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COMPARING DIFFRACTION PROBLEMS FOR AN ELECTROMAGNETIC WAVE ON A STRIP AND AN ELASTIC WAVE ON A DEFECT

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Abstract: In this paper, diffraction problems of the harmonious TE-wave on an ideally conductive infinitely thin metallic strip, placed on the media interface of two homogeneous isotropic dielectric media are considered. Also, diffraction problems for an elastic harmonious wave on a defect in the form of a split or a thin inclusion, disposed on the media interface of two homogeneous isotropic elastic media, in the case of the two-dimensional field, are considered. The formulations and solution of these problems are given in comparison. It is shown that diffraction problems are similar, in which the quantity of the types of waves spread in the medium, coincides.

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1. Formulation of the Problem

We consider diffraction problems for an electromagnetic TE-wave on the ideally conducting infinitely thin metallic strip $\{z = 0, \quad \alpha < x < \beta\}$. The strip is placed on the media interface of two homogeneous isotropic dielectric media.

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The diffraction problems for an elastic wave on the defect in the form of a spilt or a thin inclusion $\{z=0, \quad \alpha<x<\beta\}$ disposed on the media interface of two homogeneous isotropic elastic media. Assume that the body forces are absent.

Moreover, we consider the plane problem when the field is two-dimensional $(\frac{\partial}{\partial y}=0)$ by omitting the time factor e^{-ikt}. The incident field for $z>0$ and $z<0$ is given. We search the field which occurs from the diffraction of a TE-wave and an elastic wave on a strip and on a defect, correspondingly.

It is possible to consider diffraction problems in the case of an electromagnetic and elastic medium from common mathematical point of view, in spite of differences of physical nature. To solve diffraction problems, we use the method of the Fourier transformation in the class S'.

2. Diffraction Problems in the Case of one Type of Waves

It is well known (see, [1]), that there exists one type of waves in a homogeneous isotropic dielectric medium and at the anti-plane deformation in a homogeneous isotropic elastic medium. Therefore, we, at first, consider the plane diffraction problem for a TE-wave on a strip and the anti-plane diffraction problem for an elastic wave on a defect.

In the case of the anti-plane deformation with previous assumptions that different from zero component $w(.,.)$ of the displacement vector, we have the Helmholtz equation

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}+k^2(z)w=0, \qquad k(z)=(k_+, z>0; \quad k_-, z<0). \tag{1}$$

To solve the diffraction problem, it is necessary to find the solution of the equation (1) for $z>0$ and $z<0$ in the class outgoing $\widetilde{S}'$ from the straight line $z=0$ into the half-planes $\{z>0\}$ and $\{z<0\}$ of solutions, see [2]. The solution satisfies the following conditions on the media interface

$$\frac{\partial w}{\partial z}(x,0\pm 0)+\frac{\partial w_0}{\partial z}(x,0\pm 0)=0, \qquad x\in(\alpha,\beta),$$

$$[w(x,z)]|_{z=0}=0, \qquad [\frac{\partial w}{\partial x}(x,z)]|_{z=0}=0, \quad x\notin(\alpha,\beta), \tag{2}$$

where $w_0(.,.)$ is the dropping wave,

$$[w(x,z)]|_{z=0}=w(x,0+0)-w(x,0-0),$$

$$[\frac{\partial w}{\partial x}(x,z)]|_{z=0}=\frac{\partial w}{\partial x}(x,0+0)-\frac{\partial w}{\partial x}(x,0-0).$$

In order to find the field which occurs from the diffraction of an elastic wave on a defect, it is necessary to consider two auxiliary problems (see, [2], [3], [4] and [5]) the Cauchy problem for the Helmholtz equation

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} + k^2 w = 0, \qquad k > 0, \quad k \in R, \tag{3}$$

in the half-plane $z > 0$ with the boundary conditions

$$w(x,0) = w_0(x), \qquad \frac{\partial w}{\partial z}(x,0) = w_1(x), \quad x \in R,$$

and the jump problem for the Helmholtz equation at the plane $z = 0$ with the conditions

$$[w(x,z)]|_{z=0} = a(x), \qquad [\frac{\partial w}{\partial x}(x,z)]|_{z=0} = b(x), \quad x \in R.$$

We obtain the integral equation equivalent to the diffraction problem for an elastic time harmonic wave on a defect in the case of the anti-plane deformation

$$\frac{i}{2\pi}\int_\alpha^\beta a(t)\int_{-\infty}^{+\infty}\frac{\gamma_+(\xi)\gamma_-(\xi)}{\gamma_+(\xi)+\gamma_-(\xi)}\exp(i\xi(t-x))d\xi dt$$
$$= -\frac{\partial w_0}{\partial x}(x,z), \quad x \in (\alpha,\beta), \tag{4}$$

where

$$\gamma_\pm(\xi) = \{|\xi| \ge k_\pm : \quad +i\sqrt{\xi^2 - k_\pm^2}; \quad |\xi| < k_\pm : \quad -\sqrt{k_\pm^2 - \xi^2}\}$$

assuming that

$$\frac{\partial w_0}{\partial z}(x, 0 \pm 0) = \frac{\partial w_0}{\partial z}(x,0), \qquad x \in (\alpha,\beta).$$

The sign "+" refers to the domain $z > 0$ and the sign "-" refers to the domain $z < 0$. Then the field occurs from the diffraction of the elastic wave on a defect, is determined by the equality

$$w_\pm(x,z) = \frac{1}{2\pi}\int_\alpha^\beta a(t)\int_{-\infty}^{+\infty}\frac{\pm\gamma_\mp(\xi)}{\gamma_+(\xi)+\gamma_-(\xi)}\exp(\pm i\gamma_\pm(\xi)z + i\xi(t-x))d\xi dt.$$

In the case of the electromagnetic medium with previous assumptions we search a solution of equation (1) for $z > 0$ and for $z < 0$ in the class $\widetilde{S}'$, satisfying the conditions (2) and the following condtion

$$w(x, 0 \pm 0) + w_0(x, 0 \pm 0) = 0, \qquad x \in (\alpha,\beta).$$

The diffraction problem for the TE-wave on the metallic strip is solved analogously with the considered diffraction problem for the elastic wave on a defect. The integral equation equivalent to the problem, similar to (4) and equivalent to the problem is obtained in [2].

3. Diffraction Problems in the Case of two Type of Waves

It is well known that there exist two types of waves in a homogeneous isotropic elastic medium in the general case. Oscillatory processes in an elastic medium run more difficult, than an electromagnetic medium. They are accompanied by mutual transformations of longitudinally and lateral (crosswise) waves into each other, if a body has the boundary.

In the case of plane deformation different from zero components $u(.,.), v(.,.)$ of the displacement vector are defined by longitudinal and crosswise potentials $\varphi(.,.)$ and $\psi(.,.)$

$$u = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial z}, \qquad v = \frac{\partial \varphi}{\partial z} - \frac{\partial \psi}{\partial x}.$$

Let an elastic harmonious wave with potentials $\varphi_0(.,.)$ and $\psi_0(.,.)$ fall on the defect located along the real axis interval in the homogenous isotropic elastic medium. Let the defect be hard and put into the elastic medium without tension. In this case, the boundary conditions for potentials are divided, see [6].

For a solution of the plain diffraction problem, it is necessary to find solutions of the following Helmholtz equations

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} + k_1^2 \varphi = 0, \quad \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} + k_2^2 \psi = 0,$$

when $z > 0$ and when $z < 0$ in the divergence class from the line $z = 0$ in the half-plane $\{z > 0\}$ and $\{z < 0\}$ of solutions that satisfy the following conditions

$$\frac{\partial \varphi}{\partial z}(x, 0 \pm 0) + \frac{\partial \varphi_0}{\partial z}(x, 0 \pm 0),$$

$$\psi(x, 0 \pm 0) + \psi_0(x, 0 \pm 0) = 0, \quad x \in (\alpha, \beta),$$

$$[\varphi(x, z)]|_{z=0} = 0, \quad [\frac{\partial \varphi}{\partial z}(x, z)]|_{z=0} = 0,$$

$$[\psi(x, z)]|_{z=0} = 0, \quad [\frac{\partial \psi}{\partial z}(x, z)]|_{z=0} = 0, \quad x \notin [\alpha, \beta].$$

Let

$$\frac{\partial \varphi_0}{\partial z}(x, 0 \pm 0) = \frac{\partial \varphi_0}{\partial z}(x, 0), \quad \psi_0(x, 0 \pm 0) = \psi_0(x, 0), \quad x \in (\alpha, \beta).$$

The problem for the longitudinal potential $\varphi(.,.)$ completely coincides with the anti-plane diffraction problem in case of similar media while the problem for the crosswise potentials $\psi(.,.)$ is solved in the same way. We can show that the plane diffraction problem of the elastic harmonious wave on the defect in case in the same media is equivalent to the two integral equations

$$\frac{i}{4\pi}\int_{\alpha}^{\beta} a(t)\int_{-\infty}^{+\infty}\gamma_1(\xi)\exp(i\xi(t-x))d\xi dt = -\frac{\partial\varphi_0}{\partial z}(x,0), \quad x\in(\alpha,\beta),$$

$$\frac{i}{4\pi}\int_{\alpha}^{\beta} b(t)\int_{-\infty}^{+\infty}\frac{1}{\gamma_2(\xi)}\exp(i\xi(t-x))d\xi dt = \psi_0(x,0), \quad x\in(\alpha,\beta),$$

where

$$\gamma_j(\xi) = \{|\xi|\geq k_j: \quad +i\sqrt{\xi^2-k_j^2}; \quad |\xi|<k_j: \quad -\sqrt{k_j^2-\xi^2}\}, \quad j=1,2.$$

Here, the longitudinal and crosswise potentials are defined by the formulas

$$\frac{\partial\varphi_\pm}{\partial z}(x,z) = \frac{i}{4\pi}\int_{\alpha}^{\beta} a(t)\int_{-\infty}^{+\infty}\gamma_1(\xi)\exp[\pm iz\gamma_1(\xi)+i\xi(t-x)]d\xi dt,$$

$$\psi_\pm(x,z) = -\frac{i}{4\pi}\int_{\alpha}^{\beta} b(t)\int_{-\infty}^{+\infty}\frac{1}{\gamma_2(\xi)}\exp[\pm iz\gamma_2(\xi)+i\xi(t-x)]d\xi dt.$$

Integral equations are equivalent to the diffraction problem. This is similar to the equations of the diffraction problem for an electromagnetic wave on the unclosed screens of cylindrical plane, disposed one plane, obtained in [7].

Consider the plane diffraction problem for an elastic harmonious wave on the defect in the case of different media. The boundary conditions for the longitudinally and crosswise potentials are not separated, therefore it is conveniently to use the formulation of the problem in the displacement.

To solve the problem it is necessary to find a solution of the Lamé equations

$$(\lambda(z)+2\mu(z))\frac{\partial^2 u}{\partial x^2} + (\lambda(z)+\mu(z))\frac{\partial^2 v}{\partial x\partial z} + \mu(z)\frac{\partial^2 u}{\partial z^2} + \rho(z)k^2(z)u = 0,$$

$$\mu(z)\frac{\partial^2 v}{\partial x^2} + (\lambda(z)+\mu(z))\frac{\partial^2 u}{\partial x\partial z} + (\lambda(z)+2\mu(z))\frac{\partial^2 v}{\partial z^2} + \rho(z)k^2(z)v = 0,$$

where $u(.,.), v(.,.)$ are displacement components $\{k(z)=k_+; z>0, \quad k_-; z<0\}$ are Lamé constants $\lambda(.), \mu(.)$ and body density $\rho(.)$ are defined analogously to $k(.)$ when $z>0$ and when $z<0$ in the class S' satisfying the following conjugation conditions

$$u(x,0\pm 0)+u_0(x,0\pm 0)=0, \quad v(x,0\pm 0)+v_0(x,0\pm 0)=0, \quad x\in(\alpha,\beta),$$

$$[u(x,z)]|_{z=0}=0, \qquad [\frac{\partial u}{\partial z}(x,z)]|_{z=0}=0,$$

$$[v(x,z)]|_{z=0}=0, \quad [\frac{\partial v}{\partial z}(x,z)]|_{z=0}=0, \quad x\notin[\alpha,\beta],$$

where $u(.,.), v(.,.)$ determine the dropping field.

Two integral equations equivalent to the plane diffraction problem for the elastic time harmonic wave on a the defect can be obtained by the method used for a solving of the diffraction problem in the case of the anti-plane deformation. Integral equations look more simpler in case of similar media. Then, the following theorem is proven.

Theorem. *The plane diffraction problem of the elastic harmonious wave on the defect in case in the same media (boundary conditions are not divided for longitudinal and crosswise potentials) is equivalent to the two integral equations*

$$\frac{i}{4\pi(\lambda+2\mu)}\int_{\alpha}^{\beta} b_u(t)\int_{-\infty}^{+\infty}[\frac{\mu}{\gamma_2(\xi)}+\frac{\lambda+\mu}{\gamma_1+\gamma_2(\xi)}]e^{i(t-x)\xi}d\xi dt$$
$$= u_0(x,0), \qquad x(\alpha,\beta),$$

$$\frac{i}{2\pi(\lambda+\mu)}\int_{\alpha}^{\beta}\int_{-\infty}^{+\infty}[(\mu\pm(\lambda+2\mu))\frac{\gamma_2(\xi)}{\gamma_1(\xi)})\frac{\lambda+\mu}{2\mu(\gamma_1(\xi)+\gamma_2(\xi))}b_v(t)$$
$$+\frac{1}{\xi}((\lambda+2\mu)\frac{\gamma_2(\xi)}{\gamma_1(\xi)}+\mu\frac{1\pm 1}{2})b_u(t)]e^{i(t-x)\xi}d\xi dt = v_0(x,0), \qquad x(\alpha,\beta),$$

where $b_u(.), b_v(.)$ are unknown functions,

$$\gamma_j(\xi)=\{|\xi|\geq k_j: \quad +i\sqrt{\xi^2-k_j^2}; \quad |\xi|<k_j: \quad -\sqrt{k_j^2-\xi^2}\}, \quad j=1,2.$$

4. Conclusion

In [8] integral equations with logarithmic singularities in the kernels of static problems in the elasticity theory plane for bodies with defects are solved numerically by Galerkin method. It is possible to use this method also for solving the diffraction problems, the integral equations have a logarithmic singularity in the kernels.

Note that, if there exist points in an electromagnetic medium with identical physical characteristics, then in the elastic medium, by the existence of two types of waves, this corrects only by some conditions, see [1].

It is well known that there are not longitudinally waves in the Maxwell theory. But for lateral waves take place the polarization: TE- and TM-polarized waves are considered. For lateral waves in an isotropic medium one distinguishes the horizontal and vertical polarizations: the SH- and SV-waves are considered correspondingly.

In the case of a homogeneous an isotropic electromagnetic or elastic medium problem becomes more difficult. This is connected to the increase of the quantity of types of waves in the medium. Thus, the similarities of the diffraction problems are in the case of electromagnetic and elastic medium, in which the quantities of types of waves coincide.

Note that the general approach to diffraction problems in the case of elastic and electromagnetic media is possible because of the consideration of general diffraction problems, scattered waves are not introduced from physical considerations.

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