International Journal of Pure and Applied Mathematics
Volume 61 No. 2 2010, 193-210

COMPATIBLE MAPPINGS OF TYPE (I) AND (II) ON INTUITIONISTIC MENGER SPACES IN CONSIDERATION OF COMMON FIXED POINT

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Abstract: In this paper, we first formulate the definitions of compatible mappings of type (I) and (II) in the settings of intuitionistic Menger space and then prove a common fixed point theorem by using the conditions of compatible mappings of type (I) and (II) in complete intuitionistic Menger spaces.

AMS Subject Classification: 47H10, 54H25
Key Words: intuitionistic Menger space, triangular norm, triangular conorm, compatible mappings of type (I) and (II), common fixed points

1. Introduction

Motivated by the potential applicability of fuzzy topology to quantum particle

Received: April 19, 2010

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physics particularly in connection with both string and $e^{(\infty)}$ theory developed by El Naschie (see [7], [8]), Park introduced and discussed in [22] a notion of intuitionistic fuzzy metric space which is based both on the idea of intuitionistic fuzzy set due to Atanassov [1] and the concept of fuzzy metric spaces given by George and Veeramani in [11]. Actually, Park's notion is useful in modelling some phenomena where it is necessary to study relationship between probability function. It has a direct physics motivation in the context of the two slit experiment as foundation of $E-$infinity of high energy physics, recently studied by El Naschie in (see [9], [10]).

Alaca et al [2], using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space as Park [22] with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [17]. Further, they introduced the notion of Cauchy sequences in intuitionistic fuzzy metric spaces and proved the well known fixed point theorems of Banach [3] and Edelstein [6] extended to intuitionistic fuzzy metric spaces with the help of Grabiec [12].

Gregory et al [13], Saadati and Park [25] studied the concept of intuitionistic fuzzy metric spaces and its applications. Turkoglu et al [32], Sharma and Deshpande [29] proved fixed point theorems in intuitionistic fuzzy metric spaces.

Many generalizations of the concept of metric space can be modified by the requirements placed on the distance function. One such generalization is that of Menger space, first introduced by Menger [19] and developed by Schweizer and Sklar (see [26], [27], [28]), Chang et al [4], and others (see [14], [15], [16], [20], [23], [24]). In Menger's theory, the concept of distance $d(x, y)$ between two points x and y is considered as probabilistic, namely, the non-negative number $d(x, y)$ is replaced by a distance distribution function $F_{x,y} : \mathbb{R} \to \mathbb{R}^+$. Then, for any real number t, the value $F_{x,y}(t)$ is interpreted as the degree of nearness between x and y with respect to t. Modifying the idea of Kramosil and Michalek [17], George and Veeramani [11] introduced fuzzy metric spaces which are very similar to that of Menger space (see [21], [23], [24]).

Kutukcu et al [18] defined the notion of intuitionistic Menger spaces with the help of t-norms and $t-$conorms as a generalization of Menger spaces due to Menger [19].

On the other hand, Cho et al [5] introduced the definitions of compatible mappings of type (I) and (II) in fuzzy metric spaces and proved some common fixed point theorems for four mappings under the condition of compatible mappings of type (I) and (II) in complete fuzzy metric spaces. They extended, generalized and improved the corresponding results given by many authors.

Sharma and Deshpande [30] formulated the definition of compatible mappings of type (I) and (II) in intuitionistic fuzzy metric spaces and proved a common fixed point theorem under the condition of compatible mappings of type (I) and (II) in intuitionistic fuzzy metric spaces.

In this paper, we define the notion of compatible mappings of type (I) and (II) in the settings of intuitionistic Menger spaces and prove a common fixed point theorem by using the conditions of compatible mappings of type (I) and (II) in complete intuitionistic Menger spaces.

2. Preliminaries

Definition 1. A binary operation $T : [0,1] \times [0,1] \to [0,1]$ is a $t-$norm if T satisfies the following conditions:

(a) T is commutative and associative,

(b) $T(a,1) = a$ for all $a \in [0,1]$,

(c) $T(a,b) \leq T(c,d)$ whenever $a \leq c$ and $b \leq d$ and $a,b,c,d \in [0,1]$.

Definition 2. A binary operation $S : [0,1] \times [0,1] \to [0,1]$ is a $t-$conorm if S satisfies the following conditions:

(a) S is commutative and associative,

(b) $S(a,0) = 0$ for all $a \in [0,1]$,

(c) $S(a,b) \leq S(c,d)$ whenever $a \leq c$ and $b \leq d$ and $a,b,c,d \in [0,1]$.

Remark 1. The concepts of $t-$norm T and $t-$conorm S are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions respectively. These concepts were originally introduced by Menger [19] in his study of statistical metric spaces. Several examples for these concepts were proposed by many authors (see [11], [26], [21], [16], [14], [15], [23]). In [9], we also have

(a) for any $a, b \in (0,1)$, $a > b$, $\exists\ c, d \in (0,1)$ s.t. $T(a,c) \geq b, S(b,d) \leq a$.

(b) for any $a \in (0,1)$, $\exists\ b, c \in (0,1)$ s.t. $T(b,b) \geq a, S(c,c) \leq a$.

Throughout this paper, we will denote $\mathbb{R} = (-\infty, \infty)$ and $\mathbb{R}^+ = [0, \infty)$.

Definition 3. (see [27]) A distance distribution function is a function $F{:}\mathbb{R} \to \mathbb{R}^+$, which is left continuous on $\mathbb{R}$, non-decreasing and $\inf_{t\in\mathbb{R}} F(t) = 0, \sup_{t\in\mathbb{R}} F(t) = 1$. We will denote by D, the family of all distance distribution functions and by H a special element of D defined by

$$H(t) = \begin{cases} 0, & \text{if } t \leq 0, \\ 1, & \text{if } t > 0. \end{cases}$$

If X is a nonempty set, $F : X \times X \to D$ is called a probabilistic distance on X and $F(x, y)$ is usually denoted by $F_{x,y}$.

Definition 4. (see [27]) A non-distance distribution function is a function $L : \mathbb{R} \to \mathbb{R}^+$ which is right continuous on $\mathbb{R}$, non-increasing and $\inf_{t\in\mathbb{R}} L(t) = 1, \sup_{t\in\mathbb{R}} L(t) = 0$. We will denote by E, the family of all non-distance distribution functions and by G a special element of E defined by

$$G(t) = \begin{cases} 1, & \text{if } t \leq 0, \\ 0, & \text{if } t > 0. \end{cases}$$

If X is a nonempty set, $L : X \times X \to E$ is called a probabilistic non-distance on X and $L(x, y)$ is usually denoted by $L_{x,y}$.

Definition 5. A triple (X, F, L) is said to be an intuitionistic probabilistic metric space if X is a nonempty set, F is a probabilistic distance and L is probabilistic non-distance on X satisfying the following conditions: for $x, y, z \in X$, and $t, s \geq 0$,

(a) $F_{x,y}(t) + L_{x,y}(t) \leq 1$;

(b) $F_{x,y}(0) = 0$;

(c) $F_{x,y}(t) = H(t)$ if and only if $x = y$;

(d) $F_{x,y}(t) = F_{y,x}(t)$;

(e) If $F_{x,z}(t) = 1$ and $F_{z,y}(t) = 1$, then $F_{x,y}(t + s) = 1$;

(f) $L_{x,y}(0) = 1$;

(g) $L_{x,y}(t) = G(t)$ if and only if $x = y$;

(h) $L_{x,y}(t) = L_{y,x}(t)$;

(i) If $L_{x,z}(t) = 0$ and $L_{z,y}(t) = 0$, then $L_{x,y}(t + s) = 0$;

If in addition, we have the triangle inequalities:

(j) $F_{x,y}(t + s) \geq T(F_{x,z}(t), F_{z,y}(t))$;

(k) $L_{x,y}(t + s) \leq S(L_{x,z}(t), L_{z,y}(t))$.

Here T is a $t-$norm and S is a $t-$conorm. Then (X, F, L, T, S) is said to be an intuitionistic Menger space. The functions $F_{x,y}(t)$ and $L_{x,y}(t)$ denote the degree of nearness and degree of non-nearness between x and y with respect to t, respectively.

Remark 2. Every Menger space (X, F, T) is an intuitionistic Menger space of the form $(X, F, 1 - F, T, S)$ such that $t-$norm T and $t-$conorm S are associated [14], i.e. $S(x,y) = 1 - T(1-x, 1-y)$ for any $x, y \in X$.

Example 1. (Induced Intuitionistic Probabilistic Metric) Let (X,d) be a metric space. Then the metric d induces a distance distribution function F defined by $F_{x,y}(t) = H(t - d(x,y))$ and a non-distance distribution function L defined by $L_{x,y}(t) = G(t - d(x,y))$ for all $x, y \in X$ and $t \geq 0$. Then (X, F, L) is an intuitionistic probabilistic metric space. We call this intuitionistic probabilistic metric space induced by a metric d the induced intuitionistic probabilistic metric space. If $t-$norm T is $T(a,b) = \min\{a,b\}$ and $t-$conorm S is $S(a,b) = \min\{1, a+b\}$ for all $a, b \in [0,1]$, then (X, F, L, T_M, S_M) is an intuitionistic Menger space.

Remark 3. Note that the above examples hold even with the $t-$norm $T(a,b) = \min\{a,b\}$ and $t-$conorm $S(a,b) = \max\{a,b\}$, and hence (X, F, L, T, S) is an intuitionistic Menger space with respect to any $t-$norm and $t-$conorm. Also note that, in the above example the $t-$norm T and $t-$conorm S are not associated.

On the line of [31], we can prove the following:

Lemma 1. *Let $\{y_n\}$ be a sequence in intuitionistic Menger space (X, F, L, T, S), where T is continuous $t-$norm and S is continuous $t-$conorm with $T(a,a) \geq a$ and $S(1-a, 1-a) \leq 1-a$ respectively for all $a \in [0,1]$. If $\exists$ a constant $k \in (0,1)$ such that $F_{y_n, y_{n+1}}(kt) \geq F_{y_{n-1}, y_n}(t)$ and $L_{y_n, y_{n+1}}(kt) \leq L_{y_{n-1}, y_n}(t)$ for all $t > 0$ and $n \in N$, then $\{y_n\}$ is a Cauchy sequence in X.*

Lemma 2. *Let (X, F, L, T, S) be an intuitionistic Menger space. If $\exists$ a constant $k \in (0,1)$ such that for $x, y \in X, t > 0$,(2.1) $F_{x,y}(kt) \geq F_{x,y}(t)$ and $L_{x,y}(kt) \leq L_{x,y}(t)$, then $x = y$.*

Proof. Since $t > 0$ and $k \in (0,1)$,we get $t > kt$. By Definitions 3 and 4 (distance distribution function is non-decreasing and non distance distribution function is non-increasing), we have $F_{x,y}(t) \geq F_{x,y}(kt)$ and $L_{x,y}(t) \leq L_{x,y}(kt)$. Using (2.1) and the definition of intuitionistic Menger space, we have $x = y$.

Lemma 3. *Let (X, F, L, T, S) be an intuitionistic Menger space. Then $F_{x,y}(t)$ and $L_{x,y}(t)$ are continuous functions on $X \times X \to (0, \infty)$.*

3. Main Results

Let Φ be the set of all continuous and increasing functions $\phi : [0,1]^5 \to [0,1]$ in any coordinate and $\phi(t,t,t,t,t) > t$ for all $t \in [0,1)$. Also let Ψ be the set of all continuous and decreasing functions $\psi : [0,1]^5 \to [0,1]$ in any coordinate and

$\psi(t,t,t,t,t) < t$ for all $t \in [0,1)$.

Example 2. Consider the function $\phi : [0,1]^5 \to [0,1]$ defined as follows:

(i) $\phi(x_1,x_2,x_3,x_4,x_5) = (\min\{x_i\})^h$ for some $0 < h < 1$.

(ii) $\phi(x_1,x_2,x_3,x_4,x_5) = x_i^h$ for some $0 < h < 1$.

(iii) $\phi(x_1,x_2,x_3,x_4,x_5) = \max\{x_1^{\alpha_1}, x_2^{\alpha_2}, x_3^{\alpha_3}, x_4^{\alpha_4}, x_5^{\alpha_5}\}$, where $0 < \alpha_i < 1$ for $i = 1,2,3,4,5$.

Consider the function $\psi : [0,1]^5 \to [0,1]$ defined as follows:

(i) $\psi(x_1,x_2,x_3,x_4,x_5) = (\max\{x_i\})^h$ for some $h > 1$.

(ii) $\psi(x_1,x_2,x_3,x_4,x_5) = x_i^h$ for some $h > 1$.

(iii) $\psi(x_1,x_2,x_3,x_4,x_5) = \min\{x_1^{\alpha_1}, x_2^{\alpha_2}, x_3^{\alpha_3}, x_4^{\alpha_4}, x_5^{\alpha_5}\}$, where $\alpha_i > 1$ for $i = 1,2,3,4,5$.

Throughout this paper (X,F,L,T,S) will denote an intuitionistic Menger space with continuous $t-$norm T and continuous $t-$conorm S defined by $T(a,a) \geq a$ and $S(1-a,1-a) \leq 1-a$ respectively for all $a \in (0,1)$.

Definition 6. Let A, B be mappings from an intuitionistic Menger space (X,F,L,T,S) into itself. Then the pair (A,B) is said to be compatible of type (I) if for all $t > 0$, $\lim_{n\to\infty} F_{ABx_n,x}(t) \geq F_{Bx,x}(t)$ and $\lim_{n\to\infty} L_{ABx_n,x}(t) \leq L_{Bx,x}(t)$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x \in X$.

Definition 7. Let A, B be mappings from an intuitionistic Menger space (X,F,L,T,S) into itself. Then the pair (A,B) is said to be compatible of type (II) if and only if (B,A) is compatible of type (I).

Proposition 1. *Let A, B be mappings from an intuitionistic Menger space (X,F,L,T,S) into itself. Suppose that the pair (A,B) is compatible of type (I) (respectively (II)) and $Az = Bz$ for some $z \in X$. Then for all $t > 0$, $F_{Az,BBz}(t) \geq F_{Az,ABz}(t)$ and $L_{Az,BBz}(t) \leq L_{Az,ABz}(t)$ (respectively $F_{Bz,AAz}(t) \geq F_{Bz,BAz}(t)$ and $L_{Bz,AAz}(t) \leq L_{Bz,BAz}(t)$).*

Proof. See Proposition 2.16 of [5]. □

Theorem 1. *Let (X,F,L,T,S) be a complete intuitionistic Menger space. Let A, B, P and Q be mappings from X into itself such that:*

(1.1) $A(X) \subseteq Q(X)$, $B(X) \subseteq P(X)$,

(1.2) there exists a constant $k \in (0,1/2)$ such that

$$F_{Ax,By}(kt) \geq \phi\left(F_{Px,Qy}(t), F_{Ax,Px}(t),\ F_{By,Qy}(t),\ F_{Ax,Qy}(\alpha t),\ F_{By,Px}((2-\alpha)t)\right)$$

and

$$L_{Ax,By}(kt) \leq \psi\left(L_{Px,Qy}(t), L_{Ax,Px}(t), L_{By,Qy}(t), L_{Ax,Qy}(\alpha t), L_{By,Px}((2-\alpha)t)\right)$$

for all $x, y \in X, \alpha \in (0,2), t > 0$ *and* $\phi \in \Phi, \psi \in \Psi$. *If the mappings* A, B, P *and* Q *satisfy one of the following conditions:*

(1.3) the pairs (A, P) *and* (B, Q) *are compatible of type (II) and* A *or* B *is continuous,*

(1.4) the pairs (A, P) *and* (B, Q) *are compatible of type (I) and* P *or* Q *is continuous, then the mappings* A, B, P *and* Q *have a unique common fixed point in* X.

Proof. Let $x_0 \in X$ be an arbitrary point. Since $A(X) \subseteq Q(X), B(X) \subseteq P(X)$, there exist $x_1, x_2 \in X$ such that $Ax_0 = Qx_1, Bx_1 = Px_2$. Inductively, construct the sequences $\{y_n\}$ and $\{x_n\}$ in X such that

$$y_{2n} = Ax_{2n} = Qx_{2n+1}, \quad y_{2n+1} = Bx_{2n+1} = Px_{2n+2} \text{ for } n = 0, 1, 2, 3, \dots .$$

If for $\alpha = 1 - q$ and $q \in (0,1)$, we set $d_m(t) = F_{y_m, y_{m+1}}(t)$ and $d'_m(t) = L_{y_m, y_{m+1}}(t)$ for all $t > 0$, then we prove that $\{d_m(t)\}$ is increasing with respect to m and $\{d'_m(t)\}$ is decreasing with respect to m. Setting $m = 2n$, we have

$$\begin{aligned}
d_{2n}(kt) &= F_{Ax_{2n}, Bx_{2n+1}}(kt) \\
&\geq \phi\begin{pmatrix} F_{Px_{2n}, Qx_{2n+1}}(t), F_{Ax_{2n}, Px_{2n}}(t), F_{Bx_{2n+1}, Qx_{2n+1}}(t), \\ F_{Ax_{2n}, Qx_{2n+1}}((1-q)t), F_{Bx_{2n+1}, Px_{2n}}((1+q)t) \end{pmatrix} \\
&= \phi\begin{pmatrix} F_{y_{2n-1}, y_{2n}}(t), F_{y_{2n}, y_{2n-1}}(t), F_{y_{2n+1}, y_{2n}}(t), \\ F_{y_{2n}, y_{2n}}((1-q)t), F_{y_{2n+1}, y_{2n-1}}((1+q)t) \end{pmatrix} \\
&= \phi\left(d_{2n-1}(t), d_{2n-1}(t), d_{2n}(t), 1, F_{y_{2n+1}, y_{2n-1}}((1+q)t)\right) \\
&\geq \phi\left(d_{2n-1}(t), d_{2n-1}(t), d_{2n}(t), 1, T(F_{y_{2n-1}, y_{2n}}(t), F_{y_{2n}, y_{2n+1}}(qt))\right) \\
&= \phi\left(d_{2n-1}(t), d_{2n-1}(t), d_{2n}(t), 1, T(d_{2n-1}(t), d_{2n}(qt))\right),
\end{aligned}$$

that is

$$d_{2n}(kt) = F_{y_{2n}, y_{2n+1}}(kt) \geq \phi\begin{pmatrix} d_{2n-1}(qt), d_{2n-1}(qt), \\ d_{2n}(qt), 1, T(d_{2n-1}(t), d_{2n}(qt)) \end{pmatrix}. \qquad (1.5)$$

The above inequality is true since ϕ is an increasing function.

Similarly,

$$d'_{2n}(kt) = L_{y_{2n}, y_{2n+1}}(kt) \leq \psi\begin{pmatrix} d'_{2n-1}(qt), d'_{2n-1}(qt), \\ d'_{2n}(qt), 0, S(d'_{2n-1}(t), d'_{2n}(qt)) \end{pmatrix}. \qquad (1.6)$$

The above inequality is true since ψ is a decreasing function.

We claim that for all $n \in N$, $d_{2n}(t) \geq d_{2n-1}(t)$. In fact, if $d_{2n}(t) < d_{2n-1}(t)$, then since

$$T(d_{2n}(t), d_{2n-1}(t)) \geq T(d_{2n}(qt), d_{2n}(qt)) = d_{2n}(qt),$$

by inequality (1.5), we have

$$d_{2n}(qt) \geq \phi\left(d_{2n-1}(qt), d_{2n-1}(qt), d_{2n}(qt), 1, d_{2n}(qt)\right)$$
$$> \phi\left(d_{2n}(qt), d_{2n}(qt), d_{2n}(qt), 1, T\left(d_{2n-1}(t), d_{2n}(qt)\right)\right) > d_{2n}(qt),$$

which is a contradiction. Hence $d_{2n}(t) \geq d_{2n-1}(t)$ for all $n \in N, t > 0$.

Similarly for $m = 2n + 1$, we have $d_{2n+1}(t) \geq d_{2n}(t)$ and so $\{d_m(t)\}$ is an increasing sequence in $[0, 1]$.

Now we claim that for all $n \in N$, $d^{/}_{2n}(t) \leq d^{/}_{2n-1}(t)$. In fact, if $d^{/}_{2n}(t) > d^{/}_{2n-1}(t)$, then since

$$S\left(d^{/}_{2n}(t), d^{/}_{2n-1}(t)\right) \leq S\left(d^{/}_{2n}(qt), d^{/}_{2n}(qt)\right) = d^{/}_{2n}(qt),$$

by inequality (1.6), we have

$$d^{/}_{2n}(qt) \leq \psi\left(d^{/}_{2n-1}(qt), d^{/}_{2n-1}(qt), d^{/}_{2n}(qt), 0, d^{/}_{2n}(qt)\right)$$
$$< \psi\left(d^{/}_{2n}(qt), d^{/}_{2n}(qt), d^{/}_{2n}(qt), 0, S\left(d^{/}_{2n-1}(t), d^{/}_{2n}(qt)\right)\right) < d^{/}_{2n}(qt),$$

which is a contradiction. Hence $d^{/}_{2n}(t) \leq d^{/}_{2n-1}(t)$ for all $n \in N, t > 0$.

Similarly for $m = 2n + 1$, we have $d^{/}_{2n+1}(t) \leq d^{/}_{2n}(t)$ and so $\{d^{/}_m(t)\}$ is an decreasing sequence in $[0, 1]$. As $t-$norm T is continuous, letting $q \to 1$, we get from (1.5)

$$F_{y_{2n}, y_{2n+1}}(kt) \geq \phi\left(d_{2n-1}(t), d_{2n-1}(t), d_{2n}(t), 1, T(d_{2n-1}(t), d_{2n}(t))\right).$$

Since $\{d_m(t)\}$ is an increasing sequence and $T(a, a) \geq a$, we have $F_{y_{2n}, y_{2n+1}}(kt) \geq d_{2n-1}(t)$, that is $F_{y_{2n}, y_{2n+1}}(kt) \geq F_{y_{2n-1}, y_{2n}}(t)$.

Similarly, since $t-$conorm S is continuous, letting $q \to 1$, we get from (1.6)

$$L_{y_{2n}, y_{2n+1}}(kt) \leq \psi\left(d^{/}_{2n-1}(t), d^{/}_{2n-1}(t), d^{/}_{2n}(t), 0, S(d^{/}_{2n-1}(t), d^{/}_{2n}(t))\right).$$

Since $\left\{d^{/}_m(t)\right\}$ is a decreasing sequence and $S(1-a, 1-a) \leq 1-a$, we have $L_{y_{2n}, y_{2n+1}}(kt) \leq L_{y_{2n-1}, y_{2n}}(t)$.

In general, we have for $m = 1, 2, 3...$ $F_{y_m, y_{m+1}}(kt) \geq F_{y_{m-1}, y_m}(t)$ and $L_{y_m, y_{m+1}}(kt) \leq L_{y_{m-1}, y_m}(t)$. Then by Lemma 2, $\{y_n\}$ is a Cauchy sequence in X and by completeness of X, $\{y_n\}$ converges to a point in X. Let $y_n \to z$ as $n \to \infty$. Hence we have

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} Ax_{2n} = \lim_{n\to\infty} Qx_{2n+1} = \lim_{n\to\infty} y_{2n+1}$$
$$= \lim_{n\to\infty} Bx_{2n+1} = \lim_{n\to\infty} Px_{2n+2} = z.$$

Now suppose that Q is continuous and the pair (B, Q) is compatible of type (I). Hence we have

$$\lim_{n\to\infty} QQx_{2n+1} = Qz,\ \ F_{Qz,z}(t) \geq \lim_{n\to\infty} F_{BQx_{2n+1},z}(t)$$

and

$$L_{Qz,z}(t) \leq \lim_{n\to\infty} L_{BQx_{2n+1},z}(t).$$

Now for $\alpha = 1$, setting $x = x_{2n}$ and $y = Qx_{2n+1}$ in the inequality (1.2), we obtain

$$F_{Ax_{2n},BQx_{2n+1}}(kt) \geq \phi\left(\begin{array}{c} F_{Px_{2n},QQx_{2n+1}}(t), F_{Ax_{2n},Px_{2n}}(t), F_{BQx_{2n+1},QQx_{2n+1}}(t), \\ F_{Ax_{2n},QQx_{2n+1}}(t), F_{BQx_{2n+1},Px_{2n}}(t) \end{array}\right)$$

and

$$L_{Ax_{2n},BQx_{2n+1}}(kt) \leq \psi\left(\begin{array}{c} L_{Px_{2n},QQx_{2n+1}}(t), L_{Ax_{2n},Px_{2n}}(t), L_{BQx_{2n},QQx_{2n+1}}(t), \\ L_{Ax_{2n},QQx_{2n+1}}(t), L_{BQx_{2n+1},Px_{2n}}(t) \end{array}\right).$$

Letting $n \to \infty$, we have

$$\begin{aligned}\lim_{n\to\infty} F_{z,BQx_{2n+1}}(kt) &\geq \phi\left(F_{z,Qz}(t), F_{z,z}(t), \lim_{n\to\infty} F_{BQx_{2n+1},Qz}(t), F_{z,Qz}(t), \lim_{n\to\infty} F_{BQx_{2n+1},z}(t)\right) \\ &\geq \phi\Big(F_{z,Qz}(t/2), F_{z,z}(t/2), \lim_{n\to\infty} F_{BQx_{2n+1},Qz}(t/2), F_{z,Qz}(t/2), \\ &\qquad \lim_{n\to\infty} F_{BQx_{2n+1},z}(t/2)\Big).\end{aligned}$$

Similarly,

$$\begin{aligned}\lim_{n\to\infty} L_{z,BQx_{2n+1}}(kt) &\leq \psi\Big(L_{z,Qz}(t/2), L_{z,z}(t/2), \lim_{n\to\infty} L_{BQx_{2n+1},Tz}(t/2), L_{z,Qz}(t/2), \\ &\qquad \lim_{n\to\infty} L_{BQx_{2n+1},z}(t/2)\Big).\end{aligned}$$

Thus it follows that:

$$\lim_{n\to\infty} F_{BQx_{2n+1},Qz}(t) \geq T(\lim_{n\to\infty} F_{BQx_{2n+1},z}(t/2), F_{z,Qz}(t/2),$$

and

$$\lim_{n\to\infty} L_{BQx_{2n+1},Qz}(t) \leq S(\lim_{n\to\infty} L_{BQx_{2n+1},z}(t/2), L_{z,Qz}(t/2).$$

So,

$$\lim_{n\to\infty} F_{BQx_{2n+1},Qz}(t) \geq T(\lim_{n\to\infty} F_{BQx_{2n+1},z}(t/2))$$

and

$$\lim_{n\to\infty} L_{BQx_{2n+1},Qz}(t) \leq S(\lim_{n\to\infty} L_{BQx_{2n+1},z}(t/2)).$$

Hence since $\phi(t,t,t,t,t) > t$ and $\psi(t,t,t,t,t) < t$, by above inequalities, we have

$$\lim_{n\to\infty} F_{z,BQx_{2n+1}}(kt) > \lim_{n\to\infty} F_{z,BQx_{2n+1}}(t/2)$$

and

$$\lim_{n\to\infty} L_{z,BQx_{2n+1}}(kt) < \lim_{n\to\infty} L_{z,BQx_{2n+1}}(t/2),$$

which is a contradiction. It follows that $\lim_{n\to\infty} BQx_{2n+1} = z$.

Now using the compatibility of type (I), we have

$$F_{Tz,z}(t) \geq \lim_{n\to\infty} F_{z,BQx_{2n+1}}(t) = 1$$

and

$$L_{Tz,z}(t) \leq \lim_{n\to\infty} L_{z,BQx_{2n+1}}(t) = 0.$$

So, it follows that $Tz = z$.

Again replacing x by x_{2n} and y by z in (1.2), with $\alpha = 1$, we have

$$F_{Ax_{2n},Bz}(kt) \\ \geq \phi\left(F_{Px_{2n},Qz}(t), F_{Ax_{2n},Px_{2n}}(t), F_{Bz,Qz}(t), F_{Ax_{2n},Qz}(t), F_{Bz,Px_{2n}}(t)\right)$$

and

$$L_{Ax_{2n},Bz}(kt) \\ \leq \psi\left(L_{Px_{2n},Qz}(t), L_{Ax_{2n},Px_{2n}}(t), L_{Bz,Qz}(t), L_{Ax_{2n},Qz}(t), L_{Bz,Px_{2n}}(t)\right).$$

Letting $n \to \infty$, we have $F_{Bz,z}(kt) > F_{Bz,z}(t)$ and $L_{Bz,z}(kt) < L_{Bz,z}(t)$, which implies that $Bz = z$. Since $B(X) \subseteq P(X)$, there exists $u \in X$ such that $Su = z = Bz$. So by (1.2) with $\alpha = 1$ we have

$$F_{Au,Bz}(kt) \geq \phi\left(F_{Pu,Qz}(t), F_{Au,Pu}(t), F_{Bz,Qz}(t), F_{Au,Qz}(t), F_{Bz,Pu}(t)\right)$$

and

$$L_{Au,Bz}(kt) \leq \psi\left(L_{Pu,Qz}(t), L_{Au,Pu}(t), L_{Bz,Qz}(t), L_{Au,Qz}(t), L_{Bz,Pu}(t)\right).$$

Therefore $F_{Au,z}(kt) > F_{z,Au}(t)$ and $L_{Au,z}(kt) < L_{z,Au}(t)$, which implies that $Au = z$. Since the pair (A, S) is compatible of type (I) and $Au = Su = z$, by Proposition 1, we have $F_{Au,PPu}(t) \geq F_{Au,APu}(t)$ and $L_{Au,PPu}(t) \leq L_{Au,APu}(t)$. Thus $F_{z,Pz}(t) \geq F_{z,Az}(t)$ and $L_{z,Pz}(t) \leq L_{z,Az}(t)$.

Again by (1.2), with $\alpha = 1$, we have

$$F_{Az,Bz}(kt) \geq \phi\left(F_{Pz,Qz}(t), F_{Az,Pz}(t), F_{Bz,Qz}(t), F_{Az,Qz}(t), F_{Bz,Pz}(t)\right)$$

and

$$L_{Az,Bz}(kt) \leq \psi\left(L_{Pz,Qz}(t), L_{Az,Pz}(t), L_{Bz,Qz}(t), L_{Az,Qz}(t), L_{Bz,Pz}(t)\right).$$

Thus it follows that

$$F_{Az,Pz}(t) \geq T(F_{Az,z}(t/2), F_{z,Pz}(t/2)) \geq T(F_{Az,z}(t/2), F_{z,Az}(t/2)) = F_{z,Az}(t/2)$$

and

$$L_{Az,Pz}(t) \leq S(L_{Az,z}(t/2), L_{z,Pz}(t/2)) \leq S(L_{Az,z}(t/2), L_{z,Az}(t/2)) = L_{z,Az}(t/2).$$

Hence we have

$$F_{Az,z}(kt) \geq \phi\left(F_{Az,z}(t/2), F_{Az,z}(t/2), F_{Az,z}(t/2), F_{z,Az}(t/2), F_{z,Az}(t/2)\right) > F_{z,Az}(t/2)$$

and

$$L_{Az,z}(kt) \leq \psi\left(L_{Az,z}(t/2), L_{Az,z}(t/2), L_{Az,z}(t/2), L_{z,Az}(t/2), L_{z,Az}(t/2)\right) < L_{z,Az}(t/2).$$

And so $Az = z$. Therefore z is a common fixed point of the self mappings A, B, P and Q.

The uniqueness of the common fixed point of the mappings A, B, P and Q can be easily verified by using (1.2). In fact, if $w \in X$ be another common fixed point of A, B, P and Q then for $\alpha = 1$, we have

$$\begin{aligned} F_{z,w}(t) &= F_{Az,Bw}(kt) \\ &\geq \phi\left(F_{Pz,Qw}(t), F_{Az,Pz}(t), F_{Bw,Qw}(t), F_{Az,Qw}(t), F_{Bw,Pw}(t)\right) > F_{z,w}(t) \end{aligned}$$

and

$$\begin{aligned} L_{z,w}(t) &= L_{Az,Bw}(kt) \\ &\leq \psi\left(L_{Pz,Qw}(t), L_{Az,Pz}(t), L_{Bw,Qw}(t), L_{Az,Qw}(t), L_{Bw,Pw}(t)\right) < L_{z,w}(t). \end{aligned}$$

Thus $z = w$. □

Example 3. Let $X = [0, 1]$ with the metric d defined by $d(x, y) = |x - y|$ and for each $t \in [0, 1]$, define $F_{x,y}(t) = H(t - d(x, y))$ and $L_{x,y}(t) = G(t - d(x, y)), F_{x,y}(0) = 0, L_{x,y}(0) = 1$.

Then clearly (X, F, L, T, S) is a complete intuitionistic Menger space, where T and S are defined by $T(a, b) = \min\{a, b\}, S(a, b) = \max\{a, b\}$.

Define the self mappings A, B, P and Q on X by

$$Ax = \begin{cases} \frac{x}{5}, & 0 \leq x \leq 1/2, \\ \frac{1}{10}, & 1/2 < x \leq 1 \end{cases}, \qquad Bx = \begin{cases} \frac{x}{4}, & 0 \leq x \leq 1/2, \\ \frac{x}{8}, & 1/2 < x \leq 1 \end{cases},$$

$$Px = \begin{cases} \frac{x}{3}, & 0 \le x \le 1/2, \\ \frac{1}{6}, & 1/2 < x \le 1 \end{cases}, \qquad Qx = \begin{cases} \frac{x}{2}, & 0 \le x \le 1/2, \\ \frac{1}{8}, & 1/2 < x \le 1 \end{cases}.$$

Then $A(X) \subset Q(X), B(X) \subset P(X)$. Also A, P are continuous and Q, B are discontinuous.

Define a sequence $\{x_n\}$ in X by $x_n = \dfrac{1}{n+2}$, $n = 0, 1, 2, 3...$, then we have

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Px_n = 0, \quad \lim_{n\to\infty} F_{PAx_n,0}(t) \ge F_{A0,0}(t) = 1,$$

$$\lim_{n\to\infty} L_{PAx_n,0}(t) \ge L_{A0,0}(t) = 0, \quad \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Qx_n = 0,$$

$$\lim_{n\to\infty} F_{QBx_n,0}(t) \le F_{B0,0}(t) = 1 \text{ and } \lim_{n\to\infty} L_{QBx_n,0}(t) \le L_{B0,0}(t) = 0,$$

that is the pairs (A, P) and (B, Q) are compatible of type (II) and A, B are continuous.

Consider the functions $\phi : [0, 1]^5 \to [0, 1]$ and $\psi : [0, 1]^5 \to [0, 1]$ defined by

$$\phi(x_1, x_2, x_3, x_4, x_5) = (\min\{x_i\})^h$$

for some $0 < h < 1$ and

$$\psi(x_1, x_2, x_3, x_4, x_5) = (\max\{x_i\})^h$$

for some $h > 1$.

Then we have

$$F_{Ax,By}(kt) \ge \phi(x_1, x_2, x_3, x_4, x_5)$$

and

$$L_{Ax,By}(kt) \le \psi(x_1, x_2, x_3, x_4, x_5).$$

Therefore all the conditions of the Theorem 1 are satisfied and so A, B, P and Q have a common fixed point 0 in X.

Corollary 2. *Let (X, F, L, T, S) be a complete intuitionistic Menger space. Let A, B, P and Q be mappings from X into itself such that:*

(2.1) $A(X) \subseteq Q(X), B(X) \subseteq P(X)$,

(2.2) there exists a constant $k \in (0, 1/2)$ such that:

$$F_{Ax,By}(kt) \ge \left[\begin{array}{c} a_1(t)F_{Px,Qy}(t) + a_2(t)F_{Ax,Px}(t) + a_3(t)F_{By,Qy}(t) \\ +a_4(t)F_{Ax,Qy}(\alpha t) + a_5(t)F_{By,Px}((2-\alpha)t \end{array} \right]^{1/2}$$

and

$$L_{Ax,By}(kt) \le \left[\begin{array}{c} a_1(t)L_{Px,Qy}(t) + a_2(t)L_{Ax,Px}(t) + a_3(t)L_{By,Qy}(t) \\ +a_4(t)L_{Ax,Qy}(\alpha t) + a_5(t)L_{By,Px}((2-\alpha)t \end{array} \right]^{1/2}$$

for all $x, y \in X, \alpha \in (0, 2), t > 0$ and $a_i : \mathbb{R}^+ \to (0, 1]$ such that $\sum_{i=1}^{5} a_i(t) = 1$.

If the mappings A, B, P and Q satisfy one of the following conditions:

(2.3) the pairs (A, P) and (B, Q) are compatible of type (II) and A or B is continuous,

(2.4) the pairs (A, P) and (B, Q) are compatible of type (I) and P or Q is continuous,

then the mappings A, B, P and Q have a unique common fixed point in X.

Proof. By Theorem 1, if we define

$$\phi(x_1, x_2, x_3, x_4, x_5) \geq [a_1(t)x_1 + a_2(t)x_2 + a_3(t)x_3 + a_4(t)x_4 + a_5(t)x_5]^{1/2},$$

$$\psi(x_1, x_2, x_3, x_4, x_5) \leq [a_1(t)x_1 + a_2(t)x_2 + a_3(t)x_3 + a_4(t)x_4 + a_5(t)x_5]^{1/2},$$

then we have the conclusion. □

Corollary 3. *Let (X, F, L, T, S) be a complete intuitionistic Menger space and let A, B, R, P, H and Q be mappings from X into itself such that:*

(3.1) $A(X) \subseteq QH(X), B(X) \subseteq PR(X)$,

(3.2) there exists a constant $k \in (0, 1/2)$ such that

$$F_{Ax,By}(kt)$$
$$\geq \phi\left(F_{PRx,QHy}(t), F_{Ax,PRx}(t), F_{By,QHy}(t), F_{Ax,QHy}(\alpha t), F_{By,PRx}((2-\alpha)t)\right)$$

and

$$L_{Ax,By}(kt)$$
$$\leq \psi\left(L_{PRx,QHy}(t), L_{Ax,PRx}(t), L_{By,QHy}(t), L_{Ax,QHy}(\alpha t), L_{By,PRx}((2-\alpha)t)\right)$$

for all $x, y \in X, \alpha \in (0, 2), t > 0$ and $\phi \in \Phi, \psi \in \Psi$,

(3.3) $QH = HQ$, $AR = RA$, $BH = HB$, $PR = RP$.

If the mappings A, B, P and Q satisfy one of the following conditions:

(3.4) the pairs (A, PR) and (B, QH) are compatible of type (II) and A or B is continuous,

(3.5) the pairs (A, PR) and (B, QH) are compatible of type (I) and PR or QH is continuous,

then the mappings A, B, R, P, H and Q have a unique common fixed point in X.

Proof. By Theorem 1, A, B, QH and PR have a unique common fixed point in X. That is, there exists $z \in X$ such that $Az = Bz = QHz = PRz = z$. Now we prove that $Rz = z$. In fact, by condition (3.2), we have

$$F_{ARz,Bz}(kt) \geq \phi\left(F_{PRRz,QHz}(t), F_{ARz,PRRz}(t), F_{Bz,QHz}(t), F_{ARz,QHz}(\alpha t), F_{Bz,PRRz}((2-\alpha)t)\right)$$

and

$$L_{ARz,Bz}(kt) \leq \psi\left(L_{PRRz,QHz}(t), L_{ARz,PRRz}(t), L_{Bz,QHz}(t), L_{ARz,QHz}(\alpha t),\right.$$
$$\left.L_{Bz,PRRz}((2-\alpha)t)\right).$$

For $\alpha = 1$, we have

$$F_{Rz,z}(kt) \geq \phi\left(F_{Rz,z}(t), F_{Rz,Rz}(t), F_{z,z}(t), F_{Rz,z}t, F_{z,Rz}(t)\right) > F_{Rz,z}(t)$$

and

$$L_{Rz,z}(kt) \leq \psi\left(L_{Rz,z}(t), L_{Rz,Rz}(t), L_{z,z}(t), L_{Rz,z}t, L_{z,Rz}(t)\right) < L_{Rz,z}(t),$$

which is a contradiction. Therefore, it follows that $Rz = z$. Hence $Pz = PRz = z$. Similarly, we get $Qz = QHz = z$. □

Corollary 4. *Let (X, F, L, T, S) be a complete intuitionistic Menger space. Let P and Q be mappings from X into itself such that:*

(4.1) there exists a constant $k \in (0, 1/2)$ such that

$$F_{x,y}(kt) \geq \phi\left(F_{Px,Qy}(t), F_{x,Px}(t), F_{y,Qy}(t), F_{x,Qy}(\alpha t), F_{y,Px}((2-\alpha)t)\right)$$

and

$$L_{x,y}(kt) \leq \psi\left(L_{Px,Qy}(t), L_{x,Px}(t), L_{y,Qy}(t), L_{x,Qy}(\alpha t), L_{y,Px}((2-\alpha)t)\right)$$

for all $x, y \in X, \alpha \in (0, 2), t > 0$ and $\phi \in \Phi, \psi \in \Psi$. Then P and Q have a unique common fixed point in X.

Proof. If we set $A = B = I$ (identity mapping) in Theorem 1, then it is easy to check that the pairs (I, P) and (I, Q) are compatible of type (II) and the identity mapping is continuous. Hence by Theorem 1, P and Q have a unique common fixed point in X. □

Corollary 5. *Let (X, F, L, T, S) be a complete intuitionistic Menger space. Let A and P be mappings from X into itself such that:*

(5.1) $A^n(X) \subseteq P^m(X)$,

(5.2) there exists a constant $k \in (0, 1/2)$ such that

$$F_{A^n x,A^n y}(kt) \geq \phi\left(F_{P^m x,P^m y}(t), F_{A^n x,P^m x}(t), F_{A^n y,P^m y}(t), F_{A^n x,P^m y}(\alpha t),\right.$$
$$\left.F_{A^n y,P^m x}((2-\alpha)t)\right)$$

and

$$L_{A^n x,A^n y}(kt) \leq \psi\left(L_{P^m x,P^m y}(t), L_{A^n x,P^m x}(t), L_{A^n y,P^m y}(t), L_{A^n x,P^m y}(\alpha t),\right.$$
$$\left.L_{A^n y,P^m x}((2-\alpha)t)\right)$$

for all $x, y \in X, \alpha \in (0, 2), t > 0$, and $\phi \in \Phi, \psi \in \Psi$ and for some $m, n \in \mathbb{N}$,

(5.3) $A^n P = PA^n$ and $AP^m = P^m A$.

If the mappings A^n and P^m satisfy one of the following conditions:

(5.4) the pair (A^n, P^m) is compatible of type (II) and A^n is continuous,

(5.5) the pair (A^n, P^m) is compatible of type (I) and P^m is continuous,

then A and P have a unique common fixed point in X.

Proof. If we set $A = B = A^n$ and $P = Q = P^m$ in Theorem 1, then A^n and P^m have a unique common fixed point in X. That is there exists $z \in X$ such that $A^n z = AP^m z = z$. Since $A^n Az = AA^n z = Az$ and $P^m Az = AP^m z = Az$. It follows that Az is a fixed point of A^n and P^m and hence $Az = z$. Similarly we have $Sz = z$. □

Corollary 6. *Let (X, F, L, T, S) be a complete intuitionistic Menger space. Let P, Q and two sequences $\{A_i\}, \{B_j\}$ for all $i, j \in \mathbb{N}$ be mappings from X into itself such that:*

(6.1) there exists $i_0, j_0 \in \mathbb{N}$ such that $A_{i_0}(X) \subseteq T(X), B_{j_0}(X) \subseteq S(X)$,

(6.2) there exists a constant $k \in (0, 1/2)$ such that

$$F_{A_i x, B_j y}(kt) \geq \phi\left(F_{Px,Qy}(t), F_{A_i x,Px}(t), F_{B_j y,Qy}(t), F_{A_i x,Qy}(\alpha t), F_{B_j y,Px}((2-\alpha)t)\right)$$

and

$$L_{A_i x, B_j y}(kt) \leq \phi\left(L_{Px,Qy}(t), L_{A_i x,Px}(t), L_{B_j y,Qy}(t), L_{A_i x,Qy}(\alpha t), L_{B_j y,Px}((2-\alpha)t)\right),$$

for all $x, y \in X, \alpha \in (0, 2), t > 0$, and $\phi \in \Phi, \psi \in \Psi$.

If the mappings A_{i_0}, B_{j_0}, P and Q satisfy any one of the following conditions:

(6.3) the pairs (A_{i_0}, P) and (B_{j_0}, Q) are compatible of type (II) and A_{i_0} or B_{j_0} is continuous,

(6.4) the pairs (A_{i_0}, P) and (B_{j_0}, Q) are compatible of type (I) and P or Q is continuous, then A_i, B_j, P and Q have a unique common fixed point in X for $i = j = 1, 2....$

Proof. By Theorem 1, the mappings P, Q, A_{i_0}, B_{j_0} for some $i_0, j_0 \in \mathbb{N}$ have a unique common fixed point in X. That is, there exists a unique point $z \in X$ such that $P(z) = Q(z) = A_{i_0}(z) = B_{j_0}(z) = z$. Suppose that there exists $i \in N$ such that $i \neq j_0$. Then by (6.2), with $\alpha = 1$, we have

$$F_{A_i z,z}(kt) = F_{A_i z,B_{j_0} z}(kt)$$
$$\geq \phi\left(F_{Pz,Qz}(t), F_{A_i z,Pz}(t), F_{B_{j_0} z,Qz}(t), F_{A_i z,Qz}(\alpha t), F_{B_{j_0} z,Pz}(t)\right) > F_{A_i z,z}(kt)$$

and

$$L_{A_i z, z}(kt) = L_{A_i z, B_{j_0} z}(kt)$$
$$\leq \phi\left(L_{Pz,Qz}(t), L_{A_i z,Pz}(t), L_{B_{j_0} z,Qz}(t), L_{A_i z,Qz}(\alpha t), L_{B_{j_0} z,Pz}(t)\right) < L_{A_i z,z}(kt),$$

which is a contradiction. Hence for all $i \in N$, it follows that $A_i(z) = z$. Similarly for all $j \in N$, we have $B_j(z) = z$. Therefore for all $i, j \in N$, we have $P(z) = Q(z) = A_i(z) = B_j(z) = z$.

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