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EXISTENCE AND EXPONENTIAL STABILITY OF PERIODIC SOLUTIONS OF IMPULSIVE HIGH-ORDER HOPFIELD NEURAL NETWORK BOTH WITH DISCRETE AND DISTRIBUTED DELAYS ON TIME SCALES

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Abstract: On time scales, by using the continuation theorem of coincidence degree theory and constructing some suitable Lyapunov functions, the periodicity and the exponential stability are investigated for a class of delayed high-order Hopfield neural networks with impulse on time scales, which are new and complement of previously known results. Finally, an example is given to show the effectiveness of the proposed method and results.

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1. Introduction

Since high-order Hopfield neural networks (HHNNs) have stronger approximation property, faster convergence rate, greater storage capacity, and higher fault tolerance than lower-order Hopfield neural networks, the study of high-order Hopfield neural networks has recently gained a lot of attention, moreover there have been extensive results on the problem of the existence and stability of equilibrium points, periodic solutions and almost periodic solutions of

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high-order Hopfield neural networks in the literature. We refer the reder to [1]-[7] and the references cited therein. Moreover, time delays may occur in neural procession and signal transmission, which can cause instability and oscillations in system and the distributed delays should be incorporated in the model. In other words, it is the often the case that the neural networks model possesses both bounded and unbounded delays (distributed delays). In recent years, there have been some results on global asymptotical stability, global exponential stability and periodic solutions for the neural networks with bounded or unbounded delays (see [8]-[10]).

On the other hand, most neural networks can be classified as either continuous or discrete. However, there are many real-world systems and neural processes that behave in a piecewise continuous style interlaced with instantaneous and abrupt changes (impulses). Motivated by this fact, several new neural networks with impulses have recently been proposed and studied (see [11]-[13]). Various types of conditions have been obtained for the existence, uniqueness and global exponential stability of equilibrium point of these impulsive neural networks. The theory of calculus on time scales (see [14], [15] and references cited therein) was initiated by Stefan Hilger in his Ph.D. thesis in 1988 in order to unify continuous and discrete analysis, and it has a tremendous potential for application and has recently received much attention since his foundational work. In fact, both continuous and discrete systems are very important in implementing and applications. But it is troublesome to study the existence and stability of periodic solutions for continuous and discrete systems respectively. Therefore, it is meaningful to study that on time scales which can unify the continuous and discrete situations. To our best knowledge, few papers have been published on existence and stability of periodic solutions of high-order Hopfield neural networks with impulse and distributed delays on time scales.

Our purpose of this paper is to consider the following model

$$\begin{cases} x_i^{\Delta}(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t) f_j(x_j(t-\gamma_{ij})) \\ + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t) \int_0^{\infty} k_{ij}(\theta) g_j(x_j(t-\theta))\Delta\theta \int_0^{\infty} k_{il}(\theta) g_l(x_l(t-\theta))\Delta\theta \\ + I_i(t), \quad i = 1, 2, \ldots, n, \quad t \in \mathbb{T}, \\ \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-) = e_{ik}(x_i(t_k)), \quad i = 1, 2, \ldots, n, \quad k \in \mathbb{N}, \end{cases} \tag{1.1}$$

where $\mathbb{T}$ is an ω-periodic time scale which has the subspace topology inherited from the standard topology on $\mathbb{R}$. For each interval L of $\mathbb{R}$ we denote by $L_{\mathbb{T}} = L \cap \mathbb{T}$, $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$, $x_i(t_k^+)$, $x_i(t_k^-)(i = 1, 2, \ldots, n)$, represent the right and left limit of $x_i(t_k)$ in the sense of time scales. $\{t_l\}$ is a sequence of real numbers such that $0 < t_1 < t_2 < \ldots < t_l \to \infty$ as $l \to \infty$. There exists a positive integer q such that $t_{l+q} = t_l + \omega$, $e_{i(k+q)} = e_{ik}$, $l \in \mathbb{Z}$, $i = 1, 2, \ldots, n$. Without loss of generality, we also assume that $[0, \omega)_{\mathbb{T}} \cap \{t_l : l \in \mathbb{Z}\} = \{t_1, t_2, \ldots, t_q\}$. n corresponds to the number of units in a neural network, $x_i(t)$ corresponds to the state vector of the i-th unit at the time t, $c_i(t)$ represents the rate with which the i-th unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs, $a_{ij}(t)$ and $b_{ijl}(t)$ are the first- and second-order connection weights of the neural network, $\gamma_{ij} \geq 0$, correspond to the time delay required in processing and transmitting a signal from the j-th unit to the i-th unit at time t, k_{ij} is the kernel and $I_i(t)$ denote the external inputs at time t, f_j and g_j are the activation functions of signal transmission. The system (1.1) is supplemented with initial values given by

$$x_i(s) = \varphi_i(s), \quad s \in (-\infty, 0]_{\mathbb{T}},$$

where $\varphi_i(\cdot)$ denote continuous ω-periodic function defined on $(-\infty, 0]_{\mathbb{T}}$.

Throughout this paper, we assume that:

(H_1) $c_i(t) \in (0, +\infty)$ and $c_i(t)$, $a_{ij}(t)$, $b_{ijl}(t)$, $I_i(t)$ are positive continuous periodic functions with period ω, where $\min\limits_{t \in [0,\omega]_{\mathbb{T}}} c_i(t) > 0$, and $c_i(t)$ is right-dense continuous and regressive.

(H_2) There exist positive constants M_j, N_j, $j = 1, 2, \ldots, n$ such that $|f_j(x)| \leq M_j$, $|g_j(x)| \leq N_j$ for $j = 1, 2, \ldots, n$, $x \in \mathbb{R}$.

(H_3) Functions $f_j(u)$, $g_j(u)(j = 1, 2, \ldots, n)$ satisfy the Lipschitz condition, that is, there exist constants $L_j, H_j > 0$ such that $|f_j(u_1) - f_j(u_2)| \leq L_j|u_1 - u_2|$, $|g_j(u_1) - g_j(u_2)| \leq H_j|u_1 - u_2|$, $j = 1, 2, \ldots, n$.

(H_4) $e_{ik} \in C(\mathbb{R}, \mathbb{R})$ are bounded functions, that is, there exist positive numbers E_{ik} such that $|e_{ik}(u)| \leq E_{ik},\ u \in \mathbb{R},\ i = 1, 2, \ldots, n,\ k = 1, 2, \ldots$.

(H_5) The delay kernels $k_{ij} : [0, \infty)_{\mathbb{T}} \to \mathbb{R}^+$ are real-valued piecewise continuous, and there exists a $\alpha_0 > 0$ such that functions

$$K_{ij}(\alpha) = \sum_{m=1}^{\infty} e_\alpha(0, -m\omega) \int_{(m-1)\omega}^{m\omega} k_{ij}(\theta)\Delta\theta,$$

are right-dense continuous for $\alpha \in [0, \alpha_0)_{\mathbb{T}}$ and $K_{ij}(0) = 1$, $i, j = 1, 2, \ldots, n$, and

$$\sum_{m=1}^{\infty} e_{\alpha}(0, -m\omega) \int_{(m-1)\omega}^{m\omega} k_{ij}(\theta)\theta\Delta\theta < \infty,$$

for $\alpha \in [0, \alpha_0)_{\mathbb{T}}$.

The purpose of this paper is by using the continuation theorem of coincidence degree theory and constructing some suitable Lyapunov functions to study the stability and existence of periodic solutions of (1.1).

The paper is organized as follows: In Section 2, we present some basic definitions concerning the calculus on time scale. In Section 3, by using the continuation theorem of coincidence degree theory, we study the existence of periodic solutions of (1.1). In Section 4, by constructing some suitable Lyapunov functions, we study the exponential stability of the periodic solution of (1.1). In Section 5, an example is given to illustrate the effectiveness of our main results.

2. Preliminaries

In this section, we shall first recall some basic definitions, lemmas which are used in what follows.

Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+$ are defined, respectively, by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}, \quad \rho(t) = \sup\{s \in \mathbb{T} : s < t\} \quad \text{and} \quad \mu(t) = \sigma(t) - t.$$

A point $t \in \mathbb{T}$ is called left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T} \setminus \{m\}$; otherwise $\mathbb{T}^k = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m, then $\mathbb{T}_k = \mathbb{T} \setminus \{m\}$; otherwise $\mathbb{T}_k = \mathbb{T}$.

A function $f : \mathbb{T} \to \mathbb{R}$ is right-dense continuous provided it is continuous at right-dense point in $\mathbb{T}$ and its left-side limits exist at left-dense points in $\mathbb{T}$. If f is continuous at each right-dense point and each left-dense point, then f is said to be continuous function on $\mathbb{T}$.

For $y : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^k$, we define the delta derivative of $y(t)$, $y^{\Delta}(t)$, to be the number (if it exists) with the property that for a given $\varepsilon > 0$, there exists a neighborhood U of t such that

$$|[y(\sigma(t)) - y(s)] - y^{\Delta}(t)[\sigma(t) - s]| < \varepsilon|\sigma(t) - s|$$

for all $s \in U$.

If y is continuous, then y is right-dense continuous, and if y is delta differentiable at t, then y is continuous at t.

Definition 2.1. (see [16]) If $a \in \mathbb{T}, \sup \mathbb{T} = \infty$, and f is rd-continuous on $[a, \infty)$, then we define the improper integral by

$$\int_a^\infty f(t)\Delta t = \lim_{b\to\infty} \int_a^b f(t)\Delta t$$

provided this limit exists, and we say that the improper integral converges in this case. If this limit does not exist, then we say that the improper integral diverges.

Definition 2.2. (see [17]) For each $t \in \mathbb{T}$, let N be a neighborhood of t, then, for $V \in C_{rd}[\mathbb{T} \times \mathbb{R}^n, \mathbb{R}^+]$, define $D^+V^\Delta(t, x(t))$ to mean that, given $\varepsilon > 0$, there exists a right neighborhood $N_\varepsilon \subset N$ of t such that

$$\frac{[V(\sigma(t), x(\sigma(t))) - V(s, x(\sigma(t))) - \mu(t,s)f(t, x(t))]}{\mu(t,s)} < D^+V^\Delta(t, x(t)) + \varepsilon$$

for each $s \in N_\varepsilon$, $s > t$, where $\mu(t, s) \equiv \sigma(t) - s$. If t is rd and $V(t, x(t))$ is continuous at t, this reduces to

$$D^+V^\Delta(t, x(t)) = \frac{V(\sigma(t), x(\sigma(t))) - V(t, x(\sigma(t)))}{\sigma(t) - t}.$$

Definition 2.3. (see [18]) We say that a time scale $\mathbb{T}$ is periodic if there exists $p > 0$ such that if $t \in \mathbb{T}$, then $t \pm p \in \mathbb{T}$. For $\mathbb{T} \neq \mathbb{R}$, the smallest positive p is called the period of the time scale.

Definition 2.4. (see [18]) Let $\mathbb{T} \neq \mathbb{R}$ be a periodic time scale with period p. We say that the function $f : \mathbb{T} \to \mathbb{R}$ is periodic with period ω if there exists a natural number n such that $\omega = np$, $f(t + \omega) = f(t)$ for all $t \in \mathbb{T}$ and ω is the smallest number such that $f(t + \omega) = f(t)$.

If $\mathbb{T} = \mathbb{R}$, we say that f is periodic with period $\omega > 0$ if ω is the smallest positive number such that $f(t + \omega) = f(t)$ for all $t \in \mathbb{T}$.

A function $r : \mathbb{T} \to \mathbb{R}$ is called regressive if

$$1 + \mu(t)r(t) \neq 0$$

for all $t \in \mathbb{T}^k$.

If r is regressive function, then the generalized exponential function e_r is defined by

$$e_r(t, s) = \exp\left\{ \int_s^t \xi_{\mu(\tau)}(r(\tau))\Delta\tau \right\}, \text{ for } s, t \in \mathbb{T},$$

with the cylinder transformation

$$\xi_h(z) = \begin{cases} \frac{\mathrm{Log}(1+hz)}{h} & \text{if } h \neq 0, \\ z & \text{if } h = 0. \end{cases}$$

Let $p, q : \mathbb{T} \to \mathbb{R}$ be two regressive functions, we define

$$p \oplus q := p + q + \mu pq, \quad \ominus p := -\frac{p}{1+\mu p}, \quad p \ominus q := p \oplus (\ominus q).$$

Then the generalized exponential function has the following properties.

Lemma 2.1. (see [14]) *Assume that $p, q : \mathbb{T} \to \mathbb{R}$ are two regressive functions, then:*

(i) $e_0(t,s) \equiv 1$ *and* $e_p(t,t) \equiv 1$;

(ii) $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t,s)$;

(iii) $e_p(t, \sigma(s)) = \frac{e_p(t,s)}{1+\mu(s)p(s)}$;

(iv) $\frac{1}{e_p(t,s)} = e_{\ominus p}(t,s)$;

(v) $e_p(t,s) = \frac{1}{e_p(s,t)} = e_{\ominus p}(s,t)$;

(vi) $e_p(t,s)e_p(s,r) = e_p(t,r)$;

(vii) $e_p(t,s)e_q(t,s) = e_{p\oplus q}(t,s)$;

(viii) $\frac{e_p(t,s)}{e_q(t,s)} = e_{p\ominus q}(t,s)$.

Lemma 2.2. (see [14]) *Assume $f, g : \mathbb{T} \to \mathbb{R}$ are delta differentiable at $t \in \mathbb{T}^k$. Then*

$$(fg)^\Delta(t) = f^\Delta(t)g(t) + f(\sigma(t))g^\Delta(t) = f(t)g^\Delta(t) + f^\Delta(t)g(\sigma(t)).$$

Let y be right-dense continuous. If $Y^\Delta(t) = y(t)$, then we define the delta integral by

$$\int_a^t y(s)\Delta s = Y(t) - Y(a).$$

Lemma 2.3. *If $a, b \in \mathbb{T}$, $\alpha, \beta \in \mathbb{R}$ and $f, g \in C(\mathbb{T}, \mathbb{R})$, then:*

(1) $\int_a^b [\alpha f(t) + \beta g(t)]\Delta t = \alpha \int_a^b f(t)\Delta t + \beta \int_a^b g(t)\Delta t$;

(2) *If $f(t) \geq 0$ for all $a \leq t < b$, then $\int_a^b f(t)\Delta t \geq 0$;*

(3) *If $|f(t)| \leq g(t)$ on $[a,b) := \{t \in \mathbb{T} : a \leq t < b\}$, then $|\int_a^b f(t)\Delta t| \leq \int_a^b g(t)\Delta t$.*

Clearly, from Lemma 2.3, we can obtain Lemma 2.4.

Lemma 2.4. *If $f, g \in C(\mathbb{T}, \mathbb{R})$, and $f(t) \leq g(t)$ on $t \in [0, \omega)_{\mathbb{T}}$, then*

$\int_0^\omega f(t)\Delta t \le \int_0^\omega g(t)\Delta t$.

In the proof of our main result, we will use the following three lemmas which can be found in [19], [20].

Lemma 2.5. (see [19]) *Let $t_1, t_2 \in [0,\omega]_{\mathbb{T}}$, and $t \in \mathbb{T}$. If $f : \mathbb{T} \longrightarrow \mathbb{R}$ is ω-periodic, then: $f(t) \le f(t_1) + \int_0^\omega |f^\Delta(s)|\Delta s$, and $f(t) \ge f(t_2) - \int_0^\omega |f^\Delta(s)|\Delta s$.*

Lemma 2.6. (Cauchy-Schwarz Inequality on Time Scales, see [20]) *Let $a, b \in \mathbb{T}$. For rd-continuous functions: $f, g : [a, b] \to \mathbb{R}$, we have*

$$(\int_a^b |f(t)g(t)|\Delta t)^2 \le \int_a^b |f(t)|^2\Delta t \int_a^b |g(t)|^2\Delta t.$$

Lemma 2.7. (see [21]) *Assume that $\{f_n\}_{n\in N}$ is a function sequences on J such that*

(i) $\{f_n\}_{n\in N}$ is uniformly bounded on J;

(ii) $\{f_n^\Delta\}_{n\in N}$ is uniformly bounded on J.

Then there is a subsequence of $\{f_n\}_{n\in N}$ converges uniformly on J.

Lemma 2.8. *Let $\mathbb{T}$ be a ω-periodic time scale, then $\sigma(t+\omega) = \sigma(t) + \omega$, for all $t \in \mathbb{T}$.*

Proof. By using the definition of forward jump operator, we have $\sigma(t)+\omega \ge t+\omega$, then $\sigma(t)+\omega \ge \sigma(t+\omega)$, now we claim that $\sigma(t)+\omega = \sigma(t+\omega)$. If it is not true, we assume that $\sigma(t+\omega) = t_1^* < \sigma(t)+\omega$, from the definition of infimum (inf), we know that there exist a $t_2^* \in \mathbb{T}$, $t_2^* > t+\omega$, such that

$$t_2^* < t_1^* + \frac{\sigma(t)+\omega-t_1^*}{2} = \frac{\sigma(t)+\omega+t_1^*}{2} < \sigma(t)+\omega. \tag{2.1}$$

From (2.1), we obtain $t_2^*-\omega < \sigma(t)$, on the other hand, since $t_2^* > t+\omega$, $t_2^*-\omega \ge \sigma(t)$, which is a contradiction. The proof of Lemma 2.8 is complete. □

From Lemma 2.8, we obtain the following lemma.

Lemma 2.9. *Let $\mathbb{T}$ be a ω-periodic time scale, then $\mu(t)$ is a ω-periodic function.*

Proof.

$$\mu(t+\omega) = \sigma(t+\omega) - t - \omega = \sigma(t) + \omega - t - \omega = \sigma(t) - t = \mu(t). \qquad \square$$

For the sake of convenience, we introduce the following notations:

$$\overline{c_i} = \frac{1}{\omega}\int_0^\omega c_i(t)\Delta t, \quad \overline{a_{ij}} = \frac{1}{\omega}\int_0^\omega a_{ij}(t)\Delta t, \quad \overline{I_i} = \frac{1}{\omega}\int_0^\omega I_i(t)\Delta t,$$

$$\overline{b_{ijl}} = \frac{1}{\omega}\int_0^\omega b_{ijl}(t)\Delta t,\ c_i^+ = \max_{t\in[0,\omega]_{\mathbb{T}}} |c_i(t)|,\ c_i^- = \min_{t\in[0,\omega]_{\mathbb{T}}} |c_i(t)|,\ \gamma = \max_{1\le i,j\le n}\{\gamma_{ij}\},$$

$$a_{ij}^{+}=\max_{t\in[0,\omega]_{\mathbb{T}}}|a_{ij}(t)|,\quad b_{ijl}^{+}=\max_{t\in[0,\omega]_{\mathbb{T}}}|b_{ijl}(t)|,$$

$$I_i^{+}=\max_{t\in[0,\omega]_{\mathbb{T}}}|I_i(t)|,\quad E_i=\max_{1\le k\le q}E_{ik},\quad \mu=\max_{t\in[0,\omega]_{\mathbb{T}}}\mu(t),$$

$$\begin{aligned}A_i(t)&=-c_i(t)x_i(t)+\sum_{j=1}^{n}a_{ij}(t)f_j(x_j(t-\gamma_{ij}))\\&+\sum_{j=1}^{n}\sum_{l=1}^{n}b_{ijl}(t)\int_0^{\infty}k_{ij}(\theta)k_{ij}(\theta)g_j(x_j(t-\theta))\Delta\theta\int_0^{\infty}k_{il}(\theta)g_l(x_l(t-\theta))\Delta\theta\\&\qquad+I_i(t),\quad i,j,l=1,2,\ldots n.\end{aligned}$$

For the system (1.1), finding the periodic solutions is equivalent to finding those of the following boundary-value problem:

$$\begin{cases}x_i^{\Delta}(t)=-c_i(t)x_i(t)+\sum_{j=1}^{n}a_{ij}(t)f_j(x_j(t-\gamma_{ij}))\\+\sum_{j=1}^{n}\sum_{l=1}^{n}b_{ijl}(t)\int_0^{\infty}k_{ij}(\theta)g_j(x_j(t-\theta))\Delta\theta\int_0^{\infty}k_{il}(\theta)g_l(x_l(t-\theta))\Delta\theta\\+I_i(t),\\t\in[0,\omega]_{\mathbb{T}},\quad t\neq t_k,\quad k=1,2,\ldots,q,\quad i=1,2,\ldots,n,\\\Delta x_i(t_k)=x_i(t_k^{+})-x_i(t_k^{-})=e_{ik}(x_i(t_k)),\\x_i(0)=x_i(\omega),\quad i=1,2,\ldots,n.\end{cases}\tag{2.2}$$

Now, we restate Mawhins Continuous Theorem.

Theorem 2.1. *Let $\mathbb{X}$ and $\mathbb{Z}$ be two Banach spaces and L be a Fredholm mapping of index zero. $\mathbf{\Omega}\subset\mathbb{X}$ be an open bounded set and let $N:\overline{\mathbf{\Omega}}\to\mathbb{Z}$ be a continuous operator which is L-compact on $\overline{\mathbf{\Omega}}$. Assume:*

(1) *for each* $\lambda\in(0,1), x\in\partial\mathbf{\Omega}\cap\mathrm{Dom}\ L, Lx\neq\lambda Nx$,

(2) *for each* $x\in\partial\mathbf{\Omega}\cap\mathrm{Ker}\ L, QNx\neq 0$;

(3) $\deg(JNQx,\mathbf{\Omega}\cap\mathrm{Ker}\ L,0)\neq 0$, *where* $JQN:\mathrm{Ker}\,L\longrightarrow\mathrm{Ker}\,L$.

Then $Lx=Nx$ *has at least one solution in* $\overline{\mathbf{\Omega}}\cap\mathrm{Dom}\ L$.

In order to apply Theorem 2.1 to system (2.1), we first take

$$\mathbb{X}=\{x\in C[0,\omega,t_1,t_2,\ldots,t_q]_{\mathbb{T}}:x(t+\omega)=x(t),\,t\in\mathbb{T}\},\ \mathbb{Z}=\mathbb{X}\times\mathbb{R}^{n\times(q+1)}$$

and

$$||x||=||(x_1,x_2,\ldots,x_n)^T||=\sum_{i=1}^{n}\max_{t\in[0,\omega]_{\mathbb{T}}}|x_i(t)|$$

for any $x \in \mathbb{X}$. Then $\mathbb{X}$ is Banach spaces with the norm $||\cdot||$. Set

$$L : \operatorname{Dom} L \cap \mathbb{X} \to \mathbb{Z},\ x \to (x^{\Delta}, \Delta x(t_1), \dots, \Delta x(t_q), 0), \tag{2.3}$$

where $\operatorname{Dom} L = \{x \in C^1[0, \omega, t_1, t_2, \dots, t_q]_{\mathbb{T}}\}$, and $N : \mathbb{X} \to \mathbb{Z}$,

$$Nx = \left(\begin{pmatrix} A_1(t) \\ \vdots \\ A_n(t) \end{pmatrix}, \begin{pmatrix} \Delta x_1(t_1) \\ \vdots \\ \Delta x_n(t_1) \end{pmatrix}, \begin{pmatrix} \Delta x_1(t_2) \\ \vdots \\ \Delta x_n(t_2) \end{pmatrix}, \dots, \begin{pmatrix} \Delta x_1(t_q) \\ \vdots \\ \Delta x_n(t_q) \end{pmatrix}, \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} \right). \tag{2.4}$$

Obviously

$$\operatorname{Ker} L = \{x \in \mathbb{X} : x = h \in \mathbb{R}^n\},$$

$$\operatorname{Im} L = \Big\{(f, C_1, C_2, \dots, C_q, d) \in \mathbb{Z} : \int_0^{\omega} f(s)\Delta s + \sum_{k=1}^{q} C_k + d = 0\Big\} = \mathbb{X} \times \mathbb{R}^{nq} \times \{0\},$$

and $\dim \operatorname{Ker} L = \operatorname{codim} \operatorname{Im} L = n$. Define two projectors

$$Px = \frac{1}{\omega} \int_0^{\omega} x(t)\Delta t,\ x \in \mathbb{X},$$

$$Qz = Q(f, C_1, C_2, \dots, C_q, d) = \left(\frac{1}{\omega} \Big[\int_0^{\omega} f(s)\Delta s + \sum_{k=1}^{q} C_k + d \Big], 0, \dots, 0, 0 \right),$$
$$z \in \mathbb{Z}.$$

It is not difficult to show that P and Q are continuous and satisfy $\operatorname{Im} P = \operatorname{Ker} L$, $\operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I - Q)$. It is easy to see that $\operatorname{Im} L$ is closed in $\mathbb{Z}$, which leads to the following lemma.

Lemma 2.10. *Let L and N be defined by (2.3) and (2.4); then L is a Fredholm operator of index zero.*

Lemma 2.11. *Let L and N be defined by (2.3) and (2.4), respectively, suppose that Ω is an open bounded subset of* $\operatorname{Dom} L$, *then N is L-compact on $\overline{\Omega}$.*

Proof. First, we can find that the inverse $K_P : \operatorname{Im} L \to \operatorname{Ker} P \cap \operatorname{Dom} L$ of L_P has the form

$$(K_P z)(t) = \int_0^t f(s)\Delta s - \frac{1}{\omega} \int_0^{\omega} \int_0^t f(s)\Delta s \Delta t + \sum_{t > t_k} C_k - \sum_{k=1}^{q} C_k.$$

Thus, the expression of QNx is

$$\left(\begin{pmatrix} \frac{1}{\omega}\int_0^{\omega} A_1(t)\Delta t + \frac{1}{\omega}\sum\limits_{k=1}^{q} e_{1k}(x_1(t_k)) \\ \vdots \\ \frac{1}{\omega}\int_0^{\omega} A_n(t)\Delta t + \frac{1}{\omega}\sum\limits_{k=1}^{q} e_{nk}(x_n(t_k)) \end{pmatrix}, 0, \ldots, 0, 0\right),$$

and then

$$K_P(I-Q)Nx = \begin{pmatrix} \displaystyle\int_{\overline{k}}^{t} A_1(s)\Delta s + \sum_{t>t_k} e_{1k}(x_1(t_k)) \\ \vdots \\ \displaystyle\int_{\overline{k}}^{t} A_n(s)\Delta s + \sum_{t>t_k} e_{nk}(x_n(t_k)) \end{pmatrix}$$

$$- \begin{pmatrix} \displaystyle\frac{1}{\omega}\int_0^{\omega}\int_0^{t} A_1(s)\Delta s\Delta t + (\frac{t}{\omega}-\frac{1}{2})\int_0^{\omega} A_1 s\Delta s + \sum_{k=1}^{q} e_{1k}(x_1(t_k)) \\ \vdots \\ \displaystyle\frac{1}{\omega}\int_0^{\omega}\int_0^{t} A_n(s)\Delta s\Delta t + (\frac{t}{\omega}-\frac{1}{2})\int_0^{\omega} A_n s\Delta s + \sum_{k=1}^{q} e_{nk}(x_n(t_k)) \end{pmatrix}.$$

Clearly, QN and $K_P(I-Q)N$ are both continuous. Using Lemma 2.7, it is easy to show that $QN(\overline{\Omega})$, $K_P(I-Q)N(\overline{\Omega})$ are compact for any open bounded set $\Omega \subset \mathbb{X}$. Therefore, N is L-compact on $\overline{\Omega}$ for any open bounded set $\Omega \subset \mathbb{X}$. □

3. Existence of Periodic Solution

In this section, we study the existence of periodic solution of (1.1) based on Mawhins continuation theorem.

Theorem 3.1. *Assume that* $(H_1), (H_2)$ *and* $(H_4), (H_5)$ *hold, then system (1.1) has at least one* ω*-periodic solution.*

Proof. Based on Lemma 2.10 and Lemma 2.11, now, what we need to do is just to search for an appropriate open, bounded subset Ω for the application of the continuation theorem. Corresponding to the operator equation $Lx =$

λNx, $\lambda \in (0,1)$, we have

$$
\begin{cases}
x_i^{\Delta}(t) = \lambda\Big[- c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t-\gamma_{ij})) + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t) \\
\times \int_0^{\infty} k_{ij}(\theta)g_j(x_j(t-\theta))\Delta\theta \int_0^{\infty} k_{il}(\theta)g_l(x_l(t-\theta))\Delta\theta + I_i(t)\Big], \\
t \in [0,\omega]_{\mathbb{T}}, \quad t \neq t_k, \quad i = 1,2,\dots,n, \\
\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-) = \lambda e_{ik}(x_i(t_k)), \quad k = 1,2,\dots, \\
x_i(0) = x_i(\omega), \quad i = 1,2,\dots,n.
\end{cases} \tag{3.1}
$$

For the sake of convenience, defined $||x||_2$ by

$$||x||_2 = \left(\int_0^{\omega} |x(t)|^2 \Delta t \right)^{\frac{1}{2}}, \tag{3.2}$$

for $x \in C(\mathbb{T},\mathbb{R})$. Suppose that $(x_1(t), x_2(t), \dots, x_n(t))^T \in \mathbb{X}$ is a solution of system (3.1) for a certain $\lambda \in (0,1)$. Integrating (3.1) over $[0,\omega]_{\mathbb{T}}$, we obtain

$$\int_0^{\omega} A_i(t)\Delta t + \sum_{k=1}^{q} e_{ik}(x_i(t_k)) = 0.$$

Hence

$$
\begin{aligned}
\int_0^{\omega} c_i(s)x_i(s)\Delta s = \int_0^{\omega} \Big[\sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t-\gamma_{ij})) \\
+ \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t) \int_0^{\infty} k_{ij}(\theta)g_j(x_j(t-\theta))\Delta\theta \int_0^{\infty} k_{il}(\theta)g_l(x_l(t-\theta))\Delta\theta + I_i(t)\Big]\Delta t \\
+ \sum_{k=1}^{q} e_{ik}(x_i(t_k)).
\end{aligned} \tag{3.3}
$$

Let ζ_i, $\eta_i(\neq t_k) \in [0,\omega]_{\mathbb{T}}$, $k = 1,2,\dots,q$, such that $x_i(\zeta_i) = \inf\limits_{t\in[0,\omega]_{\mathbb{T}}} x_i(t)$, $x_i(\eta_i) = \sup\limits_{t\in[0,\omega]_{\mathbb{T}}} x_i(t)$, $i = 1,2,\dots,n$. Then by (3.3), and Lemma 2.4, we have

$$
\begin{aligned}
\omega\overline{c_i}x_i(\zeta_i) \leq \int_0^{\omega} \Big[\Big| \sum_{j=1}^{n} a_{ij}(s)f_j(x_j(s-\gamma_{ij})) \\
+ \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(s) \int_0^{\infty} k_{ij}(\theta)g_j(x_j(s-\theta))\Delta\theta \int_0^{\infty} g_l(x_l(s-\theta))\Delta\theta
\end{aligned}
$$

$$+ I_i(s)\Big|\Big]\Delta s + \sum_{k=1}^{q} |e_{ik}(x_i(t_k))|$$

$$\leq \sum_{j=1}^{n} \int_0^{\omega} [|a_{ij}(s)||f_j(x_j(s-\gamma_{ij}))]\Delta s$$

$$+\sum_{j=1}^{n}\sum_{l=1}^{n}\int_0^{\omega}\Big[|b_{ijl}(s)|\int_0^{\infty}k_{ij}(\theta)|g_j(x_j(s-\theta))|\Delta\theta\int_0^{\infty}k_{il}(\theta)|g_l(x_l(s-\theta))|\Delta\theta\Big]\Delta s$$

$$+ \sum_{k=1}^{q} E_{ik} + \int_0^{\omega} |I_i(s)|\Delta s$$

$$\leq \omega\Big[\sum_{j=1}^{n} a_{ij}^{+}M_j + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+}N_jN_l + I_i^{+} + \frac{q}{\omega}E_i\Big], \quad i = 1, 2, \ldots, n. \tag{3.4}$$

Hence $x_i(\zeta_i) \leq \frac{1}{\overline{c_i}}\{\big[\sum\limits_{j=1}^{n} a_{ij}^{+}M_j + \sum\limits_{j=1}^{n}\sum\limits_{l=1}^{n} b_{ijl}^{+}N_jN_l + I_i^{+} + \frac{q}{\omega}E_i\big]\} := B_i$, $i = 1, 2, \ldots, n$. By (3.3), and Lemma 2.4, we can also have

$$\omega\overline{c_i}x_i(\eta_i) \geq -\int_0^{\omega}\Big[\Big|\sum_{j=1}^{n} a_{ij}(s)f_j(x_j(s-\gamma_{ij}))$$

$$+\sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(s)\int_0^{\infty}k_{ij}(\theta)g_j(x_j(s-\theta))\Delta\theta\int_0^{\infty}k_{il}(\theta)g_l(x_l(s-\theta))\Delta\theta$$

$$+ I_i(s)\Big|\Big]\Delta s - \sum_{k=1}^{q} |e_{ik}(x_i(t_k))|$$

$$\geq -\sum_{j=1}^{n} \int_0^{\omega} [|a_{ij}(s)||f_j(x_j(s-\gamma_{ij}))]\Delta s$$

$$-\sum_{j=1}^{n}\sum_{l=1}^{n}\int_0^{\omega}\Big[|b_{ijl}(s)||\int_0^{\infty}k_{ij}(\theta)|g_j(x_j(s-\theta))|\Delta\theta\int_0^{\infty}k_{il}(\theta)|g_l(x_l(s-\theta))|\Delta\theta\Big]\Delta s$$

$$- \sum_{k=1}^{q} E_{ik} - \int_0^{\omega} |I_i(s)|\Delta s$$

$$\geq -\omega\Big[\sum_{j=1}^{n} a_{ij}^{+}M_j + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+}N_jN_l + I_i^{+} + \frac{q}{\omega}E_i\Big], \quad i = 1, 2, \ldots, n. \tag{3.5}$$

Hence $x_i(\eta_i) \geq -\frac{1}{\overline{c_i}}\{\big[\sum\limits_{j=1}^{n} a_{ij}^{+}M_j + \sum\limits_{j=1}^{n}\sum\limits_{l=1}^{n} b_{ijl}^{+}N_jN_l + I_i^{+} + \frac{q}{\omega}E_i\big]\} = -B_i$, $i =$

$1, 2, \ldots, n$. From (3.1), (3.3) and Lemma 2.6, we have

$$\int_0^\omega |x_i^\Delta(t)|\Delta t \le \int_0^\omega |c_i(t)||x_i(t)|\Delta t + \sum_{j=1}^n \int_0^\omega [|a_{ij}(t)||f_j(x_j(t-\gamma_{ij}))]\Delta t$$
$$+\sum_{j=1}^n\sum_{l=1}^n \int_0^\omega \left[|b_{ijl}(t)|\int_0^\infty k_{ij}(\theta)|g_j(x_j(t-\theta))|\Delta\theta\int_0^\infty k_{il}(\theta)|g_l(x_l(t-\theta))|\Delta\theta\right]\Delta t$$
$$+\int_0^\omega |I_i(t)|\Delta t + \sum_{k=1}^q |e_{ik}(x_i(t_k))|$$
$$\le \left(\int_0^\omega |c_i(t)|^2\Delta t\right)^{\frac{1}{2}}\left(\int_0^\omega |x_i(t)|^2\Delta t\right)^{\frac{1}{2}} + \sum_{j=1}^n \left(\int_0^\omega |a_{ij}(t)|^2\Delta t\right)^{\frac{1}{2}}\left(\int_0^\omega M_j^2\Delta t\right)^{\frac{1}{2}}$$
$$+\sum_{j=1}^n\sum_{l=1}^n b_{ijl}^+\left(\int_0^\omega N_j^2\Delta t\right)^{\frac{1}{2}}\left(\int_0^\omega N_l^2\Delta t\right)^{\frac{1}{2}}$$
$$+ I_i^+\omega + qE_i$$
$$\le (\omega)^{\frac{1}{2}}c_i^+||x_i||_2 + \sum_{j=1}^n \omega a_{ij}^+ M_j + \sum_{j=1}^n\sum_{l=1}^n (\omega) b_{ijl}^+ N_j N_l + I_i^+\omega + qE_i,\ i = 1, 2, \ldots, n. \tag{3.6}$$

Integrating (3.1) from 0 to t, we can obtain

$$x_i(t) = e_{-\lambda c_i(t)}(t, 0)x_i(0) + \int_0^t \lambda e_{-\lambda c_i(t)}(t, \sigma(s))\left[\sum_{j=1}^n a_{ij}(s)f_j(x_j(s-\gamma_{ij}))\right.$$
$$\left.+\sum_{j=1}^n\sum_{l=1}^n b_{ijl}(s)\int_0^\infty k_{ij}(\theta)g_j(x_j(s-\theta))\Delta\theta\int_0^\infty k_{il}(\theta)g_l(x_l(s-\theta))\Delta\theta + I_i(s)\right]\Delta s$$
$$+\lambda\sum_{0<t_k<t} e_{ik}(x_i(t_k)), \quad i = 1, 2, \ldots, n.$$

So

$$|x_i(t)| \le |x_i(0)| + \sum_{j=1}^n \int_0^\omega [|a_{ij}(s)||f_j(x_j(s-\gamma_{ij}))|]\Delta s$$
$$+\sum_{j=1}^n\sum_{l=1}^n \int_0^\omega \left[|b_{ijl}(s)|\int_0^\infty k_{ij}(\theta)|g_j(x_j(s-\theta))|\Delta\theta\int_0^\infty k_{il}(\theta)|g_l(x_l(s-\theta))|\Delta\theta\right]\Delta s$$
$$+\int_0^\omega |I_i(s)|\Delta s + qE_i$$

$$\leq |x_i(0)| + \sum_{j=1}^{n} \overline{a_{ij}} M_j \omega + \sum_{j=1}^{n}\sum_{l=1}^{n} \overline{b_{ijl}} N_j N_l \omega + \overline{I_i}\omega + qE_i$$

$$=: u_i, \quad i = 1, 2, \ldots, n,$$

that is,

$$||x_i||_2 = \left(\int_0^{\omega} |x_i(t)|^2 \Delta t \right)^{\frac{1}{2}} \leq u_i(\omega)^{\frac{1}{2}},\ i = 1, 2, \ldots, n. \tag{3.7}$$

Substituting (3.7) into (3.6), we have

$$\int_0^{\omega} |x_i^{\Delta}(t)| \Delta t \leq \omega c_i^{+} u_i + \sum_{j=1}^{n} \omega a_{ij}^{+} M_j + \sum_{j=1}^{n}\sum_{l=1}^{n} (\omega) b_{ijl}^{+} N_j N_l + I_i^{+}\omega + qE_i,$$

$$i = 1, 2, \ldots, n. \tag{3.8}$$

From Lemma 2.5, we have

$$x_i(t) \leq x_i(\zeta_i) + \int_0^{\omega} |x_i^{\Delta}(t)| \Delta t, \quad i = 1, 2, \ldots, n.$$

$$x_i(t) \geq x_i(\eta_i) - \int_0^{\omega} |x_i^{\Delta}(t)| \Delta t, \quad i = 1, 2, \ldots, n.$$

From (3.4), (3.5), (3.8), and the two above inequalities, there exist positive constants $\xi_i (i = 1, 2, \ldots, n)$, such that for $t \in [0, \omega]_{\mathbb{T}}$,

$$|x_i(t)| \leq \xi_i,\ i = 1, 2, \ldots, n.$$

Clearly, $\xi_i (i = 1, 2, \ldots, n)$ is independent of λ. Denote $H^* = \sum_{i=1}^{n} \xi_i + C$, where $C > 0$ is taken sufficiently large so that

$$\min_{1 \leq i \leq n} \overline{c_i} H^* > n \max_{1 \leq i \leq n} \left(|\overline{I_i}| + \sum_{j=1}^{n} |\overline{a_{ij}}| M_j + \sum_{j=1}^{n}\sum_{l=1}^{n} |\overline{b_{ijl}}| N_j N_l + \frac{q}{\omega} E_i \right).$$

Now we take

$$\Omega = \{(x_1(t), x_2(t), \ldots, x_n(t))^T : ||(x_1(t), x_2(t), \ldots, x_n(t))^T|| < H^*\}.$$

Thus (1) of Theorem 2.1 is satisfied. When

$$(x_1(t), x_2(t), \ldots, x_n(t))^T \in \partial\Omega \bigcap \mathbb{R}^n,\ (x_1(t), x_2(t), \ldots, x_n(t))^T$$

is a constant vector in $\mathbb{R}^n$ with $|x_1| + |x_2| + \ldots + |x_n| = H^*$. Then

$$QN(x_1(t), x_2(t), \ldots, x_n(t))^T = \bigg(- \overline{c_i} x_i(t) + \sum_{j=1}^{n} \overline{a_{ij}} f_j(x_j(t - \gamma_{ij}))$$

$$+\sum_{j=1}^{n}\sum_{l=1}^{n}\overline{b_{ijl}}\int_{0}^{\infty}k_{ij}(\theta)g_j(x_j(t-\theta))\Delta\theta\int_{0}^{\infty}k_{il}(\theta)g_l(x_l(t-\theta))\Delta\theta$$
$$\left.+\overline{I_i}+\frac{1}{\omega}\sum_{k=1}^{q}e_{ik}(x_i(t_k))\right)_{n\times 1}.$$

Therefore,

$$\begin{aligned}||QN(x_1(t),x_2(t),\dots,x_n(t))^T|| &= \sum_{i=1}^{n}\Bigg|\overline{c_i}x_i(t)-\sum_{j=1}^{n}\overline{a_{ij}}f_j(x_j(t-\gamma_{ij}))\\ &-\sum_{j=1}^{n}\sum_{l=1}^{n}\overline{b_{ijl}}\int_{0}^{\infty}k_{ij}(\theta)g_j(x_j(t-\theta))\Delta\theta\int_{0}^{\infty}k_{il}(\theta)g_l(x_l(t-\theta))\Delta\theta\\ &-\overline{I_i}-\frac{1}{\omega}\sum_{k=1}^{q}e_{ik}(x_i(t_k))\Bigg|\\ &\geq \sum_{i=1}^{n}\overline{c_i}|x_i(t)|-\sum_{i=1}^{n}\sum_{j=1}^{n}|\overline{a_{ij}}|M_j-\frac{q}{\omega}\sum_{i=1}^{n}E_i-\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n}\overline{b_{ijl}}N_jN_l-\sum_{i=1}^{n}|\overline{I_i}|\\ &\geq \sum_{i=1}^{n}\left(\overline{c_i}|x_i(t)|\right)-\sum_{i=1}^{n}\left(|\overline{I_i}|+\sum_{j=1}^{n}|\overline{a_{ij}}|M_j+\sum_{j=1}^{n}\sum_{l=1}^{n}\overline{b_{ijl}}N_jN_l+\frac{q}{\omega}E_i\right)\\ &\geq \min_{1\leq i\leq n}\overline{c_i}\sum_{i=1}^{n}|x_i(t)|-n\max_{1\leq i\leq n}\Bigg(|\overline{I_i}|+\sum_{j=1}^{n}|\overline{a_{ij}}|M_j\\ &+\sum_{j=1}^{n}\sum_{l=1}^{n}|\overline{b_{ijl}}|N_jN_l+\frac{q}{\omega}E_i\Bigg)>0.\end{aligned}$$

Consequently,

$$QN(x_1(t),x_2(t),\dots,x_n(t))^T\neq(0,0,\dots,0)^T$$

for

$$(x_1(t),x_2(t),\dots,x_n(t))^T\in\partial\Omega\cap\operatorname{Ker}L.$$

This satisfies condition (2) of Theorem 2.1. Define $\Psi:\operatorname{Ker}L\times[0,1]\to\mathbb{X}$ by $\Psi(x_1,x_2,\dots,x_n,\mu)=-\mu(x_1,x_2,\dots,x_n)^T+(1-\mu)QN(x_1(t),x_2(t),\dots,x_n(t))^T$. When $(x_1(t),x_2(t),\dots,x_n(t))^T\in\partial\Omega\cap\operatorname{Ker}L$, $(x_1,x_2,\dots,x_n)^T$ is a constant vector in $\mathbb{R}^n$ with $\sum_{i=1}^{n}|x_i|=H^*$, we easily have $\Psi(x_1,x_2,\dots,x_n,\mu)\neq(0,0,\dots,0)^T$. Therefore

$$\deg\left(QN(x_1(t),x_2(t),\dots,x_n(t))^T,\Omega\cap\operatorname{Ker}L,(0,0,\dots,0)^T\right)$$

$$= \deg\big((-x_1(t), -x_2(t), \dots, -x_n(t))^T, \Omega \cap \operatorname{Ker} L, (0,0,\dots,0)^T\big) \neq 0.$$

Condition (3) of Theorem 2.1 is also satisfied. Thus, by Theorem 2.1 we can obtain that $Lx = Nx$ has at least one solution in $\mathbb{X}$. That is, system (1.2) has at least one ω-periodic solution. The proof is complete. □

4. Global Exponential Stability of Periodic Solution

In this section, we will construct suitable Lyapunov functions to study the global exponential asymptotic stability of the periodic solution of (1.1). So first we shall introduce the following definition.

Definition 4.1. The periodic solution $x^*(t)$ of system (1.1) is said to be exponentially stable if there exists a positive constant α such that for every $\delta \in \mathbb{T}$, there exists $N = N(\delta) \geq 1$ such that the solution of (1.1) satisfies

$$||x(t) - x^*(t)|| \leq N||\varphi - x^*||e_{\ominus\alpha}(t, \delta),\ t \in \mathbb{T}^+,$$

where

$$||\varphi - x^*|| = \sum_{i=1}^{n} \max_{\delta \in (-\infty, 0]_{\mathbb{T}}} |\varphi_i(\delta) - x_i^*(\delta)|.$$

Theorem 4.1. *Assume that* (H_1)*-*(H_5) *hold. Suppose further that*

(H_6) $c_i^- - L_i \sum_{j=1}^{n} a_{ji}^+ - H_i \sum_{j=1}^{n}\sum_{l=1}^{n}[b_{jil}^+ N_l + b_{lji}^+ N_j] > 0,\ i = 1, 2, \dots, n.$

(H_7) *Impulsive operators* $e_{ik}(x_i(t))$ *satisfy*

$$e_{ik}(x_i(t_k)) = -\tau_{ik} x_i(t_k),\ 0 < \tau_{ik} < 2,\ i = 1, 2, \dots, n,\ k \in \mathbb{Z}^+.$$

Then the ω*-periodic solution of system (1.1) is globally exponentially stable.*

Proof. According to Theorem 3.1, we know that (1.1) has an ω-periodic solution $x^*(t) = \big(x_1^*(t), x_2^*(t), \dots, x_n^*(t)\big)^T$. Suppose that $x(t) = \big(x_1(t), x_2(t), \dots, x_n(t)\big)^T$ is an arbitrary solution of (1.1). Then it follows from system (1.1) that

$$\begin{aligned}
&\big(x_i(t) - x_i^*(t)\big)^\Delta \\
&\quad = -c_i(t)(x_i(t) - x_i^*(t)) + \sum_{j=1}^{n} a_{ij}(t)\Big(f_j(x_j(t-\gamma_{ij})) - f_j(x_j^*(t-\gamma_{ij}))\Big) \\
&\qquad + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t)\Big[\int_0^\infty k_{ij}(\theta) g_j(x_j(t-\theta))\Delta\theta \int_0^\infty k_{il}(\theta) g_l(x_l(t-\theta))\Delta\theta
\end{aligned}$$

$$- \int_0^\infty k_{ij}(\theta) g_j(x_j^*(t-\theta))\Delta\theta \int_0^\infty k_{il}(\theta) g_l(x_l^*(t-\theta))\Delta\theta \Big], \quad (4.1)$$

for $i = 1, 2, \ldots, n$, the initial condition of (4.1) is

$$\psi_i(s) = \varphi_i(s) - x_i^*(s), s \in (-\infty, 0]_{\mathbb{T}}, i = 1, 2, \ldots, n.$$

Therefore from (4.1) we have

$$\begin{aligned} D^+|x_i(t) - x_i^*(t)|^\Delta &= \text{sign}\,(x_i(t) - x_i^*(t))\big((x_i(t))^\Delta - (x_i^*(t))^\Delta\big) \\ &= \text{sign}\,(x_i(t) - x_i^*(t))\Big\{ - c_i(t)(x_i(t) - x_i^*(t)) \\ &+ \sum_{j=1}^n a_{ij}(t)\Big(f_j(x_j(t-\gamma_{ij})) - f_j(x_j^*(t-\gamma_{ij}))\Big) \\ &+ \sum_{j=1}^n \sum_{l=1}^n b_{ijl}(t)\Big[\int_0^\infty k_{ij}(\theta) g_j(x_j(t-\theta))\Delta\theta \int_0^\infty k_{il}(\theta) g_l(x_l(t-\theta))\Delta\theta \\ &- \int_0^\infty k_{ij}(\theta) g_j(x_j^*(t-\theta))\Delta\theta \int_0^\infty k_{il}(\theta) g_l(x_l^*(t-\theta))\Delta\theta \Big]\Big\} \\ &\le -c_i^-|x_i(t) - x_i^*(t)| + \sum_{j=1}^n a_{ij}^+ L_j |x_j(t-\gamma_{ij}) - x_j^*(t-\gamma_{ij})| \\ &+ \sum_{j=1}^n \sum_{l=1}^n b_{ijl}^+ \Big[H_j N_l \int_0^\infty k_{ij}(\theta)|x_j(t-\theta) - x_j^*(t-\theta)|\Delta\theta \\ &+ H_l N_j \int_0^\infty k_{il}(\theta)|x_l(t-\theta) - x_l^*(t-\theta)|\Delta\theta \Big], \quad i = 1, 2, \ldots, n. \quad (4.2) \end{aligned}$$

Also, for $i = 1, 2, \ldots, n,\ k \in \mathbb{Z}^+$,

$$\begin{aligned} x_i(t_k^+) - x_i^*(t_k^+)| &= x_i(t_k) + e_{ik}(x_i(t_k)) - x_i^*(t_k) - e_{ik}(x_i^*(t_k)) \\ &= (1 - \tau_{ik})(x_i(t_k) - x_i^*(t_k)). \end{aligned}$$

Hence

$$|x_i(t_k^+) - x_i^*(t_k^+)| = |1 - \tau_{ik}||x_i(t_k) - x_i^*(t_k)|,\ i = 1, 2, \ldots, n,\ k \in \mathbb{Z}^+.$$

Let F_i be defined by

$$\begin{aligned} F_i(\varepsilon_i) &= c_i^- - \varepsilon_i - L_i \sum_{j=1}^n a_{ji}^+ e_{\varepsilon_j}(\omega, -\gamma) \\ & -H_i \sum_{j=1}^n \sum_{l=1}^n \Big[b_{jil}^+ N_l \sum_{m=1}^\infty e_{\varepsilon_j}(0, -m\omega) \int_{(m-1)\omega}^{m\omega} k_{ji}(\theta)\Delta\theta \end{aligned}$$

$$+b_{lji}^{+}N_j\sum_{m=1}^{\infty}e_{\varepsilon_l}(0,-m\omega)\int_{(m-1)\omega}^{m\omega}k_{li}(\theta)\Delta\theta\Big],$$

where $\varepsilon_i\in[0,\infty),\ i=1,2,\dots,n$. It is clear that

$$F_i(0)=c_i^{-}-L_i\sum_{j=1}^{n}a_{ji}^{+}-H_i\sum_{j=1}^{n}\sum_{l=1}^{n}[b_{jil}^{+}N_l+b_{lji}^{+}N_j]>0,\ i=1,2,\dots,n.$$

Since F_i are continuous on $[0,\infty)$ and $F_i(\varepsilon_i)\to-\infty$, as $\varepsilon_i\to+\infty$, there exist $\varepsilon_i^*>0$ such that $F_i(\varepsilon_i^*)=0$ and $F_i(\varepsilon_i)>0$, for $\varepsilon_i\in(0,\varepsilon_i^*)$. By choosing $\varepsilon=\min\big\{\min_{1\le i\le n}\{\varepsilon_i\},\frac{\alpha_0}{2}\big\}$ we have

$$\begin{aligned}F_i(\varepsilon)&=c_i^{-}-\varepsilon-L_i\sum_{j=1}^{n}a_{ji}^{+}e_\varepsilon(\omega,-\gamma)\\&\quad-H_i\sum_{j=1}^{n}\sum_{l=1}^{n}\Big[b_{jil}^{+}N_l\sum_{m=1}^{\infty}e_\varepsilon(0,-m\omega)\int_{(m-1)\omega}^{m\omega}k_{ji}(\theta)\Delta\theta\\&\quad+b_{lji}^{+}N_j\sum_{m=1}^{\infty}e_\varepsilon(0,-m\omega)\int_{(m-1)\omega}^{m\omega}k_{li}(\theta)\Delta\theta\Big]\ge0,\end{aligned}$$

where $i=1,2,\dots,n$. Now we define

$$h_i(t)=e_\varepsilon(t,\delta)|x_i(t)-x_i^*(t)|,\ t\in\mathbb{T},\ i=1,2,\dots,n,\tag{4.3}$$

where $\delta\in(-\infty,0]_{\mathbb{T}}$. For $t>0,\ t\ne t_k,\ k\in\mathbb{Z}^+,\ i=1,2,\dots,n$, notice (4.2)-(4.3) and by using Lemma 2.9, we have

$$\begin{aligned}D^+h_i^{\Delta}(t)&\le\varepsilon e_\varepsilon(t,\delta)|x_i(t)-x_i^*(t)|\\&\quad+e_\varepsilon(\sigma(t),\delta)\bigg(-c_i^{-}|x_i(t)-x_i^*(t)|+\sum_{j=1}^{n}a_{ij}^{+}L_j|x_j(t-\gamma_{ij})-x_j^*(t-\gamma_{ij})|\\&\quad+\sum_{j=1}^{n}\sum_{l=1}^{n}b_{ijl}^{+}\Big[H_jN_l\int_0^{\infty}k_{ij}(\theta)|x_j(t-\theta)-x_j^*(t-\theta)|\Delta\theta\\&\quad+H_lN_j\int_0^{\infty}k_{il}(\theta)|x_l(t-\theta)-x_l^*(t-\theta)|\Delta\theta\Big]\bigg)\\&\le[1+\mu(t)\varepsilon]\bigg\{-(c_i^{-}-\varepsilon)h_i(t)+\sum_{j=1}^{n}a_{ij}^{+}L_je_\varepsilon(t,t-\gamma_{ij})h_j(t-\gamma_{ij})\\&\quad+\sum_{j=1}^{n}\sum_{l=1}^{n}b_{ijl}^{+}\Big[H_jN_l\int_0^{\infty}k_{ij}(\theta)e_\varepsilon(t,t-\theta)h_j(t-\theta)\Delta\theta\end{aligned}$$

$$+ H_l N_j \int_0^{\infty} k_{il}(\theta) e_\varepsilon(t, t-\theta) h_l(t-\theta)\Delta\theta\Big]\Big\}$$

$$\le [1+\mu(t)\varepsilon]\Big\{ -(c_i^- - \varepsilon) h_i(t) + \sum_{j=1}^n a_{ij}^+ L_j e_\varepsilon(t, t-\gamma_{ij}) h_j(t-\gamma_{ij})$$

$$+ \sum_{j=1}^n \sum_{l=1}^n b_{ijl}^+ \Big[H_j N_l \sum_{m=1}^{\infty} \int_{(m-1)\omega}^{m\omega} k_{ij}(\theta) e_\varepsilon(t, t-m\omega) h_j(t-\theta)\Delta\theta$$

$$+ H_l N_j \sum_{m=1}^{\infty} \int_{(m-1)\omega}^{m\omega} k_{il}(\theta) e_\varepsilon(t, t-m\omega) h_l(t-\theta)\Delta\theta\Big]\Big\}$$

$$\le [1+\mu(t)\varepsilon]\Big\{ -(c_i^- - \varepsilon) h_i(t) + \sum_{j=1}^n a_{ij}^+ L_j e_\varepsilon(\omega, -\gamma) h_j(t-\gamma_{ij})$$

$$+ \sum_{j=1}^n \sum_{l=1}^n b_{ijl}^+ \Big[H_j N_l \sum_{m=1}^{\infty} e_\varepsilon(0, -m\omega) \int_{(m-1)\omega}^{m\omega} k_{ij}(\theta) h_j(t-\theta)\Delta\theta$$

$$+ H_l N_j \sum_{m=1}^{\infty} e_\varepsilon(0, -m\omega) \int_{(m-1)\omega}^{m\omega} k_{il}(\theta) h_l(t-\theta)\Delta\theta\Big]\Big\}$$

$$\le [1+\mu\varepsilon]\Big\{ -(c_i^- - \varepsilon) h_i(t) + \sum_{j=1}^n a_{ij}^+ L_j e_\varepsilon(\omega, -\gamma) h_j(t-\gamma_{ij})$$

$$+ \sum_{j=1}^n \sum_{l=1}^n b_{ijl}^+ \Big[H_j N_l \sum_{m=1}^{\infty} e_\varepsilon(0, -m\omega) \int_{(m-1)\omega}^{m\omega} k_{ij}(\theta) h_j(t-\theta)\Delta\theta$$

$$+ H_l N_j \sum_{m=1}^{\infty} e_\varepsilon(0, -m\omega) \int_{(m-1)\omega}^{m\omega} k_{il}(\theta) h_l(t-\theta)\Delta\theta\Big]\Big\}.$$

Also,

$$h_i(t_k^+) = |1-\tau_{ik}| h_i(t_k) \le h_i(t_k), \quad i = 1, 2, \ldots, n, \quad k \in \mathbb{Z}^+.$$

Consider the following Lyapunov function

$$G(t) = \sum_{i=1}^n \Big(h_i(t) + [1+\mu\varepsilon] \sum_{j=1}^n a_{ij}^+ L_j e_\varepsilon(\omega, -\gamma) \int_{t-\gamma_{ij}}^t h_j(s)\Delta s$$

$$+ [1+\mu\varepsilon] \sum_{j=1}^n \sum_{l=1}^n b_{ijl}^+ \big[H_j N_l \sum_{m=1}^{\infty} e_\varepsilon(0, -\omega) \int_{(m-1)\omega}^{m\omega} k_{ij}(\theta) \big(\int_{t-\theta}^t h_j(s)\Delta s\big)\Delta\theta$$

$$+ H_l N_j \sum_{m=1}^{\infty} e_\varepsilon(0, -m\omega) \int_{(m-1)\omega}^{m\omega} k_{il}(\theta) \big(\int_{t-\theta}^t h_l(s)\Delta s\big)\Delta\theta\big]\Big), \quad (4.4)$$

and we note that $G(0) > 0$ for $t > 0$ and $G(0)$ is positive and finite. Calculating the Δ-derivative of G along solutions of (4.4) we get

$$\begin{aligned} D^+G^{\Delta}(t) \;\leq\;& [1+\mu\varepsilon]\sum_{i=1}^{n}\Big\{-(c_i^- - \varepsilon)h_i(t) + \sum_{j=1}^{n} a_{ij}^+ L_j e_\varepsilon(\omega,-\gamma)h_j(t) \\ &+\sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^+\Big[H_j N_l \sum_{m=1}^{\infty} e_\varepsilon(0,-m\omega)\int_{(m-1)\omega}^{m\omega} k_{ij}(\theta)h_j(t)\Delta\theta \\ &+H_l N_j \sum_{m=1}^{\infty} e_\varepsilon(0,-m\omega)\int_{(m-1)\omega}^{m\omega} k_{il}(\theta)h_l(t)\Delta\theta\Big]\Big\} \\ \leq\;& -[1+\mu\varepsilon]\sum_{i=1}^{n}\Big\{c_i^- - \varepsilon - L_i\sum_{j=1}^{n} a_{ji}^+ e_\varepsilon(\omega,-\gamma) \\ &-H_i\sum_{j=1}^{n}\sum_{l=1}^{n}\Big[b_{jil}^+ N_l \sum_{m=1}^{\infty} e_\varepsilon(0,-m\omega)\int_{(m-1)\omega}^{m\omega} k_{ji}(\theta)\Delta\theta \\ &+b_{lji}^+ N_j \sum_{m=1}^{\infty} e_\varepsilon(0,-m\omega)\int_{(m-1)\omega}^{m\omega} k_{li}(\theta)\Delta\theta\Big]\Big\}h_i(t) \\ =\;& -[1+\mu\varepsilon]\sum_{i=1}^{n} F_i(\varepsilon)h_i(t) \leq 0, \quad t>0,\ t\neq t_k,\ k\in\mathbb{Z}^+. \end{aligned}$$

Also,

$$\begin{aligned} G(t_k^+) =& \sum_{i=1}^{n}\Big(h_i(t_k^+) + [1+\mu\varepsilon]\sum_{j=1}^{n} a_{ij}^+ L_j e_\varepsilon(\omega,-\gamma)\int_{t_k^+-\gamma_{ij}}^{t_k^+} h_j(s)\Delta s \\ &+[1+\mu\varepsilon]\sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^+\big[H_j N_l\sum_{m=1}^{\infty} e_\varepsilon(0,-\omega)\int_{(m-1)\omega}^{m\omega} k_{ij}(\theta)\big(\int_{t_k^+-\theta}^{t_k^+} h_j(s)\Delta s\big)\Delta\theta \\ &+H_l N_j\sum_{m=1}^{\infty} e_\varepsilon(0,-m\omega)\int_{(m-1)\omega}^{m\omega} k_{il}(\theta)\big(\int_{t_k^+-\theta}^{t_k^+} h_l(s)\Delta s\big)\Delta\theta\big]\Big) \\ \leq& \sum_{i=1}^{n}\Big(h_i(t_k) + [1+\mu\varepsilon]\sum_{j=1}^{n} a_{ij}^+ L_j e_\varepsilon(\omega,-\gamma)\int_{t_k-\gamma_{ij}} t_k h_j(s)\Delta s \\ &+[1+\mu\varepsilon]\sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^+\big[H_j N_l\sum_{m=1}^{\infty} e_\varepsilon(0,-\omega)\int_{(m-1)\omega}^{m\omega} k_{ij}(\theta)\big(\int_{t_k-\theta}^{t_k} h_j(s)\Delta s\big)\Delta\theta \\ &+H_l N_j\sum_{m=1}^{\infty} e_\varepsilon(0,-m\omega)\int_{(m-1)\omega}^{m\omega} k_{il}(\theta)\big(\int_{t_k-\theta}^{t_k} h_l(s)\Delta s\big)\Delta\theta\big]\Big) = G(t_k), \quad k\in\mathbb{Z}^+. \end{aligned}$$

It follows that $G(t) \leq G(0)$ for $t > 0$ and hence from this and (4.4) we have

$$\begin{aligned}
\sum_{i=1}^{n} h_i(t) &\leq G(t) \leq G(0) \\
&= \sum_{j=1}^{n} \Big(h_i(0) + [1+\mu\varepsilon] \sum_{j=1}^{n} a_{ij}^{+} L_j e_{\varepsilon}(\omega, -\gamma) \int_{-\gamma_{ij}}^{0} h_j(s)\Delta s \\
&+ [1+\mu\varepsilon] \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+} \big[H_j N_l \sum_{m=1}^{\infty} e_{\varepsilon}(0, -m\omega) \int_{(m-1)\omega}^{m\omega} k_{ij}(\theta) \big(\int_{-\theta}^{0} h_j(s)\Delta s \big) \Delta\theta \\
&+ H_l N_j \sum_{m=1}^{\infty} e_{\varepsilon}(0, -m\omega) \int_{(m-1)\omega}^{m\omega} k_{il}(\theta) \big(\int_{-\theta}^{0} h_l(s)\Delta s \big) \Delta\theta \big] \Big),
\end{aligned}$$

for $t > 0$. In view of (4.3) and above inequality, we obtain

$$\begin{aligned}
\sum_{i=1}^{n} |x_i(t) - x_i^*(t)| &\leq e_{\Theta\varepsilon}(t, \delta) \sum_{i=1}^{n} \Big\{ e_{\varepsilon}(0, \delta) + [1+\mu\varepsilon] \sum_{j=1}^{n} a_{ij}^{+} L_j e_{\varepsilon}(\omega, -\gamma)\gamma_{ij} \\
&+ [1+\mu\varepsilon] \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+} \Big[H_j N_l \sum_{m=1}^{\infty} e_{\varepsilon}(0, -m\omega) \int_{(m-1)\omega}^{m\omega} \theta k_{ij}(\theta)\Delta\theta \\
&+ H_l N_j \sum_{m=1}^{\infty} e_{\varepsilon}(0, -m\omega) \int_{(m-1)\omega}^{m\omega} \theta k_{il}(\theta)\Delta\theta \Big] \Big\} \max_{\delta\in(-\infty,0]_{\mathbb{T}}} |\varphi_i(\delta) - x_i^*(\delta)| \\
&\leq N e_{\Theta\varepsilon}(t, \delta) \sum_{i=1}^{n} \max_{\delta\in(-\infty,0]_{\mathbb{T}}} |\varphi_i(\delta) - x_i^*(\delta)|, \ t > 0,
\end{aligned}$$

where

$$\begin{aligned}
N &= \max_{1\leq i\leq n} \Big\{ e_{\varepsilon}(0, \delta) + [1+\mu\varepsilon] \sum_{j=1}^{n} L_j a_{ij}^{+} L_j e_{\varepsilon}(\omega, -\gamma)\gamma_{ij} \\
&+ [1+\mu\varepsilon] \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+} \Big[H_j N_l \sum_{m=1}^{\infty} e_{\varepsilon}(0, -m\omega) \int_{(m-1)\omega}^{m\omega} \theta k_{ij}(\theta)\Delta\theta \\
&+ H_l N_j \sum_{m=1}^{\infty} e_{\varepsilon}(0, -m\omega) \int_{(m-1)\omega}^{m\omega} \theta k_{il}(\theta)\Delta\theta \Big] \Big\} \geq 1.
\end{aligned}$$

From Definition 4.1, the periodic solution of system (1.2) is globally exponentially stable. □

5. An Example

Consider the following neural networks system with impulses

$$\begin{cases} x_i^{\Delta}(t) = -c_i(t)x_i(t) + \sum\limits_{j=1}^{2} a_{ij}(t)f_j(x_j(t-\gamma_{ij})) \\ + \sum\limits_{j=1}^{2}\sum\limits_{l=1}^{2} b_{ijl}(t)\int_0^{\infty} k_{ij}(\theta)g_j(x_j(t-\theta))\Delta\theta \int_0^{\infty} k_{il}(\theta)g_l(x_l(t-\theta))\Delta\theta \\ +I_i(t), \qquad i=1,2,\ t\in\mathbb{T},\ t>0, \\ \Delta x(t_k) = -0.1x(t_k), \end{cases}$$

where $\mathbb{T}$ is a 1-periodic time scale, and

$$f_1(x_1) = \sin\big(\frac{1}{2^{\frac{1}{2}}}x_1\big), \quad f_2(x_2) = \sin\big(\frac{1}{2^{\frac{3}{2}}}x_2\big),$$

$$g_1(x_1) = |\arctan\big(\frac{1}{2^{\frac{1}{2}}}x_1\big)|, \quad g_2(x_2) = |\arctan\big(\frac{1}{2^{\frac{3}{2}}}x_2\big)|.$$

$$a_{11}(t) = 1+\cos(2\pi t), \quad a_{12}(t) = 2+\cos(2\pi t),$$

$$a_{21}(t) = 2+\cos(2\pi t), \quad a_{22}(t) = 3+\cos(2\pi t),$$

$$c_1(t) = 25+5\sin(2\pi t), \quad c_2(t) = 35+16\sin(2\pi t),$$

$$I_1(t) = 1+\sin(\pi t), \quad I_2(t) = 1+\cos(\pi t),$$

$$b_{111}(t) = b_{222}(t) = \frac{1}{4}+\frac{1}{4}\sin(\pi t), \quad b_{112}(t) = b_{212}(t) = \frac{1}{3}+\frac{1}{3}\cos\pi t),$$

$$b_{121}(t) = b_{221}(t) = \frac{1}{5}+\frac{1}{5}\cos(\pi t), \quad b_{122}(t) = b_{211}(t) = \frac{1}{6}+\frac{1}{6}\sin(\pi t),$$

$$(k_{ij}(\theta))_{2\times 2} = \begin{pmatrix} \frac{2}{1+2\mu(0)}e_{\ominus 2}(\theta,0) & \frac{3}{1+3\mu(0)}e_{\ominus 3}(\theta,0) \\ \frac{4}{1+4\mu(0)}e_{\ominus 4}(\theta,0) & \frac{5}{1+5\mu(0)}e_{\ominus 5}(\theta,0) \end{pmatrix},$$

$$\gamma_{11} = \gamma_{12} = \gamma_{21} = \gamma_{22} = 1.$$

By calculating, we have

$\omega = 1$, $c_1^- = 20$, $c_2^- = 19$, $a_{11}^+ = 2$, $a_{12}^+ = 3$, $a_{21}^+ = 3$, $a_{22}^+ = 4$, $b_{111}^+ = b_{222}^+ = \frac{1}{2}$, $b_{112}^+ = b_{212}^+ = \frac{2}{3}$, $b_{121}^+ = b_{221}^+ = \frac{2}{5}$, $b_{122}^+ = b_{211}^+ = \frac{1}{3}$, $L_1 = L_2 = H_1 = H_2 = 1$, $N_1 = N_2 = \frac{\pi}{2}$, $\gamma = 1$. It is not difficult to verify that (H_1)-(H_4) and (H_6)-(H_7) are satisfied.

And by calculating, we also have

$(K_{ij}(\alpha))_{2\times 2} =$

$$\begin{pmatrix} -[e_{\alpha\ominus 2}(\omega,0)-e_\alpha(\omega,0)]\frac{1}{1-e_{\alpha\ominus 2}(\omega,0)} & -[e_{\alpha\ominus 3}(\omega,0)-e_\alpha(\omega,0)]\frac{1}{1-e_{\alpha\ominus 3}(\omega,0)} \\ -[e_{\alpha\ominus 4}(\omega,0)-e_\alpha(\omega,0)]\frac{1}{1-e_{\alpha\ominus 4}(\omega,0)} & -[e_{\alpha\ominus 5}(\omega,0)-e_\alpha(\omega,0)]\frac{1}{1-e_{\alpha\ominus 5}(\omega,0)} \end{pmatrix},$$

is right-dense continuous for $\alpha \in [0,2)$ and $K_{ij}(0) = 1$, $i, j = 1, 2$, and

$$\sum_{m=1}^{\infty} e_\alpha(0,-m\omega)\int_{(m-1)\omega}^{m\omega} k_{11}(\theta)\theta\Delta\theta < \infty,$$

since

$$\begin{aligned} & \sum_{m=1}^{\infty} e_\alpha(0,-m\omega)\int_{(m-1)\omega}^{m\omega} k_{11}(\theta)\theta\Delta\theta \\ \le\ & \sum_{m=1}^{\infty} e_\alpha(0,-m\omega)m\omega\int_{(m-1)\omega}^{m\omega} k_{11}(\theta)\Delta\theta \\ =\ & \sum_{m=1}^{\infty} e_\alpha^m(\omega,0)m\omega\int_{(m-1)\omega}^{m\omega} \frac{2}{1+2\mu(0)}e_{\ominus 2}(\theta,0)\Delta\theta \\ =\ & \sum_{m=1}^{\infty} e_\alpha^m(\omega,0)m\omega[e_{\ominus 2}^{m-1}(\omega,0)-e_{\ominus 2}^m(\omega,0)], \end{aligned}$$

and

$$\lim_{m\to\infty}\frac{e_{\alpha\ominus 2}^m(\omega,0)m\omega[e_2(\omega,0)-1]}{e_{\alpha\ominus 2}^{m-1}(\omega,0)(m-1)\omega[e_2(\omega,0)-1]} = e_{\alpha\ominus 2}(\omega,0) < 1, \quad \alpha \in [0,2)_{\mathbb{T}}.$$

For the similar reason, we obtain

$$\sum_{m=1}^{\infty} e_\alpha(0,-m\omega)\int_{(m-1)\omega}^{m\omega} k_{12}(\theta)\theta\Delta\theta < \infty,$$

$$\sum_{m=1}^{\infty} e_\alpha(0,-m\omega)\int_{(m-1)\omega}^{m\omega} k_{21}(\theta)\theta\Delta\theta < \infty,$$

$$\sum_{m=1}^{\infty} e_\alpha(0,-m\omega)\int_{(m-1)\omega}^{m\omega} k_{22}(\theta)\theta\Delta\theta < \infty,$$

i.e, the condition (H_5) hold. From Theorem 3.1 and Theorem 4.1, we know that equation (5.1) has at least one 1-periodic solution, which is exponentially stable.

6. Conclusion

Using the time scale calculus theory, coincidence degree theory and the Lapunov functional method, some sufficient conditions are obtained to ensure the existence and the global exponential stability of periodic solutions for high-order Hopfield neural networks with bounded and distributed delays on time scales. The results obtained in this paper possess highly important significance and are easily checked in practice. In addition, the method in this paper may be applied to some other systems such as the BAM and DCNNs systems and so on.

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