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CONTINUOUS IMPROVEMENT IN THE QUALITY OF PRODUCTS USING FUZZY SETS

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Abstract: Improvement is a goal that is present in several of our daily activities and projects. Currently, improvement is a necessity for industrial processes. Optimization methods play a vital role to achieve this goal, as these methods allow evaluating the existence and significance of the improvement. In addition, these methodologies are relevant in the planning of the strategy which has to be followed in order to improve the process characteristics. Thus, in this paper, four optimization methods that utilize fuzzy set theory and multiple attribute decision making are proposed. These have been shown to be more efficient when comparing them through an example with other classic methods of statistical optimization. An industrial example has been used as a baseline for this comparison. The results are illustrated using a graphic technique of optimization.

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1. Introduction

Improvement is a word that is present almost in every daily activity. Also, it is a parameter that is used as reference to evaluate the advancements and

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achievements which are being accomplished in the work. In the production processes, not only the improvement but the continuous improvement plays an important role. Thus in this paper, we focus our attention to the continuous improvement in an industrial process.

Studies on the evolution of knowledge show that the acquisition of new knowledge behaves as a spiral. It is possible to say that a continuous improvement is implicitly behind this behavior. Experts on the field of total quality take this concept to illustrate how new knowledge is acquired, and consequently the quality of the products is improved.

Juran [13] uses a spiral to illustrate the advances that are obtained in sequential manner during the study of the processes. Whereas, Deming [5] proposes to use a circle with a starting point to evaluate the achievements on a process. This circle has four points: planning, doing, verifying and acting. This circle is dynamic because acting generates new knowledge meaning an improvement of the process. Then, we go back to the starting point of the circle. This cycle leads to the continuous improvement. An application of this approach in the improvement of a process was reported by Reid and Koljonen [20]. Other approach is that proposed by Box [1] which consists of an improvement cycle and iterative learning. In other words, the continuous improvement is the new knowledge that is generated by iterative learning. At each step, new information is generated and used to build the mathematical model.

Variation is around each performed activity and present in every process. Every work is a series of interconnected processes and the identification, characterization, quantification, control and reduction of the variance offer the opportunity of improving. The improvement of an industrial process is highly related to the distance between the average value of the response and a reference value, and the variation of the process. Hence, the methods of mathematical optimization seek to optimize both the mean and the variance.

There are different situations in a process which we wish to optimize or improve in a continuous way. For instance: augment the production in a industrial process; reduce the production and/or administrative costs; minimize the number of impurities in a chemistry process; reduce the indexes/levels of pollution in an industrial process; enlarge the life of a product on a shelf; improve the reliability of the equipment; developing new products to increase the positive properties and performance; obtaining a superior quality of the products.

In the context of optimization, the experimental planning plays a relevant role in the procedure to establish the optimal conditions of a process operation. In this paper, we firstly outline the idea of the robust design where the experiment is restricted to a scheme with replications. Hence, it is possible to generate the mean and variance at each experimental condition. Therefore, there are two responses that can be modelled according to the process factors. Thus, the objective of our work is to propose and to contrast five optimization procedures for both the mean and the standard deviation. Furthermore, we evaluate the impact that these responses produce in the process capability indices and the loss function. The first four methods consider the multi-objective pattern of optimization developed in the context of the fuzzy set theory. The fifth contemplate the elaboration of an index in which the weights for the variables are obtained using the approach of fuzzy multiple attribute decision making. The proposed methods have been developed in the framework of fuzzy set theory and have the advantage that both responses can be seen as a objective function. Thus, the optimization of a response is not restrictive of the other response. This gives a great flexibility to the decision making of the selection of alternative solutions.

In the literature about statistic optimization, there are several methodologies that have been proposed to model jointly the mean and the variance. These have been presented by Vining and Myers [24], Lin and Tu [18], Copeland and Nelson [4], Kim and Lin [15], Tang and Xu [23], Köksoy and Doganaksoy [17] and Jeong et al [12]. An industrial example has been used as a baseline to compare the efficiency of every new approach. Here, we are going to use this example to contrast the five proposed techniques against the optimal results of previously published methods. Finally, the differences in the results are discussed.

The rest of this paper is organized as follows: Firstly, the parameters used to measure the improvement of a process are described, followed by the experimental procedure, the statistical modelling as well as the strategy of optimization. Then, the proposed methodologies and the TOPSIS (Technique for Order-Preference by Similarity to Ideal Solution) method are explained. Afterward, using a standard example, a comparison of the performances of the optimization techniques is performed. Finally, some conclusions are given based on the results obtained by a graphic method.

2. Problem Overview

2.1. Parameters to Measure Improvement

An important parameter to evaluate the efficiency of a process and its improvements is the capacity index, C_p. This index is defined by $C_p = \frac{\text{LSL–USL}}{6\widehat{\sigma}}$, where LSL and USL stand for the lower specification and upper specification limits of the process, respectively, see [3]. These limits are established by a given criteria based on the quality necessities. Whereas $\widehat{\sigma}$ bears in mind the estimated variation. An index C_p with a value lower than 1 is unacceptable, a value between 1 and 1.33 is barely acceptable, and a value greater than 1.33 is desirable. The strategy to reach a desirable index is minimizing the variance $\widehat{\sigma}^2$ o standard deviation σ. As this index does not take into account the mean, other index more realistic is proposed. This index, which is named process capacity index, is denoted by C_{pk} and explains the deviations between the estimated mean, $\widehat{\mu}$, and an objective value, M, which defines a characteristic of optimal quality. The index C_{pk} is given by: $C_{pk} = \min\{CPI, CPS\}$, where $CPI = \frac{\widehat{\mu}-LSL}{3\widehat{\sigma}}$ and $CPS = \frac{USL-\widehat{\mu}}{3\widehat{\sigma}}$. This can be rewritten as:

$$C_{pk} = C_p(1-k), \text{ where } k = \frac{2\,|M-\mu|}{LSE-LIE}. \tag{1}$$

Accordingly, it can be observed that a process improves when it is centered (i.e., $k = 0$) or if the variance, $\widehat{\sigma}^2$ decreases. Based on this statement, an objective of the continuous improvement is to center the process with the minimum variance. However, some considerations should be considered to use this index: it is important to have customer driven or functional specification, to assume the process is in statistical control, to suppose the data follow a normal distribution, and it is best to have at least 100 observation in the data set. On the other hand, the loss function defined by Taguchi [22], it is also used as measure of quality and product improvement. The loss function is an approach to assessing process capability. It may be written as $L(y) = k(y-M)^2$. If the random variable Y has mean μ and variance σ^2, then the expected loss function $L(y)$ is $L = E(L(y)) = k(\sigma^2 + (\mu - M)^2)$. The estimation of L is given by:

$$\widehat{L} = k(\widehat{\sigma}^2 + (\widehat{\mu} - M)^2). \tag{2}$$

In this case, the loss function estimate is optimum when the variance is reduced and the difference between the quality mean μ and the quality target M is also reduced.

In every process, there are a series of factors that affect the response(s), that is, the characteristic(s) of the quality of a process. With the aim of identifying these factors, an experimental plan is proposed. From the experimental results, the responses are modelled. In this case, the mean and standard deviation are modelled. Then, we propose an optimization scheme for these models.

2.2. Experimental Design

The experimental scheme is shown in Table 1, which has the structure of a double array (Wu and Hamada [26]). Here $X = (X_1, \ldots, X_k)$ represent k control factors that are related to the process, and $Z = (Z_1, \ldots, Z_q)$ are q noise factors that are independent to the process. Every row $(x_{i1}, \ldots, x_{ik}, i = 1, \ldots, n)$ corresponds to a combination of the values of the k factors X. Equivalently, every column $(z_{j1}, \ldots, z_{jr}, j = 1, \ldots, q)$ is the combination of the q factors Z. Notice that here the combination of the noise factors are treated as replications.

			Z_1	z_{11}	$\ldots$	z_{r1}		
			.	.	$\ldots$	.		
			Z_q	z_{1q}	$\ldots$	z_{rq}		
X_1	$\ldots$	X_k					$\widehat{\mu}$	$\widehat{\sigma}$
x_{11}	$\ldots$	x_{1k}		y_{11}	$\ldots$	y_{r1}	$\overline{y}_1$	S_1
.	$\ldots$	.		.	$\ldots$	.		
x_{n1}	$\ldots$	x_{nk}		y_{1q}	$\ldots$	y_{rq}	$\overline{y}_n$	S_n

Table 1: Experimental structure of a double array design with mean $\overline{y}_i$ and standard deviation S_i

2.3. Models

With the proposed experimental structure, the mean, $\overline{y}_i$, and the standard deviations, S_i, of each treatment are calculated. To these results, a model is fitted by minimum squares. The proposed model is a second order model and is expressed by:

$$y_j = \beta_0 + x'\beta + \Phi x + \varepsilon, \quad j = 1, 2, \tag{3}$$

where β_0 is a constant, $\beta' = (\beta_1, \ldots, \beta_k)$ is a vector of parameters. The vector $x' = (x_{i1}, \ldots, x_{ik})$ gives the settings for the control factors. Φ is a symmetric matrix of order k, and its diagonal consists of the parameters that correspond to the quadratic effect. The elements outside the diagonal are the parameters that show the interaction effect. That is, $\Phi = (\beta_{11}, \ldots, \frac{1}{2}\beta_{1k}, \ldots, \frac{1}{2}\beta_{k1}, \ldots, \beta_{kk})$. Finally, y is the vector of n observations and ε is a random vector, which has a normal probability distribution.

When replications are performed in an experiment, it is possible to fit two models, one for the mean and other for the standard deviation. These two estimated models are:

$$\widehat{\mu}(x) = \widehat{\beta}_0 + x'\widehat{\beta} + x'\widehat{B}x, \tag{4}$$

$$\widehat{\sigma}(x) = \widehat{\alpha}_0 + x'\widehat{\alpha} + x'\widehat{A}x, \tag{5}$$

for the mean and for the standard deviation, respectively. Here $\widehat{\beta}_0$, $\widehat{\beta}$, and $\widehat{B}$ are the estimates of the intercept, linear, and second-order coefficients for the model (4), respectively. Similarly for the other model (5) $\widehat{\alpha}_0$, $\widehat{\alpha}$, and $\widehat{A}$ are the estimates of the intercept, linear, and second-order coefficients.

2.4. Optimization Outline

The general optimization statement for the standard deviation as objective function is:

Optimize:	$\widehat{\sigma}(x)$,
(1) subject to:	$\widehat{\mu}(x) = M$, M is an objective value,
	$x \in R(x)$, is the experimental region
(2) or	$\widehat{\mu}(x) > M$ maximize
(3)	$\widehat{\mu}(x) < M$ minimize

(6)

This objective function can also be used for the optimization of the mean, that is:

Optimize:	$\widehat{\mu}(x)$,
(1) subject to:	$\widehat{\sigma}(x) = L$, L is an objective value,
	$x \in R(x)$.

(7)

The criterion, which is used to plan the global evaluation of the optimization, consists in minimizing the mean squared error (MSE). This considers both the distance between the mean and the objective value, and the variance. The MSE is given by:

$$MSE = \widehat{\sigma}^2(x) + (\widehat{\mu}(x) - M)^2. \tag{8}$$

3. Bi-Objective Optimization Model

3.1. Membership Function Assessment

Finding a common solution for two responses is frequently hard. Then, we typically establish ideal values so the deviation to these values is minimal. Fuzzy set theory has a strong mathematical structure to study in a rigorous way this kind of problems.

The formulation in this context consists in transforming the variable of response $\widehat{y}_j$ $(j = 1, 2)$ into a scale of values. These values should assume the satisfaction level of the experimenter or the decision maker (DM). This is denoted by D_j $(j = 1, 2)$ and with values in the range between 0 and 1. The value approaches 1 when $\widehat{y}_j$ is close to the objective value M_j and decreases monotonously when $\widehat{y}_j$ move away from M_j. If $y^{\min}$ and $y^{\max}$ represent the inferior and superior aspiration levels, respectively, DM does not accept a x solution for either $\widehat{y}_j \leq y^{\min}$ or $\widehat{y}_j \geq y^{\max}$. In this case, there are two membership functions that correspond to the models 4 and 5. Then, the function $D_i(\widehat{y}_j)$ $(j = 1, 2)$ can be expressed for the mean by:

$$D_1(\widehat{y}_1(x_i)) = \begin{cases} 0 & \text{if} \quad \widehat{y}_1(x) < y_1^{\min} \quad \text{or} \quad \widehat{y}_1(x) > y_1^{\max}, \\ 1 - \frac{M_1 - \widehat{y}_1(x)}{M_1 - y_1^{\min}} & \text{if} \quad y_1^{\min} \leq \widehat{y}_1(x) \leq M_1, \\ 1 - \frac{\widehat{y}_1(x) - M_1}{y_1^{\max} - M_1} & \text{if} \quad M_1 < \widehat{y}_1(x) < y_1^{\max}, \end{cases}$$

and for the standard deviation:

$$D_2(\widehat{y}_2(x_i)) = \begin{cases} 1 & \text{if} \quad \widehat{y}_2(x_i) < y_2^{\min}, \\ 1 - \frac{\widehat{y}_2(x_i) - y_2^{\min}}{y_2^{\max} - y_2^{\min}} & \text{if} \quad y_2^{\min} \leq \widehat{y}_2(x_i) \leq y_2^{\max}, \\ 0 & \text{if} \quad \widehat{y}_2(x_i) > y_2^{\max}. \end{cases}$$

There are several criteria to determine these limits. For instance, the specification limits of a product, regulations or standards of a company, or merely in a subjective manner. If it is necessary to determine the limits based on a functional range of the process, then it is advisable to consider the minimums and maximums of the individual estimated responses, i.e.,

$$\left(y_j^{\min} = \min_{x \in R} [\widehat{y}_j(x)],\ y_j^{\max} = \max_{x \in R}\ [\widehat{y}_j(x)] \right).$$

3.2. Proposed Fuzzy Optimization

Triangular memberships are convenient for generalizing the triangular inequality of a metric. In addition, using triangular norms (t-norms) as aggregation approach results in a convex optimization problem. Furthermore, they have the advantage of being computational simple and economic, [8], and satisfy the conditions of commutativity, monotonicity and associativity, see [9]. Moreover, the Yager t-norm and Hamacher t-norm have been exhaustively studied with excellent results in the optimization process [10], [21].

By means of the Yager norm **FY**, the bi-objective optimization problem requires a global optimization, i.e., a simultaneous fulfilment of the mean and the standard deviation. For this case, Yager [27] proposed a methodology to find the best alternative. This is achieved by the following definition:

$$\begin{aligned} &\text{Minimize:} \quad F(x_i) = \left\{ \sum_{j=1}^{2} w_j \left[\tfrac{1}{2} \left(D_j(\widehat{y}_j(x_i)) \right) \right]^2 \right\}^{1/2} , \\ &\text{subject to:} \quad x \in R(x), \end{aligned} \tag{9}$$

for $i = 1, ..., n$, where w_j are the weights and generalize the function $F(x)$.

Other alternative is to utilize the Hamacher norm **HK**, particularly, the extension proposed by Kaymak and van Nauta Lemke [14]:

$$\begin{aligned} &\text{Minimize:} \quad H(x_i) = \frac{1}{1 + \sum_{j=1}^{2} w_j \frac{\left(D_j(\widehat{y}_j(x_i)) \right)}{1 - \left(D_j(\widehat{y}_j(x_i)) \right)}} , \\ &\text{subject to:} \quad x \in R(x), \end{aligned} \tag{10}$$

for $i = 1, ..., n$, where w_j are the weights, and $H(x_i) = 0$ when $D_j(\widehat{y}_j(x_i)) = 1$ for any i.

Another technique for optimization is the product norm **NP**. This expression is similar to the proposed by Derringer and Suich [6]:

$$\begin{aligned} &\text{Miminize:} \quad G(x_i) = \prod_{j=1}^{2} \left[(D_j(\widehat{y}_j(x_i))) \right]^{w_i} , \\ &\text{subject to:} \quad x \in R(x). \end{aligned} \tag{11}$$

Furthermore, optimization can be achieved by maximizing the minimum degree of satisfaction. Kim and Lin [15] apply this concept to the bi-objective optimization (KL method):

$$\begin{aligned} &\text{Maximize:} \quad \lambda , \\ &\text{subject to:} \quad D_1(\widehat{y}_1(x_i)) \geq \lambda , \\ &\qquad\qquad\quad\;\; D_2(\widehat{y}_2(x_i)) \geq \lambda , \\ &\qquad\qquad\quad\;\; x \in R(x). \end{aligned} \tag{12}$$

A variation to these decision methodologies consists of creating an index as described next.

3.3. Optimization Index

The application of fuzzy logic on decision making plays an important role in the process of optimization [16], [25]. The procedure used for the optimization strategy comes from TOPSIS (Technique for Order-Preference by Similarity to Ideal Solution). This framework proposes a technique to optimize both the mean and the standard deviation. This technique consists on building an index that bear in mind two solutions: the desired solution (i.e., ideal solution) and a negative ideal solution. Thus, the modelling of the index is reported next. With the level curves of the index model the MSE is estimated. Hence it is possible to evaluate the efficiency of the result and compare it against other methodologies.

Consider the basic matrix D:

$$D = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1p} \\ y_{21} & y_{22} & \cdots & y_{2p} \\ \cdot & \cdot & \cdots & \cdot \\ y_{n1} & y_{n2} & \cdots & y_{np} \end{bmatrix},$$

where the number of attributes or characteristics is given by y_j $(j = 1, \ldots, p)$, and n represents the number of observations (which are equivalent to the experimental results in each treatment). For the evaluated case in this work, there are two attributes ($p = 2$), one for the mean and one for the standard deviation.

3.4. Procedure to Obtain the Optimization Index

The proposed optimization algorithm is composed of the following steps:

1. Firstly, we obtain the values of y_{ij} by: $y_{i1} = \overline{y}_i$ and $y_{i2} = S_i$, then the variable d_{ij} is constructed by normalizing the response variables, y_{ij}:

$$d_{ij} = \frac{y_{ij}}{\sqrt{\sum y_{ij}^2}}. \tag{13}$$

The variable d_{ij} is pondered using the weights, w_j, and the variable, v, is constructed in the framework of fuzzy logic by $v_{ij} = w_{ij} d_{ij}$. There are different criteria to assign the weights, w_j. In the results reported, all weights have the same value.

2. At this step, an ideal optimal solution is proposed and the distance D_i^+ is calculated:

$$D_i^+ = \sqrt{\sum_{j=1}^{2}(v_{ij} - A_j^+)^2}, \tag{14}$$

where

$$\begin{aligned} A^+ &= \{(\max_i(v_{ij}) = |j\epsilon J) \text{ or} \\ &\quad (\min_i(v_{ij}) = |j\epsilon J)(\min_i(v_{ij}) = |j\epsilon J\prime) \text{ for } i = 1,\dots,n\} \\ &= \{A_1^+, A_2^+, ..., A_n^+\}. \end{aligned}$$

J $(j = 1, 2)$ is associated with some profit criterium and $J\prime$ $(j = 1, 2)$ regards to cost.

Observe that some responses are not restricted, i.e., both responses (the primary and the secondary) are on the same level of importance conversely to the multiobjective criterion. This gives more flexibility to take decisions and explore alternative solutions, see [17]. For the statement of optimization given in (6), it is necessary to minimize $\hat{\sigma}(x)$ and the optimization of $\hat{\mu}(x)$ presents three cases: minimize, maximize or reach an objective value M. For the last situation, the response y_{i1} can be rewritten as $z_{i1} = |\bar{y}_i - m|$, then we look to minimize z_{i1}. In this sense, $d_{i1} = z_{i1}/\sqrt{\sum z_{i1}^2}$.

3. Also, a negative ideal solution is planned. This solution can be seen as the worst solution. Analogous, the distance D_i^- is obtained:

$$D_i^- = \sqrt{\sum_{j=1}^{2}(v_{ij} - A_j^-)^2}, \tag{15}$$

where

$$\begin{aligned} A^- &= \{(\min_i(v_{ij}) = |j\epsilon J) \text{ or} \\ &\quad (\max_i(v_{ij}) = |j\epsilon J)(\max_i(v_{ij}) = |j\epsilon J\prime) \text{ for } i = 1,\dots,n\} \\ &= \{A_1^-, A_2^-, ..., A_n^-\}. \end{aligned}$$

4. Finally, we obtain the index:

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-}. \tag{16}$$

If the ideal optimal solution is reached (i.e., $C_i = 1$), the optimal solution for the process is achieved. This means that the positive ideal solution tends to zero. Also, it is possible to obtain alternative solutions which are suitable, for instance, if the index has a value between 0.7 and 1. When the process supports the negative ideal solution, the index tends to zero.

Equations (13) and (16) utilize the data generated by the experiments. Nevertheless, rather that using experimental data, it is possible to use data predicted by a quadratic regression model or the best model. Accordingly, the estimation of the index (equation (16)) is based on the regression models given by equations (4) and (5). The weights w_i (from the step 1 of the algorithm) give preference to some response variables. It is expected that the regression model generated by the index C_i consider the importance of the weights w_i. This methodology based on fuzzy theory (FT method) allows the possibility of selecting ranges of values which are near to the optimal one. Hence, feasible solutions which are approximated to the optimal can be generated.

3.5. The Printing Ink Example

Box and Draper [2] presented an example about the capacity of a printing office to print full-color labels. It is considered that three factors at three levels have effect on the ink printing. These factors are:

Factors	Levels:	1(-1)	2(0)	3(1)
x_1: speed		30	45	60
x_2: precision		90	110	130
x_3: distance		12	20	28

This design has been used as a reference to different authors to illustrate and compare the results obtained from applying their proposed methodologies for the optimization of both the mean and the standard deviation. Accordingly, this example is used here to make a global comparison of all the results. The design is a complete factorial 3^3. The data is shown in Table 2. Columns 8 and 9 represent the mean ($y_{i1} = \overline{y}_i$) and the standard deviation ($y_{i2} = S_i$) for every treatment, respectively. The last column reproduces the index that corresponds to the method TOPSIS applied to each treatment.

3.5.1. Full Models for $\widehat{\sigma}(x)$ and $\widehat{\mu}(x)$

The models obtained after adjusting the mean, $\overline{y}$, and the standard deviation, S, are presented next. The model for the mean is given by:

$$\begin{aligned} \widehat{y}_1(x) = \quad & 328 + 177x_1 + 109x_2 + 131.5x_3 + 32x_1^2 - 22.4x_2^2 \\ & -29.1x_3^2 + 66x_1x_2 + 75.5x_1x_3 + 43.6x_2x_3. \end{aligned} \tag{17}$$

$Trat$	x_1	x_2	x_3	r_1	r_2	r_3	y_{i1}	y_{i2}	IO
1	-1	-1	-1	34	10	28	24.0	12.5	0.66
2	0	-1	-1	115	116	130	120.3	8.4	0.72
3	1	-1	-1	192	186	263	213.7	42.8	0.78
4	-1	0	-1	82	88	88	86.0	3.7	0.70
5	0	0	-1	44	178	188	136.7	80.4	0.73
6	1	0	-1	322	350	350	340.7	16.2	0.85
7	-1	1	-1	141	110	86	112.3	27.6	0.73
8	0	1	-1	259	251	259	256.3	4.6	0.81
9	1	1	-1	290	280	245	271.7	23.6	0.79
10	-1	-1	0	81	81	81	81.0	0.0	0.70
11	0	-1	0	90	122	93	101.7	17.7	0.71
12	1	-1	0	319	376	376	357.0	32.9	0.85
13	-1	0	0	180	180	154	171.3	15.0	0.75
14	0	0	0	372	372	372	372.0	0.0	0.86
15	1	0	0	541	568	396	501.7	92.5	0.85
16	-1	1	0	288	192	312	264.0	63.5	0.80
17	0	1	0	432	336	513	427.0	88.6	0.84
18	1	1	0	713	725	754	730.7	21.1	0.69
19	-1	-1	1	364	99	199	220.7	133.8	0.74
20	0	-1	1	232	221	266	239.7	23.5	0.74
21	1	-1	1	408	415	443	422.0	18.5	0.79
22	-1	0	1	182	233	182	199.0	29.4	0.67
23	0	0	1	507	515	434	485.3	44.6	0.74
24	1	0	1	846	535	640	673.7	158.2	0.62
25	-1	1	1	236	126	168	176.7	55.5	0.59
26	0	1	1	660	440	403	501.0	138.9	0.66
27	1	1	1	878	991	1161	1010.0	142.5	0.34

Table 2: Experimental design where r_i is the number of replications $(l = 1, 2, 3)$

The model of second order for the mean has been used widely in the literature to illustrate the different optimization methods. The model is statistically significative with $F(9, 17, 0.05) = 23.94$ but the quadratic effects are not. The model has a determination coefficient $R^2 = 91.6$ and a $CM_{error} = 5634.77$.

The standard deviation model is:

$$\begin{aligned}\widehat{y}_2(x) = \;& 34.9 + 11.5x_1 + 15.3x_2 + 29.2x_3 + 4.2x_1^2 - 1.3x_2^2 \\ & +16.8x_3^2 + 7.7x_1x_2 + 5.1x_1x_3 + 14.1x_2x_3.\end{aligned} \tag{18}$$

This model is not statistically significant and has a R^2 very low, i.e., the data is not explained by the model. However, both models have been utilized by different authors in their proposed methodologies. Nevertheless, these models are not the best statistical models, and therefore the optimization procedures lose precision.

In reference to the optimization methodology given in equation (6), the objective is to minimize the standard deviation, with restrictions on both the mean and the experimental region. Vining and Myers [24] proposed a solution to this (VM method). Their solution was based on the framework of the response surface methodology and utilizes the dual-response technique proposed by Myers and Carter [19]. Consequently, it requires that both models to be full second-order even when they have terms statistically non significative. Then, the technique optimizes some responses bearing in mind the restrictions imposed on other responses. Using the Lagrange method, the function to be optimized can be expresses as:

$$H(x) = \widehat{Y}_2(x) - \lambda(\widehat{Y}_1(x) - M). \tag{19}$$

This method form a baseline for assessment of the efficiency of using methods based on fuzzy logic.

3.6. The Best Models for $\widehat{\sigma}(x)$ and $\widehat{\mu}(x)$

As said, the models given by equations (17) and (18) are not the best models. Hence, the best models are proposed. The model for the mean is:

$$\widehat{y}_1(x) = 314.7 + 177x_1 + 109x_2 + 131.5x_3 + 66x_1x_2 + 75.5x_1x_3 + 43.6x_2x_3. \tag{20}$$

The model is significative with $F(6, 20) = 36.5$. It has a determination coefficient $R^2 = 92$ and a $CM_{error} = 5634.8$.

The standard deviations has a model given by:

$$\widehat{y}_2(x) = 48 + 11.5x_1 + 15.3x_2 + 29.2x_3 + 7.7x_1x_2 + 5.1x_1x_3 + 14.1x_2x_3 + 29.6x_1x_2x_3. \tag{21}$$

This model is significative with $F(7, 19) = 3.2$. It has a determination coefficient $R^2 = 54$ and a $CM_{error} = 1444.7$.

These models can be represented in a visual way using the contour plots. These graphics are very useful as guideline to outline the optimum of both the mean and the standard deviation.

3.7. Calculation of the Index C_i

The index C_i (equation (16)) is estimated from $\overline{y}$ and S for every treatment. The index value of C_i is exposed by the column IO (Table 2). In equation (14), A_i^+ were minimum values. In order to evaluate the global performance of the index, a regression model using the values of Table 2 is built. The best regression model for the index C_i is:

$$C(x) = 8.3 + 0.13x_1 - 0.26x_2 - 0.49x_3 - 0.8x_3^2 - 0.5x_1x_2 - 0.5x_1x_3 - 0.7x_2x_3.$$

The model coefficient are multiplied by 10^{-1}. The optimal condition of this model is for $x_1 = 0.989$, $x_2 = -0.279$, $x_3 = 0.08$ and $C_i = 0.82$. This is a global optimal solution for the process, and the optimal values for the mean and the standard deviation are not known directly. So, with this type of index, it is very hard to obtain a value for MSE. A graphic strategy is utilized in order to have an idea of the value of MSE. In addition, this graphic allows to compare results with the other methods.

4. Optimization Process Results

4.1. Full Models

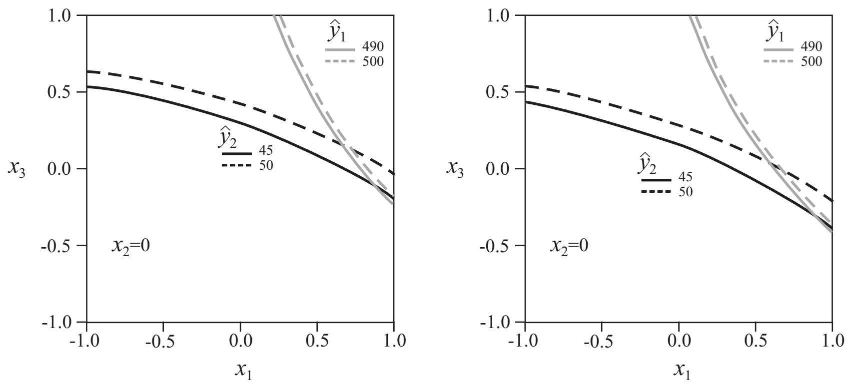
Optimization was performed using all the proposed methods and applying the technique suggested by Jang et al [11]. The methodologies were programmed using *Gauss 8.0*. The main idea is to evaluate the efficiency of every method to reach the optimum for the established objective value for the mean and minimizing the variance. The best results are those that produce the smallest MSE (equation (19)), the biggest process capacity index C_{pk} (equation (1)) and $k = 0$. In addition, the proposed methods are compared with the methodology reported by Vining and Myers [24]. Table 3 shows the results when the full models are considered.

Comparing the proposed methods with the VM methodology, the former produced a better result when the MSE values are considered. In addition, despite no being favorable for a process, the C_{pk} values were also better for the proposed methods. These small values in the capacity index lead to ascertain directly in the process of reducing the variance.

Figure 1 shows the contour plots of the models described by equations (17) and (18). Both plots are overlaid. In order to evaluate the performances, one

Method	x_1	x_2	x_3	$\widehat{y}_1(x) = \widehat{\mu}(x)$	$\widehat{y}_2(x) = \widehat{\sigma}(x)$	MSE	C_{pk}
FY	1.091	0.265	-0.442	500	44.92	2018.6	0.74
HK	0.722	0.454	-0.184	500	47.99	2299.8	0.70
NP	0.733	0.382	-0.151	500.2	48.01	2305.4	0.69
KL	1.0	-0.070	-0.250	492.3	44.18	2010.8	0.70
TOPSIS*	0.989	-0.279	0.081	499.4	46.45	2157.7	0.72
VM	0.620	0.230	0.100	500	51.7	2679.7	0.64

Table 3: Optimization process results (*using the original data)



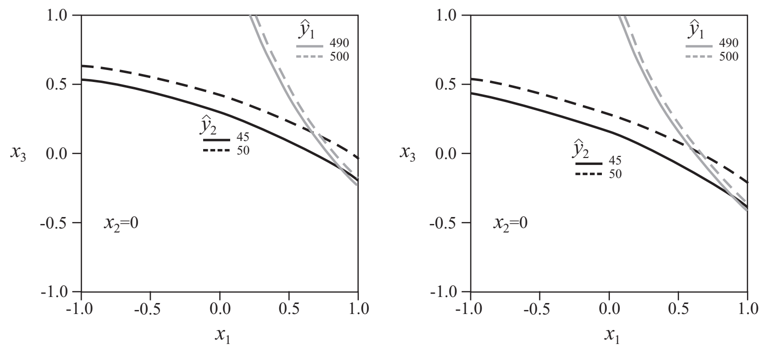
(a) Factor $x_2 = 0$ (b) Factor $x_2 = 0.3$

Figure 1: Contour plots for the models $\widehat{y}_1$, and $\widehat{y}_2$, with (a) $x_2 = 0$ and (b) $x_2 = 0.3$

factor is fixed. In this case, the factor x_2 is fixed to 0 and 0.3 as illustrated by Figures 1(a) and 1(b), respectively. Accordingly, it is possible to obtain different scenarios of the behavior of the mean, $\widehat{y}_1 = \overline{y}$, and the standard deviation, $\widehat{y}_2 = S$.

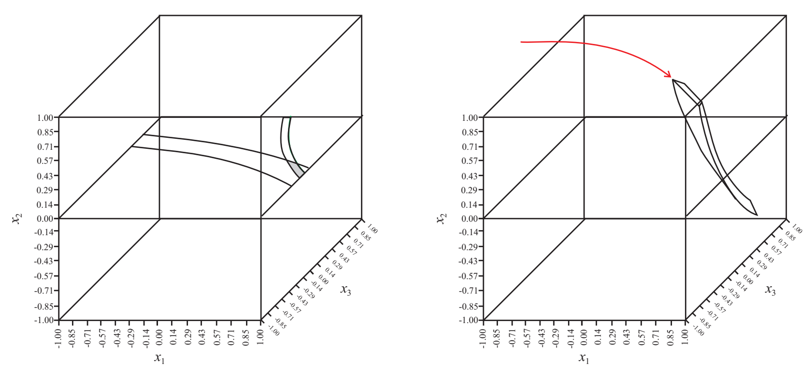
The plots shown in Figure 1 confirm the results reported in Table 3. Furthermore, the plots help to visualize other possible optimal values. These values can be used as initial points for the optimization process. In addition, the contour plots are an excellent assistant in the industrial practice because they allow to predict other optimal values. The value of the variable x_1 has to be extended and the obtained points have to be verified by experimentation (e.g., as shown in the right region of Figure 2(b)). It is observed that standard deviation lessens under 40 units.

With this strategy, a comparison of methods can be performed. Further-



(a) Factor $x_2 = 1$ (b) Factor $x_2 = -1$

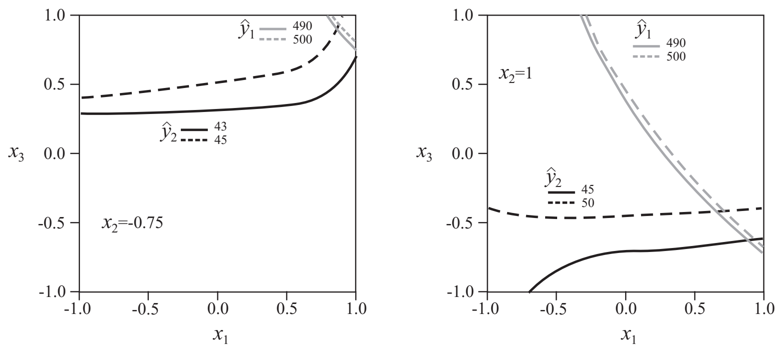
Figure 2: Contour plots for the models $\widehat{y}_1$, and $\widehat{y}_2$, with (a) $x_2 = 1$ and (b) $x_2 = -1$, with extension in the region for x_1



(a) (b)

Figure 3: Optimal region with different cross selections for x_2

more, it is possible to obtain other solution using the graphic method. For this, it is necessary to vary x_2 between -1 and 1, see [7]. In addition, the optimal region determined by the contour plots in the plane (x_1, x_3) and ($-1 \leq x_2 \leq 1$) contains the solutions of the proposed methods. These solutions approach to the results reported in Table 3.



(a) Factor $x_2 = -0.75$ (b) Factor $x_2 = 1$

Figure 4: Contour plots for the best models $\widehat{y}_1$, and $\widehat{y}_2$, with (a) $x_2 = -0.75$ and (b) $x_2 = 1$

4.2. The Best Models

Considering the models with parameters statistically significative, the results obtained after optimization are highly improved. Table 4 shows that the MSE is reduced and the method NP has the smallest MSE.

Figure 4(a) illustrates a feasible region to find an optimum, with a standard deviation that vary between 43 and 45 when x_2 is fixed to -0.75. This is reflected in a reduction on the MSE. Accordingly, all the methods produced better results than the proposed by Vining and Myers [24].

Method	x_1	x_2	x_3	$\widehat{y}_1(x) = \widehat{\mu}(x)$	$\widehat{y}_2(x) = \widehat{\sigma}^2(x)$	MSE	C_{pk}
FY	0.979	-0.758	0.835	500	43.2	1869.95	0.77
HK	0.893	-0.725	0.892	500	46.1	2125.3	0.72
NP	1.014	-0.863	0.924	500	36.29	1318.9	0.92
KL	0.958	-0.783	0.893	500.6	42.1	1766.5	0.79

Table 4: Results of the optimization process utilizing the best models

The Nelder Mead algorithm allowed to explore other regions. As illustrated in Figure 4(b), the optimum reported by each method were highly improved. In addition, when the experimental region is extended for values greater to 1 in x_1, the values of the standard deviation are reduced, e.g., least to 30. To verify this reduction in the practice, tests of experimental confirmation are carried out in these points. This situation can be seen in Figure 4.

Method	x_1	x_2	x_3	$\widehat{y}_1(x) = \widehat{\mu}(x)$	$\widehat{y}_2(x) = \widehat{\sigma}^2(x)$	MSE	C_{pk}
FY	1.340	0.785	-0.781	499	22.5	506.5	1.48
HK	0.940	0.840	-0.530	498.4	41.1	1688.9	0.81
NP	1.45	0.500	-0.670	499.9	34.51	1179.0	0.97

Table 5: Optimal obtained using the initial points suggested by Figure 3

5. Conclusions

The methods based on fuzzy logic perform well even when the regression models have a low significance level and low determination coefficient. It was expected that other optimization methodologies had a better performance when models have a high significance level. Nevertheless, the proposed methods showed to have a better performance when the significance level was high. In addition, the proposed techniques allowed to discover situations which can be very useful into practice.

Although the methodologies were utilized to solve a reference problem, they can be directly applied to any kind of design with double orthogonal array, designs with replications, or in cases in which it is possible to generate models for both the mean and the variance. Future work includes put in practice these methodologies to solve a real industrial problem in which proving tests can be performed. Hence, it would be possible to evaluate the importance of these techniques.

The TOPSIS method reflects properly the information given by the real data of Table 5. In this case, the real data points out that, for every treatment, the values are distant to be ideal. For the first response, the index demonstrates clearly that the regression model was not good enough statistically.

The information provided by the contour plots is relevant as it can be used to establish initial points for the optimization process. Also, they are useful to predict a larger region and then perform experiments in this region. Moreover, when the optimal region is determined, random points can be experimented. This can contribute with vital information to the experimenter about the process.

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