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# A RANDOM COINCIDENCE POINT THEOREM FOR MULTIFUNCTION

K. Fakhar[1], G. Mustafa[2], M. Azram[3] §

[1]Department of Mathematics
Faculty of Sciences
University of Technology of Malaysia
Skudai, Johor, 831300, MALAYSIA
e-mail: kamranfakhar@yahoo.com

[2]Department of Mathematics
The Islamia University of Bahawalpur
PAKISTAN
e-mail: mustafa_rakib@yahoo.com

[3]Department of Science in Engineering (SIE)
Faculty of Engineering
International Islamic University of Malaysia – IIUM
Kuala Lumpur, 50728, MALAYSIA
e-mail: azram50@hotmail.com

**Abstract:** A random coincidence point theorem for multifunction under the very mild conditions is established. The theorem proved here can be viewed as stochastic version of Ljubomir Ciric (*Indian J. pure appl. Math.*, **24** (1993), 145-149).

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## 1. Introduction

Random coincidence point theorems and random fixed point theorems are

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§Correspondence author

stochastic generalizations of classical coincidence point theorems and classical fixed point theorems. Random fixed point theorems for contractive mappings on separable complete metric spaces have been proved by several authors; Spacek [11] and Hans (see [2], [3]). Sehgal and Singh [10] have proved the different stochastic versions of the well-known Schauder's fixed point theorem. Ljubomir Ciric [1] investigated a class of self mappings $T$ of metric spaces $X$ which satisfies the following contractive definition:

$$d(Tx,Ty) \le a\max\{d(x,y),d(x,Tx),d(y,Ty),1/2.[d(x,Ty)+d(y,Tx)]\}$$
$$+\,b\max\{d(x,Tx),d(y,Ty)\}+c.[d(x,Ty)+d(y,Tx)],$$

for all $x, y \in X$, where $a, b, c$ are positive real numbers such that $a+b+2c = 1$ and established a fixed point theorem. The aim of this paper is to study stochastic versions of fixed point theorems for contractive type multifunctions on separable complete metric spaces by generalizing the contractive mappings of Ljubomir Ciric. For details about multivalued contractive type mappings we refer to Ghulam Mustafa (see [7], [8]).

## 2. Preliminaries

Through out this paper $(X, d)$ is a separable complete metric space and $(\Omega, \delta)$ is measurable space. Let $2^X$ be the family of all subsets of $X$, $CB(X)$ denote the family of all non empty closed and bounded subsets of $X$.

A mapping $\mu : \Omega \to 2^X$ is called *measurable* if for any open subset $C$ of $X$, $\mu^{-1}(C) = \{w \in \Omega : \mu(w) \cap C \neq \emptyset\} \in \delta$. This type of measurability is usually called weakly measurable (cf. Himmelberg [4]), but in this paper since we only use this type of measurability, we omit the term "weakly" for simplicity. A mapping $\xi : \Omega \to X$ is said to be *measurable selector* of a measurable mapping $\mu : \Omega \to 2^X$ if $\mu$ is measurable and for any $w \in \Omega$, $\xi(w) \in \mu(w)$. A mapping $f : \Omega \times X \to X$ is called a *random operator* if for any $x \in X$, $f(., x)$ is measurable. A mapping $T : \Omega \times X \to CB(X)$ is called a *multifunction* if for every $x \in X$, $T(., x)$ is measurable. A measurable mapping $\xi : \Omega \to X$ is called a *random fixed point* of a multifunction (*random operator*) $T : \Omega \times X \to CB(X)$ $(f : \Omega \times X \to X)$ if for every $w \in \Omega$, $\xi(w) \in T(w, \xi(w))$ $(f(w, \xi(w)) = \xi(w))$. A measurable mapping $\xi : \Omega \to X$ is a *random coincidence point* of $T : \Omega \times X \to CB(X)$ and $f : \Omega \times X \to X$ if for every $w \in \Omega$, $f(w, \xi(w)) \in T(w, \xi(w))$.

For the remaining part of this section $T : \Omega \times X \to CB(X)$ is multifunction, $f : \Omega \times X \to X$ is random operator and $\xi_n : \Omega \to CB(X)$ is measurable mapping for each $n = 0, 1, 2 \ldots$. For a map $\xi_0 : \Omega \to X$ if there exists a sequence $\{\xi_n(w)\}$ such that $f(w, \xi_{n+1}(w)) \in T(w, \xi_n(w))$, $n = 0, 1, 2, \ldots\ldots$ then $O_f(\xi_0(w)) = \{f(w, \xi_n(w)) : n = 1, 2, 3, \ldots.$ for each $w \in \Omega\}$ is the *orbit* for $(T, f)$ at $\xi_0(w)$. $O_f(w, \xi_0(w))$ is called a *regular orbit* for $(T, f)$ if for each $n$, for each $w \in \Omega$, $d(f(w, \xi_{n+1}(w)), f(w, \xi_{n+2}(w))) \leq H(T(w, \xi_n(w)), T(w, \xi_{n+1}(w)))$, where $H$ is a Hausdorff metric on $CB(X)$ induced by the metric $d$ of $X$; that is, for $A, B$ in $CB(X)$,

$$H(A, B) = \max\left\{\sup_{a \in A} d(a, B), \sup_{b \in B} d(b, A)\right\},$$

where $d(x, E)$ is the distance from a point $x \in X$ to a subset $E \subset X$, i.e., $d(x, E) = \inf\{d(x, y) : y \in E\}$. If there exists a measurable map $\xi : \Omega \to X$ such that $f(w, \xi_n(w)) \to f(w, \xi(w))$ for all $w \in \Omega$, then $O_f(\xi_0(w))$ converge in $X$. If $O_f(\xi_n(w))$ converge in $X$, then $X$ is called $(T, f, \xi_0(w))-$ *orbitally complete*.

## 3. Main Result

In this section, we give stochastic version of result of Ljubomir Ciric [1].

Let $T : \Omega \times X \to CB(X)$ be a multifunction and let $f : \Omega \times X \to X$ be a random operator such that,

$$\begin{aligned} &H(T(w, x), T(w, y)) \\ &\quad \leq \alpha(w) \max\{d(f(w, x), f(w, y)), d(f(w, x), T(w, x)), d(f(w, y), T(w, y)), \\ &[d(f(w, x), T(w, y)) + d(f(w, y), T(w, x))]/2\} + \beta(w) \max\{d(f(w, x), T(w, x)), \\ &\ d(f(w, y), T(w, y))\} + \gamma(w)[d(f(w, x), T(w, y)) + d(f(w, By), T(w, x))], \end{aligned} \tag{3.1}$$

for all $x, y \in X$ and for all $w \in \Omega$, where $\alpha, \beta, \gamma : \Omega \to (0, 1)$ are measurable mappings, such that

$$\alpha(w) + \beta(w) + 2\gamma(w) = 1. \tag{3.2}$$

**Lemma 3.1.** *Let $T : \Omega \times X \to CB(X)$ be a continuous multifunction and let $f : \Omega \times X \to X$ be a random operator satisfying (3.1), for all $w \in \Omega$, where $\alpha, \beta, \gamma : \Omega \to (0, 1)$ are measurable mappings, such that $\alpha(w) + \beta(w) + 2\gamma(w) = 1$. Then*

$$inf\{d(f(w, \xi_n(w)), T(w, \xi_n(w))); \quad w \in \Omega\} = 0, \tag{3.3}$$

*where $\xi_n : \Omega \to X$ are measurable mappings for $n = 0, 1, 2 \ldots.$*

*Proof.* Let $\xi_0, \xi_1 : \Omega \to X$ be measurable mappings such that $f(w, \xi_1(w)) \in T(w, \xi_0(w))$, for all $w \in \Omega$. Indeed, since $T$ is a continuous random operator, we conclude that, for every $v \in X$, the map $(w, x) \to d(v, T(w, x))$ is a caratheodory function (that is measurable in $w \in \Omega$, continuous in $x \in X$). Thus it is jointly measurable. Hence, since $\xi_0 : \Omega \to X$ is measurable, $w \to d(v, T(w, \xi_0(w))$ is measurable too, therefore $w \to T(w, \xi_0(w))$ is weakly measurable by Wagner ([12], p. 868). By K. Kuratowski, (see [5], Theorem 8), there exists a measurable map $\xi_1 : \Omega \to X$ such that $f(w, \xi_1(w)) \in T(w, \xi_0(w))$ for all $w \in \Omega$. Define a sequence of measurable mappings $\xi_n : \Omega \to X$ such that $f(w, \xi_{n+1}(w)) \in T(w, \xi_n(w))$, for $n = 0, 1, 2 \ldots$. Then by (3.1),

$$\begin{aligned}
&d(f(w,\xi_{n+1}(w)), T(w,\xi_{n+1}(w))) \le H(T(w,\xi_n(w)), T(w,\xi_{n+1}(w))) \\
&\le \alpha(w) \max \{d(f(w,\xi_n(w)), f(w,\xi_{n+1}(w))), d(f(w,\xi_n(w)), T(w,\xi_n(w))), \\
&\quad d(f(w,\xi_{n+1}(w)), T(w,\xi_{n+1}(w))), [d(f(w,\xi_n(w)), T(w,\xi_{n+1}(w))) \\
&+ d(f(w,\xi_{n+1}(w)), T(w,\xi_n(w)))]/2\} + \beta(w) \max \{d(f(w,\xi_n(w)), T(w,\xi_n(w))), \\
&\quad d(f(w,\xi_{n+1}(w)), T(w,\xi_{n+1}(w)))\} + \gamma(w)[d(f(w,\xi_n(w)), T(w,\xi_{n+1}(w))) \\
&\quad + d(f(w,\xi_{n+1}(w)), T(w,\xi_n(w)))] \\
&\le \alpha(w) \max \{d(f(w,\xi_n(w)), f(w,\xi_{n+1}(w))), d(f(w,\xi_{n+1}(w)), T(w,\xi_{n+1}(w))), \\
&\quad d(f(w,\xi_n(w)), T(w,\xi_{n+1}(w)))/2\} + \beta(w) \max \{d(f(w,\xi_n(w)), T(w,\xi_n(w))), \\
&\quad d(f(w,\xi_{n+1}(w)), T(w,\xi_{n+1}(w)))\} + \gamma(w) d(f(w,\xi_n(w)), T(w,\xi_{n+1}(w))).
\end{aligned}$$

Hence, applying the triangle inequality, we get

$$\begin{aligned}
&d(f(w,\xi_{n+1}(w)), T(w,\xi_{n+1}(w))) \\
&\le \alpha(w) \max \{d(f(w,\xi_n(w)), T(w,\xi_n(w))), d(f(w,\xi_{n+1}(w)), T(w,\xi_{n+1}(w)))\} \\
&+ \beta(w) \max \{d(f(w,\xi_n(w)), T(w,\xi_n(w))), d(f(w,\xi_{n+1}(w)), T(w,\xi_{n+1}(w)))\} \\
&+ \gamma(w)[d(f(w,\xi_n(w)), T(w,\xi_n(w))) + d(f(w,\xi_{n+1}(w)), T(w,\xi_{n+1}(w)))] \\
&\le [\alpha(w) + \beta(w)] \max \{d(f(w,\xi_n(w)), T(w,\xi_n(w))), d(f(w,\xi_{n+1}(w)), \\
&\quad T(w,\xi_{n+1}(w)))\} + \gamma(w)[d(f(w,\xi_n(w)), T(w,\xi_n(w))) \\
&\quad + d(f(w,\xi_{n+1}(w)), T(w,\xi_{n+1}(w)))].
\end{aligned}$$

If $d(f(w,\xi_{n+1}(w)), T(w,\xi_{n+1}(w))) > d(f(w,\xi_n(w)), T(w,\xi_n(w)))$ for some $n$, then we have,

$$\begin{aligned}
&d(f(w,\xi_{n+1}(w)), T(w,\xi_{n+1}(w))) \\
&\qquad < [\alpha(w) + \beta(w) + 2\gamma(w)] d(f(w,\xi_{n+1}(w)), T(w,\xi_{n+1}(w)))
\end{aligned}$$

a contradiction. Thus

$$d(f(w,\xi_{n+1}(w)), T(w,\xi_{n+1}(w))) \le d(f(w,\xi_n(w)), T(w,\xi_n(w))).$$

Hence

$$d(f(w,\xi_n(w)), T(w,\xi_n(w))) \le d(f(w,\xi_0(w)), T(w,\xi_0(w))) \tag{3.4}$$

for all positive integer $n$. Using (3.1) again, by (3.4) we have

$$\begin{aligned}
&d(f(w,\xi_1(w)),T(w,\xi_2(w))) \le H(T(w,\xi_0(w)),T(w,\xi_2(w)))\\
&\le \alpha(w)\max\{d(f(w,\xi_0(w)),f(w,\xi_2(w))),d(f(w,\xi_0(w)),T(w,\xi_0(w))),\\
&d(f(w,\xi_2(w)),T(w,\xi_2(w))),[d(f(w,\xi_0(w)),T(w,\xi_2(w)))\\
&+d(f(w,\xi_2(w)),T(w,\xi_0(w)))]/2\}+\beta(w)\max\{d(f(w,\xi_0(w)),T(w,\xi_0(w))),\\
&d(f(w,\xi_2(w)),T(w,\xi_2(w)))\}+\gamma(w)[d(f(w,\xi_0(w)),T(w,\xi_2(w)))\\
&+d(f(w,\xi_2(w)),T(w,\xi_0(w)))]\le \alpha(w)\max\{d(f(w,\xi_0(w)),T(w,\xi_1(w))),\\
&d(f(w,\xi_0(w)),T(w,\xi_0(w))),d(f(w,\xi_2(w)),T(w,\xi_2(w))),[d(f(w,\xi_0(w)),\\
&T(w,\xi_2(w)))+d(f(w,\xi_2(w)),T(w,\xi_0(w)))]/2\}+\beta(w)\max\{d(f(w,\xi_0(w)),\\
&T(w,\xi_0(w)))+d(f(w,\xi_2(w)),T(w,\xi_2(w)))\}\\
&+\gamma(w)[d(f(w,\xi_0(w)),T(w,\xi_2(w)))+d(f(w,\xi_2(w)),T(w,\xi_0(w)))],
\end{aligned}$$

$$\begin{aligned}
&d(f(w,\xi_1(w)),T(w,\xi_2(w)))\\
&\le \alpha(w)\max\{d(f(w,\xi_0(w)),T(w,\xi_1(w))),d(f(w,\xi_0(w)),T(w,\xi_0(w))),\\
&[d(f(w,\xi_0(w)),T(w,\xi_2(w)))+d(f(w,\xi_0(w)),T(w,\xi_0(w)))]/2\}\\
&+\beta(w)d(f(w,\xi_0(w)),T(w,\xi_0(w)))+\gamma(w)[d(f(w,\xi_0(w)),T(w,\xi_2(w)))\\
&+d(f(w,\xi_0(w)),T(w,\xi_0(w)))]. \qquad (3.5)
\end{aligned}$$

Since by (3.4) and the triangle inequality, we have

$$\begin{aligned}
&(1/2)[d(f(w,\xi_0(w)),T(w,\xi_2(w)))+d(f(w,\xi_0(w)),\\
&T(w,\xi_0(w)))]\le[d(f(w,\xi_0(w)),f(w,\xi_1(w)))+d(f(w,\xi_1(w)),f(w,\xi_2(w)))\\
&+d(f(w,\xi_2(w)),T(w,\xi_2(w)))+d(f(w,\xi_0(w)),T(w,\xi_0(w)))]/2\\
&\le[d(f(w,\xi_0(w)),T(w,\xi_0(w)))\\
&+d(f(w,\xi_1(w)),T(w,\xi_1(w)))+d(f(w,\xi_2(w)),T(w,\xi_2(w)))\\
&+d(f(w,\xi_0(w)),T(w,\xi_0(w)))]/2=2d(f(w,\xi_0(w)),T(w,\xi_0(w)))\\
&d(f(w,\xi_0(w)),T(w,\xi_2(w)))+d(f(w,\xi_0(w)),T(w,\xi_0(w)))\\
&\le d(f(w,\xi_0(w)),f(w,\xi_1(w)))+d(f(w,\xi_1(w)),T(w,\xi_2(w)))\\
&+d(f(w,\xi_0(w)),T(w,\xi_0(w)))\\
&\le d(f(w,\xi_0(w)),T(w,\xi_0(w)))+d(f(w,\xi_1(w)),T(w,\xi_2(w)))\\
&+d(f(w,\xi_0(w)),T(w,\xi_0(w)))\le 2d(f(w,\xi_0(w)),T(w,\xi_0(w)))\\
&+d(f(w,\xi_1(w)),T(w,\xi_2(w))).
\end{aligned}$$

From (3.5) and (3.2) we have

$$d(f(w,\xi_1(w)),T(w,\xi_2(w)))$$

$$\leq \alpha(w)\max\{d(f(w,\xi_0(w)),T(w,\xi_1(w))),d(f(w,\xi_0(w)),T(w,\xi_0(w))),$$
$$2d(f(w,\xi_0(w)),T(w,\xi_0(w)))\}+\beta(w)d(f(w,\xi_0(w)),T(w,\xi_0(w)))$$
$$+2\gamma(w)d(f(w,\xi_0(w)),T(w,\xi_0(w)))+\gamma(w)d(f(w,\xi_1(w)),T(w,\xi_2(w)))$$
$$\leq 2\alpha(w)d(f(w,\xi_0(w)),T(w,\xi_0(w)))+\beta(w)d(f(w,\xi_0(w)),T(w,\xi_0(w)))$$
$$+2\gamma(w)d(f(w,\xi_0(w)),T(w,\xi_0(w)))+\gamma(w)d(f(w,\xi_1(w)),T(w,\xi_2(w))).$$

$$d(f(w,\xi_1(w)),T(w,\xi_2(w)))\leq[(1+\alpha(w))/(1-\gamma(w))]d(f(w,\xi_0(w)),T(w,\xi_0(w)))$$

$$d(f(w,\xi_1(w)),T(w,\xi_2(w)))\leq[2-\beta(w)]d(f(w,\xi_0(w)),T(w,\xi_0(w))) \quad (3.6)$$

Using (3.1), (3.4) and (3.6) we have

$$d(f(w,\xi_2(w)),T(w,\xi_2(w)))\leq H(T(w,\xi_1(w)),T(w,\xi_2(w)))$$
$$\leq \alpha(w)\max\{d(f(w,\xi_1(w)),f(w,\xi_2(w))),d(f(w,\xi_1(w)),T(w,\xi_1(w))),$$
$$d(f(w,\xi_2(w)),T(w,\xi_2(w))),[d(f(w,\xi_1(w)),T(w,\xi_2(w)))+d(f(w,\xi_2(w)),$$
$$T(w,\xi_1(w)))]/2\}+\beta(w)\max\{d(f(w,\xi_1(w)),T(w,\xi_1(w))),d(f(w,\xi_2(w)),$$
$$T(w,\xi_2(w)))\}+\gamma(w)[d(f(w,\xi_1(w)),T(w,\xi_2(w)))+d(f(w,\xi_2(w)),T(w,\xi_1(w)))]$$
$$\leq \alpha(w)\max\{d(f(w,\xi_0(w)),T(w,\xi_0(w))),d(f(w,\xi_1(w)),$$
$$T(w,\xi_2(w)))/2\}+\beta(w)\max\{d(f(w,\xi_0(w)),T(w,\xi_0(w)))$$
$$+\gamma(w)d(f(w,\xi_1(w)),T(w,\xi_2(w))),$$

$$d(f(w,\xi_2(w)),T(w,\xi_2(w)))\leq \alpha(w)\{d(f(w,\xi_0(w)),T(w,\xi_0(w)))$$
$$+\beta(w)d(f(w,\xi_0(w)),T(w,\xi_0(w)))$$
$$+\gamma(w)[2-\beta(w)]d(f(w,\xi_0(w)),T(w,\xi_0(w))), \quad (3.7)$$

and hence

$$d(f(w,\xi_2(w)),T(w,\xi_2(w)))\leq[1-\beta(w)\gamma(w)]d(f(w,\xi_0(w)),T(w,\xi_0(w))).$$

It is easily shown by induction that (3.7) implies

$$d(f(w,\xi_n(w)),T(w,\xi_n(w)))$$
$$\leq[1-\beta(w)\gamma(w)]^{[n/2]}d(f(w,\xi_0(w)),T(w,\xi_0(w))) \quad (3.8)$$

Here $[n/2]$ means the greatest integer not exceeding $n/2$. Since by hypothesis $0<\beta(w)\gamma(w)<1$ from (3.8) we conclude that (3.3) holds. This completes the proof. □

**Theorem 3.1.** *Let $T:\Omega\times X\to CB(X)$ be a continuous multifunction and let $f:\Omega\times X\to X$ be a random operator, such that for a measurable map $\xi_0:\Omega\to X$, $f(w,X)$ is $(T,f,\xi_0(w))-$ orbitally complete, for every $w\in\Omega$, and satisfy (3.1) for all $w\in\Omega$, where $\alpha,\beta,\gamma:\Omega\to(0,1)$ are measurable mappings, such that (3.2) holds. Then $T$ and $f$ have a random coincidence* *point. Moreover, if $f$ is one-to-one such that $f^{-1}$ exists, then $T$ and $f$ have a random coincidence point.*

*Proof.* We define a sequence of measurable mappings $\xi_n : \Omega \to X$, such that $f(w, \xi_{n+1}(w)) \in T(w, \xi_n(w)))$, for all $w \in \Omega$, for $n = 0, 1, 2, \ldots$ Since, $d(f(w, \xi_n(w)), f(w, \xi_{n+1}(w))) = d(f(w, \xi_n(w)), T(w, \xi_n(w)))$ then by using (3.8), we conclude that, for any $w \in \Omega$, $\{f(w, \xi_n(w))\}$ is a Cauchy sequence in $f(w, X)$. The orbital completeness of $f(w, X)$ allows us to obtain a measurable map $\xi : \Omega \to X$ such that $f(w, \xi_n(w)) \to f(w, \xi(w))$ for all $w \in \Omega$. Using the triangle inequality and (3.1) we have

$$\begin{aligned}
&d(f(w,\xi(w)),T(w,\xi(w))) \leq d(f(w,\xi(w)),f(w,\xi_{n+1}(w))) + d(f(w,\xi_{n+1}(w)),\\
&T(w,\xi(w))) \leq d(f(w,\xi(w)),f(w,\xi_{n+1}(w))) + H(T(w,\xi_n(w)),T(w,\xi(w)))\\
&\leq d(f(w,\xi(w)),f(w,\xi_{n+1}(w))) + \alpha(w)\max\{d(f(w,\xi_n(w)),f(w,\xi(w))),\\
&d(f(w,\xi_n(w)),T(w,\xi_n(w))),d(f(w,\xi(w)),T(w,\xi(w))),[d(f(w,\xi_n(w)),\\
&T(w,\xi(w))) + d(f(w,\xi(w)),T(w,\xi_n(w)))]/2\} + \beta(w)\max\{d(f(w,\xi_n(w)),\\
&T(w,\xi_n(w))),d(f(w,\xi(w)),T(w,\xi(w)))\} + \gamma(w)[d(f(w,\xi_n(w)),T(w,\xi(w)))\\
&\qquad + d(f(w,\xi(w)),T(w,\xi_n(w)))].
\end{aligned}$$

Taking the limit as $n \to \infty$, yields.

$$\begin{aligned}
&d(f(w,\xi(w)),T(w,\xi(w))) \leq \alpha(w)d(f(w,\xi(w)),T(w,\xi(w)))+\\
&\beta(w)d(f(w,\xi(w)),T(w,\xi(w))) + \gamma(w)d(f(w,\xi(w)),T(w,\xi(w)))\\
&\qquad \leq [1-\gamma(w)]d(f(w,\xi(w)),T(w,\xi(w))),
\end{aligned}$$

which implies $d(f(w,\xi(w)),T(w,\xi(w))) = 0$, as $0 < \gamma(w) < 1$. Thus

$$f(w,\xi(w)) \in T(w,\xi(w))).$$

This completes the proof. □

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