NUMERICAL SIMULATION OF GRANULAR FLOW
DURING FILLING AND DISCHARGING OF A SILO

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Abstract: In this study, we simulate the static and dynamic processes of granular flow during filling and discharging of a vertical-sided silo with conical hopper bottom. The granular material is an assembly of 7,500 soybeans. Based on the discrete element method, the governing equations for the granular flow are solved by the centred finite difference scheme. The effects of inlet flow rate and bottom angles on pressure distribution on the hopper wall throughout the static process of material filling into the silo are investigated. Influences of bottom angle on the discharging problem in a varied mass-flow silo and a constant mass-flow silo are also discussed.

AMS Subject Classification: 93A30, 76T25
Key Words: mathematical modelling, discrete element method, granular flow, right-conical silo

Received: June 24, 2010 © 2010 Academic Publications

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1. Introduction

Silos have been used as devices for storing granular solids in many industries over the last ten decades. In industrial practices, many problems still frequently occur during the filling and discharging processes. These problems include flow blockage, segregation, dead zones and silo collapse. As industrial competition is high, these problems, especially the silo collapse and the flow blockage, are expected to become more and more critical to the success of the process. It is well known that the pressures acting on the walls of a silo play an essential role of the silo structural design.

Over the last few decades, extensive research has been carried out to study the physics of granular materials including the properties of granular flow, the silo pressure, the flow rates of granular materials flowing out of silos, etc. Early research mainly focused on experimental investigation (see [1, 16, 23]) and deriving approximate analytical methods (see [7, 17, 18, 8, 9]). Generally, due to a large number of particles used in the system, experiment is limited and numerical methods have thus been the major approach used in this study. The numerical methods including continuum mechanic methods and discrete element method (DEM) have been developed to predict the pressure distribution on the silo walls. Since late 1970s, the continuum mechanics models based on the finite element method (FEM) have been developed to study rapid flows of granular material. Taking into account the micro-mechanics characteristics to the model, granular material is identified as a continuum and the particulate nature of the flowing bulk is ignored completely. The FEM based on the plastic flow rule theory has been used to study stress fields in silo (see [15, 21, 4, 3, 10]). The results obtained in general do not agree well with experimental results. Thus, the FEM is not able to yield accurate predictions on granular flow in silos. Due to discontinuous physical properties of granular material with respect to position and time, the DEM, originally developed by Cundall and Strack (1979) (see [2]), has been recognized as an efficient method for studying the physics of granular materials in silos. The DEM considers a finite number of discrete interacting particles and can describe granular flow at the particle scale based on Newton’s equations of motion related to translational and rotational motions. The gradients of translation and rotation of a particle are determined in terms of the forces and the torques exerted on it. The contact forces for each contact can be estimated by the Voigt model (see [2]), simplified Hertz-Mindlin and Deresiewicz model (see [11, 12, 13]) and Walton and Braun model (see [20, 19]). The torque causing a particle to rotate can be conducted by the tangential force, namely the tangential torque, and the rolling friction force,
namely the rolling torque. In the granular models, either the tangential torque (see [21]), rolling torque (see [5, 6]), or the tangential torque together with the rolling torque (see [22, 14]) is applied. These studies have resulted in a basic understanding of the physics of the granular material and provided some basic guidelines for the silo designed and operation. However, many phenomena such as pressure distribution on the silo wall, the formation of arching and the flow behaviour have not been fully understood nor well controlled. Thus, further study of the filling and discharging processes and development of robust mathematical models for simulating the flow of granular materials become more and more important for the design of silos and optimization of these processes.

In this study, we develop an efficient DEM model to study the granular flow in a vertical-sided silo with a conical hopper bottom during the filling and discharging processes. In the filling process, effects of inlet flow rate and hopper angle on the pressure on the bottom wall are investigated. In the discharge process, we investigate the pressure of the bottom wall of the varied-mass flow silo and constant mass-flow silo under various hopper angles. Section 2 concerns the mathematical model and the method of solution for the problem. A numerical example is given in Section 3. Conclusions and discussion are given in Section 4.

2. Mathematical Model

The governing equations describing the motion of a particle $i$ are the principle of linear momentum and the principle of angular momentum, namely

$$m_i \frac{d^2 \mathbf{r}_i(t)}{dt^2} = \mathbf{F}_i(t) + m_i \mathbf{g},$$

$$I_i \frac{d^2 \theta_i(t)}{dt^2} = M_i(t),$$

where $\mathbf{r}_i$ and $\theta_i$ are the position vector and the rotation vector of the center of the particle; $m_i$ and $I_i$ denote respectively the mass and the moment of inertia of the particle; $\mathbf{F}_i(t)$ and $M_i(t)$ represent the total force and torque acting on particle $i$ at time $t$, respectively.

The total force, $\mathbf{F}_i(t)$ in equation (1) acting on particle $i$, is defined by

$$\mathbf{F}_i(t) = \sum_{j=1,j\neq i}^{q_p(t)} \mathbf{F}_{i,j} + \sum_{w=1}^{q_w(t)} \mathbf{F}_{i,w},$$

in which the first and the second terms on the RHS of equation (3) are the
interaction force acting on particle $i$ by all contact particles $j = 1, \ldots, q_p$ and the wall contact force acting on particle $i$ by all contact walls $w = 1, \ldots, q_w$, respectively.

Based on a dynamic process and a viscoelastic model as shown in Figure 1(b), the interaction force is the sum of the normal force and the tangential force:

$$ F_{i,j}(t) = F_{ijn}(t)n + F_{ijs}(t)s, $$

where $n$ and $s$ are unit vectors in the normal and tangential directions of the contact plane as shown in Figure 1(a).

Once the contact condition

$$ \delta_n = R_i + R_j - \| r_i - r_j \| \geq 0, $$

is met, two particles $i$ and $j$ with radii $R_i$ and $R_j$ are in contact and

$$ F_{ijn}(t) = -k_n\delta_n + \eta_n(v_{ij} \cdot n), $$

where $k_n$ and $\eta_n$ denote respectively the stiffness of the spring and the damping coefficient in the normal directions, $\delta_n$ represents the amount of overlap as defined in equation (5), and $v_{ij}$ is the relative velocity determining the interaction force via a contact force law and is defined by

$$ v_{ij} = \left( \frac{d}{dt}r_i - \frac{d}{dt}r_j \right) + \left( R_i \frac{d}{dt}\theta_i + R_j \frac{d}{dt}\theta_j \right) s. $$

The tangential force in equation (4) can be determined if the slip condition

$$ \mu|F_{ijn}| < k_s|v_{ij} \cdot s| $$

is met and

$$ F_{ijs}(t) = \left[ \text{sign}(\delta_s)\mu|F_{ijn}|(v_{ij} \cdot s) \right] s, $$

where $\mu$ and $k_s$ denote respectively the friction coefficient and the stiffness of the spring in the tangential direction, and $\delta_s$ is defined by

$$ \delta_s = \left[ (v_i - v_j) \cdot s - R_j \frac{d}{dt}\theta_j - R_i \frac{d}{dt}\theta_i \right] \triangle t $$

for the small time step $\triangle t$.

The wall contact force acting on particle $i$ by the wall $w$ can be expressed as

$$ F_{i,w}(t) = F_{iwn}(t)n + F_{iws}(t)s, $$

where $n$ and $s$ are unit vectors in the normal and tangential directions of the wall contact plane as shown in Figure 1(d).

To determine the above forces, we firstly convert coordinates of the particle
### Table 1: Values of the model parameters used in the simulation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Silo geometry</strong></td>
<td></td>
</tr>
<tr>
<td>height, (m)</td>
<td>1.2</td>
</tr>
<tr>
<td>width, (m)</td>
<td>0.4</td>
</tr>
<tr>
<td><strong>Granular material</strong></td>
<td></td>
</tr>
<tr>
<td>Particle diameter, (mm)</td>
<td>6 and 7.5</td>
</tr>
<tr>
<td>Number of particles with size of 6 mm</td>
<td>3,500</td>
</tr>
<tr>
<td>Number of particles with size of 7.5 mm</td>
<td>4,000</td>
</tr>
<tr>
<td>Particle density, $\rho$ (kg/m$^3$)</td>
<td>1.033</td>
</tr>
<tr>
<td>Simulation time step, $\Delta t$ (s)</td>
<td>$5.2711 \times 10^{-6}$</td>
</tr>
<tr>
<td><strong>Properties</strong></td>
<td><strong>Particle-particle</strong></td>
</tr>
<tr>
<td>Normal contact stiffness, $k_n$ (N/m)</td>
<td>$2.8322 \times 10^4$</td>
</tr>
<tr>
<td>Tangential contact stiffness, $k_s$ (N/m)</td>
<td>$2.5740 \times 10^4$</td>
</tr>
<tr>
<td>Normal damping constant, $\eta_n$ (N/m)</td>
<td>$1.3048 \times 10^2$</td>
</tr>
<tr>
<td>Frictional coefficient, $\mu$</td>
<td>0.33</td>
</tr>
</tbody>
</table>
into local wall coordinates. The new coordinates of the particle can be expressed as
\[
\begin{bmatrix}
x_n \\
y_n
\end{bmatrix} = \begin{bmatrix}
\cos \beta & \sin \beta \\
-\sin \beta & \cos \beta
\end{bmatrix} \begin{bmatrix}
x_i - x_1 \\
y_i - y_1
\end{bmatrix},
\] (12)
where $\beta$ is an angle between the wall $w$ and the $y$-axis, $x_1$ and $y_1$ are the minimum values of relevant coordinates of the wall. The particle $i$ and the wall are in contact if the following conditions are satisfied:
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(i) the contact with the top part of the wall occurs when \( y_n > L \) and \( \delta_{mag} = x_n^2 + (y_n - L)^2 \leq R_i^2 \);

(ii) the contact with the end part of the wall occurs when \( y_n < 0 \) and \( \delta_{mag} = x_n^2 + y_n^2 \leq R_i^2 \);

(iii) the contact with the middle part of the wall occurs when \( \delta_{mag} = |x_n| < R_i \).

Once one of the above wall contact conditions is met, the wall contact force can be calculated by

\[
F_{iwn}(t) = -k_n \gamma_n + \eta_n (v_w - v_i) \cdot n,
\]

where the amount of overlap \( \gamma_n \) can be determined by

\[
\gamma_n = R_i - \sqrt{\delta_{mag}}
\]

for the conditions (i) and (ii) and

\[
\gamma_n = R_i - \delta_{mag}
\]

for the condition (iii). The parameter \( \gamma_s \) in equation (14) is given by

\[
\gamma_s = \left[ (v_w - v_i) \cdot s - R_i \frac{d}{dt} \theta_i \right] \Delta t,
\]

where \( v_w \) and \( v_i \) denote velocities of the wall and particle \( i \), respectively.

The total force acting on the wall at time \( t \) is calculated by

\[
F_w(t) = -\frac{1}{L} \sum_{i=1}^{Q} F_{i,w}(t),
\]

where \( L \) is the length of the wall and \( Q \) is the number of particles in contact with the wall at time \( t \).

In this study, we consider only the tangential torque causing a particle to rotate. The moment \( M_i(t) \) in equation (2) is then determined by

\[
M_i(t) = R_i F_{ijs}.
\]

To simulate the granular flow of \( N \) particles, we let

\[
\frac{d}{dt} r_i(t) = v_i, \quad \frac{d}{dt} \theta(t) = \omega,
\]

and from equations (1) and (2), we obtain

\[
m_i \frac{d}{dt} v_i(t) = F_i(t) + m_i g,
\]

\[
I_i \frac{d}{dt} \omega_i(t) = M_i(t).
\]
Assembling the equations of motions of all particles yields a system of \(8N\) first order differential equations in 2-D cases, namely
\[
\frac{d}{dt} z(t) = w(t), \tag{23}
\]
\[
\frac{d}{dt} w(t) = P(t), \tag{24}
\]
where
\[
z(t) = [r_1(t), \theta_1(t), ..., r_N(t), \theta_N(t)]^T,
\]
\[
w(t) = [v_1(t), \omega_1(t), ..., v_N(t), \omega_N(t)]^T,
\]
\[
P = \left[ \frac{F_1}{m_1} + g, \frac{F_1}{m_1I_1}, ..., \frac{F_N}{m_N} + g, \frac{M_N}{I_N} \right]^T.
\]

Using the centered difference formulation, we can calculate the unknown variables as follows
\[
w^{n+1} = w^{n-1} + 2\Delta t P^n, \tag{25}
\]
\[
z^{n+1} = z^{n-1} + 2\Delta t w^{n+1}. \tag{26}
\]

3. Numerical Example

Using the DEM and numerical scheme as presented in the previous section, a numerical investigation has been conducted to analyze the effect of silo configuration on the flow behavior in the static and dynamic processes of material filling and discharging of a silo. The examples under consideration include vertical-side silos with hopper bottom of various angles of 15°, 30°, 45°, and 60° and various outlet widths of 0.04 m, 0.06 m, and 0.08 m. Granular materials are soybean having two sizes of 0.006 m and 0.0075 m and the silo is made of a steel-sheet. In each of the simulations shown here, an assembly of 7,500 particles is considered. The computational region as shown in Figure 2 represents a two dimensional (2D) silo geometry. The height and the width of the silo are 1.2 m and 0.4 m, respectively. To determine the velocity field and pressure field, the model parameters listed in Table 1 are used.

To study the static process of granular filling in the silo, we use the silo with the outlet width of 0.08 m, and choose two inlet flow rates of 32 ml/s and 172 ml/s and four bottom angles of 15°, 30°, 45°, and 60°. To investigate pressure distribution on the bottom wall of the hopper, we recorded the pressure distribution at 10 segments of each wall as shown in Figure 2. Influence of inlet flow rate on the wall pressure are as shown in Figures 3 and 4.
At $t = 0$ s, particles start to flow at a given inlet flow rate into the empty silo. Filling patterns of particles in the silo with $45^\circ$ bottom angle at 30 s, 70 s and 110 s for two different inlet flow rates are shown in Figure 3. Figure 4 shows the variation of normal pressure with time on each segment of the hopper wall. The pressure variation is large at the early stage of the filling process and it then becomes smaller as the height of the bulk material increases. It is noted that the bottom angle has significant effect on the pressure distribution only on the base of the hopper. The model with higher inlet flow rate and bigger bottom angle give higher pressure on the base of the hopper as shown in Figure 5.

To study the dynamic process of granular discharging from the silo, we used two models of silos including a varied mass-flow silo and a constant mass-flow silo. At the initial time $t = 0$ s, an assembly of 7,500 particles rests on the base of the silo with outlet width of 0.06 m, the discharging process starts when the outlet of the silo is opened. In the varied mass-flow silo, particles are allowed to flow out of the silo under gravitational force. In the constant mass-flow silo, particles that have discharged out of the silo re-enter at the top of the silo and fall on to the top surface of particles.

For a varied mass-flow model, to determine the relative rate of discharge

Figure 2: Computational domain
from movement of interfaces between shaded strata, particles are colored into many layers using two different colors. Four bottom angles of $15^\circ$, $30^\circ$, $45^\circ$
Figure 4: The variations of normal pressure with time along some hopper wall segments in the process of filling materials into the silo with 45° bottom angle and 0.08 m outlet width.

Figure 5: Effect of inlet flow rate and bottom angle on the average normal pressure (a solid line for 15°, a dashed line for 45° and a dotted line for 60°) in the process of filling the silo: (a) \( Q = 32 \text{ ml/s} \); (b) \( Q = 172 \text{ ml/s} \).
and 60° have been used in computation to investigate their impact on the flow pattern, the deformation of the interfaces and the wall pressure. The computed flow pattern, the stable arch, the zig-zag flow and the wall pressure are compared in Figures 6, 7, 8, and 11(a), respectively. The results show that the movement of particles through a conical bottomed silo with a higher bottom angle is faster and its interface is clearly v-shape as shown in Figure 6. A stable arch (bridge) may appear over the hopper outlet of the silo. Here, it is found that the flow of the bulk solid is stopped at time $t = 19\, s$, once the arch is formed over the hopper outlet of the silo having 15° bottom angle and 0.04 m outlet width as shown in Figure 7. It is recognized that using large outlet width can prevent arching. Thus in the following investigations, we only use the silo having 0.06m outlet width. Figure 8 shows that particles flow out of the silo with a zig-zag behaviour. Figures 9 and 10 show the variation of normal pressure with time on some segments of the hopper wall with an angle of 30° during discharging particles from a varied mass-flow silo and a constant mass-flow silo, respectively. It indicates that in the varied mass-flow silo, the pressure distribution reduces its fluctuation as the height of the bulk materials in the silo decreases. In the constant mass-flow silo, the pressure distribution
Figure 7: Formulation of arching above the outlet of the silo with 15° bottom angle and 0.04 m outlet width at three different times: (a) $t = 10\,\text{s}$; (b) $t = 19\,\text{s}$; (c) $t = 22\,\text{s}$

Figure 8: Zig-Zag flow pattern in the process of discharging the silo with 30° bottom angle at three instants of times: (a) $t = 42\,\text{s}$; (b) $t = 47\,\text{s}$; (c) $t = 52\,\text{s}$

always fluctuates with time.

Influence of bottom angle on the bottom wall pressure is investigated for
the varied mass-flow silo. The computed average normal pressures for different bottom angle are compared in Figure 11(a). The comparison indicates that
Figure 11: Average normal pressure along the bottom wall (from point C to point D in Figure 2) in the first 90 s of discharge process under two investigations: (a) effect of three bottom angles of the varied mass-flow silo; (b) effect of two different mass-flow silos with 45° bottom angle.

Bottom angle of the hopper wall is one of the important factors dominating the pressure on the hopper wall. With the increase of the wall angle from 30° to 60°, the wall pressure decreases from 3.3 $kN/m^2$ to 2.5 $kN/m^2$. In this study, the computed average normal wall pressure for different mass-flow silo are also compared in Figure 11(b). It is indicated that the average wall pressure of the constant mass-flow silo is higher than the wall pressure of the varied mass-flow silo.

4. Conclusion

This paper focus on the development of the mathematical model for the study of granular flow in the filling and discharging processes. The model is used to study the effect of the inlet flow rate and bottom angle on pressure distribution on the hopper wall throughout the static process of material filling into the silo and also study the effect of bottom angle on the flow pattern and normal pressure distribution for the discharging process in the varied mass-flow and
constant mass-flow silos. The results gained from the study show that the bottom angle of the hopper wall is one of the important factors dominating the pressure on the hopper wall. Thus, it is important to choose proper bottom angle in the design of silos. In our simulation, dead zone of particles is found near the outlet. It has been known that the outlet width and the vibration of the silo also have significant influence on the pattern of the solid flow and the wall pressure. Therefore, further research will be carried out to include the effect of the outlet width and the silo vibration.

Acknowledgments

The first author gratefully acknowledges the financial support from the Commission on higher Education, Thailand, and the Australia research council.

References


