \overline{\nu_e}$, e^- an electron, $\overline{\nu_e}$ an antineutrino, the d-charge $-1/3$ is changed to the u, e^- charges $(+2/3) - 1$. The u, d-decay is similar. On S^2 at projective infinity of a system Q, I set at its northpole ∞ the magnetic point, at the opposite pole $0 \in \mathbb{C} = S^2 - \{\infty\}$ the mass m of Q as accelerating Newtonian force vector $F = m \cdot a$, also integrated in the form of the momentum $p = m\vec{v}$ of Q. For the electrical charge, for instance e_0, I assume that it rotates (not carrying mass) as a point on discrete latitude circles $\theta = const.$, where the angle $0 \leq \theta \leq (\pi/2)$ is measured towards the positive z-axis through $0, \infty$. The latitude circles as currents have radii $r > 0$ in $\mathbb{C}$ according to the energy of the electrical charged part C_Q of Q. This is best investigated for electrons in the shell of a hydrogen atom where the size of r is presented through the scaled main quantum numbers $n \in \mathbb{N}$ of e^-.

Using $st_3 \circ h^{-1}$, where $\infty \in S^3$ as north pole is deleted in this $\mathbb{R}^3$ projection, part of the generated field lines are similar to those of a permanent magnet but with a rotation z-axis and with concentric tori, rotated about this axis. The tori have in their inner most location the image of $0 \in S^2$, a circle in the plane $(re^{i\varphi}) \sim z_1$, carrying the mass of C_Q. The 1-dimensional e_0-location is a 45^0 leaning circle on one of the discrete tori (belonging to the main quantum numbers discrete radii, for instance of e^-) which winds around the z-axis. Through the projection of the deleted $\infty \in S^2$ magnetic field quantums $\phi_0 = h/(2e_0) = \frac{h}{2} \cdot \sigma_1(e_0)$, h is the Planck constant, are generated. They are oriented, circular-conic whirls with tip $0 \in \mathbb{C}$ in the projected z_1-plane and with a leaning angle of 45^0 towards the z-axis as rotation axis of C_Q. The Pauli-spin $S = (S_x, S_y, S_z)$ is rotating on the cones surface parallel to the magnetic momentum μ of C_Q, generated by the ϕ_0. The gyromagnetic relation and constant arise this way. Through magnetic induction, the rotation of the e_0 charged circle on its torus location combine the orientations for ± 1 electrical charges and μ. The toroidal orientation can be in the clockwise direction for e^- and in the counterclockwise direction for e^+.

The frequency, momentum p and angular momentum $J = r \times p$ of a charged lepton are computed through complex contour integrations on S^2 (at projective infinity and as boundary of the e^- energy location) about poles of the Moebius transformations, associated with the SU(2) generators. Set $z = (1/z)$ for $id, \sigma_1 \in$SU(2) and get as denominator for the contour integration $z^2 - 1 = (z+1)(z-1)$ with poles at $z = \pm 1$. With a suitable scaling and residues:

(i) at $0 \in [-1, 1]$ the momentum p is obtained.

(ii) at -1 the angular momentum J is set with a suitably scaled radius, for instance of the leptons bag. The circular image of -1 in the above blow up to $\mathbb{R}^3$ is on a torus which contains also the circular boundary of the ϕ_0, s, μ cone.

(iii) If the coordinates are energetically normed such that $+1 \in \mathbb{P} \subset S^2$ is always the rotating point for e_0 on a latitude circle of S^2, the frequency $f = (mc^2)/h$, m mass of e^-, is set there for inner rotations and also for the EM potential field, spreading out with speed of light in the e^- spacetime environment from this point on a Minkowski light-cone $r^2 = c^2t^2$ with tip at $+1$. In space holds $r^2 = x^2 + y^2 + z^2$ for the potential field lines $\frac{e_0}{r 4\pi\epsilon_0}$ of e^-. Since I explain below the gravitational GR field in a similar form, their EM/GR formulas (GR: $\frac{\gamma_G m}{r}$), in physics are similar.

The EM preservation theorems for electrical charges, magnetic momentum, Lorentz-force (replacing Newtonian force and mass through electrical charge) belong as invariants to the SU(2) three (Pauli spin) Moebius transformations MT $-z, \pm\frac{1}{z}$. For the MT z, belonging to $id \in$SU(2), one can set a scaling of spin lengths in multiples $n \in \mathbb{N}$ of the fermion spin length $\hbar$. Further details for C_Q, concerning for instance the fine structure constant and spectral series, using also intermediately generated neutral WI-bosons Z^0, can be found in [9]. The length $\hbar$ can be computed through Planck numbers and the following newly introduced color charge group D_3.

2. Gravity GR, the Color Charge Group and SI

I treat SI and gravity with the color charge group D_3 of quarks, described below, extending the quantum chromo dynamics QCD approach. The group D_3 acts as invariants (complex cross ratios) under all MT of S^2 on a real 6-dimensional, complex 3-dimensional operator space $\mathbb{C}^3$, consisting of Gleason-operators T. The T have extended inertial coordinates EIS: $(r, \varphi, \theta, ict, iu, iw)$, where an energy plane (iu, iw) is orthogonaly adjoint to the (spherical) space-time coordinates. This plane contains the Einstein line $hf = mc^2$, transforming frequencies f and masses m of systems. According to physics, systems can show in different experiments wave (with f) or particle (with m) properties, called the wave-particle duality.

r	φ	θ	ict	iu	iw
$x \in \mathbb{R}$	$iy \in i\mathbb{R}$	$z \in \mathbb{R}$			
r	g	$\overline{g}$	$\overline{b}$	b	$\overline{r}$
z	$\frac{z}{-z-1}$	$\frac{-z-1}{z}$	$(-z-1)$	$\frac{1}{-z-1}$	$\frac{1}{z}$
$\frac{1}{z}$	-$\frac{1}{z}$	$-z$	z		
$id; \sigma_1$	$\alpha\sigma_1; \sigma_2$	$\alpha^2; \sigma_3$	$\alpha^2\sigma_1; id$	α	$\sigma_1; id$
1	4	3	2	6	5
length λ_P	temp. T_P	dens. ρ_P	time t_P	ener. E_P	mass m_P
	C	T			P

Table 1:

In the combined Table 1 I further add in the second line of Table 1 the affine space-coordinates of SU(2), the third line is a distribution on six quarks (enumerated in line 7 by 1-6) possible color charges, the fourth and fifth line contain also the cross ratios or MT's for D_3 and the three Pauli matrices of SU(2), the sixth line their matrix-names used, the second to last line the six Planck numbers, generated at time $10^{-43}s$, and the last line the three operators C,P,T of physics for changing all quantum numbers, space- or time-reversal through their $\pm$ orientations in spacetime. The CPT-product is preserved. The matrices α, α^2 are for rotations in space. Mathematically[1] $\alpha + \alpha^2 = -id$ allows scalar multiplications with minus, division of numbers can use on a projective circle $\mathbb{P}^1$ the σ_1 matrix in $[x, 1]\sigma_1 = [1, x]^{tr} = [1/x, 1]^{tr}$.

[1]The mathematical/physics evolution of numbers is by main quantum numbers in nucleons extended to $\mathbb{N}$, and extensions by $-$ to $\mathbb{Z}$, by division to rational numbers, by waves/superpositions to $\mathbb{R}$, by rotations/U(1) or S^2-stereographic projection to $re^{i\varphi} \in \mathbb{C}$ and by the 3-dimensional cross-products $L = r \times p$ or the EM $F_L = Q(v \times B)$ to the SU(2) quaternions.

The degenerate orbits of D_3 as MT's are $\{(-1), 0, \infty\}$ which I use as reference points for cross ratios on a $\mathbb{P}$ circle. On such a projective line as boundary of a color charge whirl four points are needed in order to obtain a measure. For the fourth point I use the degenerate spin-lengths D_3 orbits as points $\{(-2), 1, (-1/2)\}$ for the color charges $\{r, g, b\}$. These orbits, as well as the complex roots of unities as another degenerate D_3 orbit, are obtained by setting two of the D_3 MT's equal and solving the obtained equations. The cubic behaviour of GR shows up this way. Concerning the generation of leptonic and fermionic particle series, they belong to the symmetry group $\mathbb{Z}_2 \times D_3$ of order 12. It contains as subgroup the projective $\mathbb{Z}_2 \times \mathbb{Z}_2$ version of SU(2).

For the location of a nucleons 3 quarks I set 3 points as vertices of a triangle whose sides can be stretched or squeezed, as observed in general relativity, but in a bag mostly generated by the exchange of gluons as carriers of the strong interaction SI between paired quarks. In the acutal exchange I postulate that the distance between the 2 participating quarks is temporarily shortened and the distance to the third quark becomes larger. This way the triangle is never symmetric. The increase (or decrease) of distances can consume in experiments so much energy that 2 nucleons are generated from one (or a gravitational collapse occurs, stopping the SI gluon exchange). Gluons are superpositions of two color charges $x\overline{y},\ x \neq y$, or of 6 color charges. I refer to QCD with 3-dimensional blown up Pauli-matrices of SU(2). The blown up S^3 geometry is $S^3 \times S^5$ for the SI symmetry group SU(3), which is a toroidal product. It contains a GR/color charge sphere $S^5 \subset \mathbb{C}^3$ of the Gleason operators T in space $\mathbb{C}^3$ without absolute system-coordinates which is for the single T the measure-norming of EIS unit vectors on S^5. Systems/particles mass measures for instance are due to its EIS units. The Minkowski measurements – without a GR interaction between *two* systems P, Q gets is relativistic factor $\gamma = \sqrt{1 - (v^2/c^2)}$ determined on a bounding common sphere S^2 for P, Q (see [8])[2].

For energy carrying particles (or systems) I always assume geometrically a bag boundary S^2 at projective infinity where its main QM probability distribution $\psi\psi^*$ can be found inside. For potential fields, also for GR, or wave descriptions I use for instance the Minkowski light cone (its metric scaled according to Einstein through the Schwarzschild factor). Or for light/photons I use geometrical a circular cylindrical expansion with two energy carrying helix lines on the surface winding and rotating in time about a central z-axis which

[2] γ rescales all measured coordinates, is multiplicative for ict, iw as $\triangle t, m$ or $\vec{p}$, is by division for r, iu as length and frequency f where the last one has also the $\frac{(c \pm v)^{1/2} f}{c^2 \gamma}$ rescalings. The rescalings of speeds and accelerations are more complicated and for 3-dimensional space coordinates.

coincides with the time-axis. Since light is spreading out from mass systems in gravitational fields, it changes through red shift proportional to the changing gravitational fields potential on a light-cone with speed of light. According to Einstein, angular accelerations of lights maximal possible speed c in the universe are observed, when it passes by other mass systems (double lenses, deviation, diffraction) or more generally when a system Q is captured by a huge central mass system (sun) as a planet (rosette rotation with an angle (for the GR-acceleration of the speed of Q) added after one revolution instead of Kepler ellipses).

I repeat my formulas for the two Einstein metrics: Minkowski is $ds^2 = dr^2 - c^2dt^2$ for differentials ds as common length measurement, dr for radius in space with $r^2 = x^2 + y^2 + z^2$ and cdt for time. The Schwarzschild factor $\cos^2\beta$ arises only when a system Q is in a gravitational field of a second system A. The Copenhagen interpretation of one system, here A as measuring apparatus and Q as measured system, applies to their two EIS, where the one A carries the GR-scaling of general relativity in the projective form $ds^2 = (dr^2/\cos^2\beta) - \cos^2\beta c^2dt^2$. Details can be found in [7, 8].

The factor $\cos^2\beta$ can be obtained through the MT $\frac{-z-1}{z} \sim \alpha^2 \in D_3$, belonging to the θ-coordinate of an EIS. The other members of D_3 in Table 1 are the symmetries of an equilateral triangle as a $\pm 120^0$ oriented rotational angle of α, α^2, or id, and 3 reflections $\sigma_1, \alpha\sigma_1, \alpha^2\sigma_1$.

r	φ	iu	iw
$x \in \mathbb{R}$	$iy \in i\mathbb{R}$		
r	g	b	$\overline{r}$
z	$\frac{z}{-z-1}$	$\frac{1}{-z-1}$	$\frac{1}{z}$
$\frac{1}{z}$	-$\frac{1}{z}$		
$id; \sigma_1$	$\alpha\sigma_1; \sigma_2$	α	$\sigma_1; id$
1	4	6	5
length λ_P	temp. T_P	ener. E_P	mass m_P
	C		P

Table 2:

In extrem collisions or explosions (of a black hole SL for instance), two (θ, ict) coordinates of EIS are complex-projectively normed to 1 and newly set with angular momenta and eigen-time for new particles and systems generated.

In comparison to EM in Section 1, there are 4 D_3 poles for color charges and GR, while EM has only 2. I remark, that also EM with SU(2) could have four poles by setting $z = (-1/z)$ with the solutions $\pm i$. The contour integration for EM is repeated for the complex solutions of $(1/z) = (z/(-z-1)$ for instance where the denominator for a GR-computing with residues at the complex third roots of unity is $1/(z^2+z+1)$. At the third real root $1 \in \mathbb{C}$ the GR vector as momentum of a nucleon is set through a suitable choice for the two complex residues, at $p_+ = \frac{-1+i\sqrt{3}}{2}$ the frequency of the nucleon is set through $hf = mc^2$, m the mass of the nucleon, and at $p_- = \frac{-1-i\sqrt{3}}{2}$ its angular momentum as vector is set. As in the EM case, the GR potential expands with speed of light (possibly Schwarzschild scaled) on a Minkowski light-cone from p_+. The 3 points stand in scaled form for 3 quarks centers of color charge whirls $x = r, g, b$ red, green blue – as combined color white in a nucleon with 3 quarks, for instance as uud proton or udd neutron. Their couplings of u, d spins is used as in physics. I postulate (as for magnetic field quantums) a circular, conic whirl with upper bound $\mathbb{P}$ for the six color charges $x, \overline{x}$ which in this pairing have the same rotational speed/frequency $v_x = 2\pi r f_x$, but opposite orientations for the circular orientation with radius r. The v_x are pairwise different which is useful for their superpositions. Equal speeds can annihilate one another.

For heat transfer of a nucleon with its environment I use the pole of the φ coordinate. If I name, as for EM and the 3 Pauli matrices, the $D_3 - \{id\}$ matrices a 5-dimensional color charge gravitational spin, then as MT invariants they generate (beside a constant energy potential and frequency (according to Planck) at 0 temperature (heat is associated with the φ coordinate)), the preservation theorems for energy (iu coordinate, $E = hf$), momentum (iw coordinate, $E = mc^2$, $p = mv$), angular momentum (θ coordinate, $J = r \times p$) and for time ict I can use preserved measures through atomic clocks or my bag cycles with a time $t_0 > 0$. The Heisenberg uncertainites HU belong to the EIS coordinates in the pairings (r, iw) position-momentum, (φ, θ) angle-angular momentum and (ict, iu) time-energy. For EM I used similarly the pairing of (ϕ_0, e_0).

Since the D_3 group allows also 3 as factor of its groups order, I postulate for the not experimentally verified gravitons with spin 2 that they are $r - g - b$ white superpositions of 3 color charge whirls. They stretch and squeeze also the quarks distances in a nucleon, not only SI. For vacuum as energy empty spacetime I assume no coordinates. Orthogonality for dimensions is not bound to coordinates (see [7, 8]). Vacuum fluctuations are obtained by temporarily generated EIS in it. They can be of electrical, gravitational or heat nature – in form of WI-bosons carrying mass, as mass-less field quantums photons, gravitons, phonons, ... SI is also observed in vacuum as gluon balls, independent of nucleons.

I mentioned above a bag cycle for the inner motion of a nucleon with 3 quarks. I set for instance the color charge cycle, moving in the time t_0 (or slightly time-streched and squeezed with a very small $\delta > 0$) as in Table 3. The D_3 symmetry (and Γ) can be used for the color charges of quarks, exchanging color during the actions through gluons. For the 6 time intervals, draw 6 triangles in equal position along a circle, oriented for the time motion between them. Give their vertices a fixed number 1, 2, 3. The motion is always generated by α, which is multipled from the left by the reflection of D_3 used to change the color charge distribution of the quark $q = 1, 2, 3$ after an application of α, measured towards the previous location of q having color charge b. Start with the distribution $1g, 2b, 3r,\ \sigma_1$ and move on in the following cycle to $1b, 2g, 3r,\ \alpha\sigma_1;\ 1r, 2g, 3b,\ \alpha^2\sigma_1;\ 1r, 2b, 3g,\ \sigma_1;\ 1b, 2r, 3g,\ \alpha\sigma_1;\ 1g, 2r, 3b,\ \alpha^2\sigma_1$ and back to the first color charge distribution[3]. The cycle is counter-clockwise with b. The $g, r - \Gamma$ whirls underly reflections and move in the clockwise direction, $1g \to 2g \to 3g \to 1g \ldots$ and $3r \to 1r \to 2r \to 3r \ldots$.

Time	GR	SI	WI(o→i)	q_{in}	WI(i→o)	q_{out}
$1t_0 + \delta$	min	$\overline{2_{o3}3_{o1}}$	1_{o3}	1_{in}	1_{o2}	
$2t_0$	max					$1_{o2} \to 1_{o3}$
$3t_0 + \delta$	min	$\overline{3_{o2}1_{o3}}$	2_{o1}	2_{in}	2_{o3}	
$4t_0$	max					$2_{o3} \to 2_{o1}$
$5t_0 + \delta$	min	$\overline{1_{o2}2_{o1}}$	3_{o1}	3_{in}	3_{o2}	
$6t_0$	max					$3_{o2} \to 3_{o1}$

Table 3:

A common GR + SI + WI time Table 3 as 6-cycle (modulo 6) for these actions is with the Γ-max, -min alternating in every row, and moving as column together with the cyclic time intervals in the first column; these $\Gamma_{\text{max,min}}$ inner bag potentials move in the time intervals $jt_0 - (j + 1)t_0$ (modulo 6) between max- and min- values discrete-wave-like. Draw two flat concentric circles where as a point $q_{in,oj=out}$ is marked for the color charge carrying handle of a quark $q = 1, 2, 3$. The quarks location on the inner and outer radii are using for a Γ_{min} location the inner circle, for the other two quarks (exchanging gluons) the outer circle. The triangular membran (which may carry no energy itself) between the triangles vertices is stretched or squeezed in (a possibly empty vacuum) space through this inner motion. SI acts through exchanged gluons

[3]If I compare the EM/WI switching circuits $0, 1$ on-off or as $W^{\pm} \sim S_x \pm iS_y$ decays with these SI switches, the orientations can also be used for off-on-(half open) switched ventiles $b - r - g$.

on the outer circles segment between the two points listed in outer position, shortening their distance. A virtual neutral Z^0 is assumed to induct the q-quarks triangular pendulum motion WI($o \rightarrow$i) from the outer quark location $q_{oj},\ j \neq q$, to its location q_{in} on the inner circle. Adding a small time interval δ in between, another virtual Z^0 WI-boson is assumed to induct the q-quarks triangular motion WI($i \rightarrow$o) from the inner quark location q_{in} to its location $q_{oj},\ j \neq q$, on the outer circle.

3. Concluding Remarks

The 6-dimensional, complex 3-dimensional operator space of the previous section is for setting coordinate systems EIS, belonging to energy carriers. The complex projective normings of this $\mathbb{C}^3$ are not always down to a 4-dimensional spacetime as vacuum. It can arise as observed scalar or vectorial fields. I mentioned potential fields which move with the speed c and the frequency of huge amounts of graviton whirl superpositions as wave. Always there is a projective closure of such a 4-dimensional space at infinity through a Riemannian sphere S^2. This is for atoms, nucleons, particles bags boundaries, but can also be used for large scaled systems in the universe. In gravitational collapses it is assumed that angular momentum and time are stopped. The systems as black holes SL have S^2 as space (with coordinates as in Table 2) and as flat stereographic projection $\mathbb{C}$ its SL aggregation disk. The common $\sigma_1 \in D_3 \cap \mathrm{SU}(2)$ matrix transfers the ratio of (radius per mass) between quark systems of the universe and its black hole systems at the circular Schwarzschild radius of the system when SL collapses or explosions occur. For more extensive descriptions, I refer to [7, 8, 9].

In the two sections above I treated first the SU(2) geometry of electromagnetism EM and the weak interaction WI. In earlier publications I used this for interpretations to unsolved EM problems and includes possible nuclear decay processes through WI. It also shows similarities to my newly introduced nucleon approach with the color charge group D_3 for bags with boundary S^2, gravity GR, the strong interaction SI, and hydrogen which adds EMI through emitted or absorbed spectral series. Einstein's two relativities are included in this treatment. It is considered as a unification of the four basic forces EMI, GR, SI, WI in physics.

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