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CONDITION OF FREE AND CONSTRAINED MOTION OF CABLE CONNECTED SATELLITES SYSTEM IN LOW ALTITUDE ORBIT

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Abstract: Combined effects of the Earth's magnetic field and oblateness of the Earth on the stability of two satellites connected by a light, flexible and inextensible cable in the central gravitational field of the Earth have been studied. The equations of motion of the system have been deduced with respect to the centre of mass, which assumed to move along a low altitude equatorial orbit. We have considered the dipole of the Earth has its axis inclined from the polar axis of the Earth by $11^0 24'$. We have transformed the equation of motion in polar-spherical co-ordinate system to analyze the problem. The Jacobi integral exists for the problem and hence with the help of this integral we have analyzed the condition of free and constrained motion. We have adopted simulation technique using *MATLAB 6.1* to analyse the free and constrained motion of the cable connected system moving in a low altitude equatorial orbit.

AMS Subject Classification: 70F15
Key Words: cable connected satellites, magnetic field of the Earth, oblateness of the Earth, free and constrained motion

1. Introduction

The present paper deals with the condition of free and constrained motion of

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two satellites connected by a light, flexible and inextensible cable with the combined influence of the Earth's magnetic field and the oblateness of the Earth in the central gravitational fields of the Earth. The satellites are considered to be charged material particles and the motion of the system is studied relative to their centre of mass, under the assumption that the later moves along equatorial orbit. While investigating the relative motion of the system of two cable connected satellites, it is supposed that the particles are subjected to impacts of absolutely non-elastic in nature, when the cable tightened up. Cable connected satellites system in space are mathematical modeling of the real space problems such as space vehicle and astronaut floating in space, two or multi sectional satellites system connected by a cable manned space capsule attached to its booster by cable and spinners to provide artificial gravity for the astronaut and finally two satellites at the same time of rendezvous in order to transport a man successfully to an orbiting station. Many space configurations of inter connected satellites system have been proposed and analyzed like two satellites are connected by a rod (dumbell satellite) (see [4]), two or more satellites connected by a tether, M. Krupa et al [7], Beletsky and E.H. Levin [2], A.K. Mishra and V.J. Modi [9]. All these authors have discussed numerous important applications of the system and stability of relative equilibrium if the system is moving on a circular orbit. Beletsky [1], and Beletesky and Novikova [3], studied the motion of a system of two satellites connected by a light, flexible and inextensible string in the central gravitational field of force, relative to the centre of mass, which is assumed to move along a Keplerian elliptical orbit under the assumption that the two satellites are moving in the plane of motion of the centre of mass. The same problem in its general form was further investigated Singh [15], [16], These works conducted the analysis of the relative motion of the system for the elliptical orbit of the centre of mass in two dimensional case as well as three dimensional case. Narayan and Singh [10], [11], [17], studied non-linear oscillations due to the solar radiation pressure provided the centre of mass of the system is moving along an elliptical orbit. Sharma and Narayan [13], [14], studied the combined effects of the solar radiation pressure and the forces of general nature on the motion and stability of the inter-connected satellites system in orbit. Singh et al [17], [18], studied the non-linear effects of the Earth oblateness in the motion and stability of cable connected satellites system in elliptical orbit. Das et al [5], studied the influence of the Earth magnetic field on the stability of the cable-connected satellites system in inclined orbit; where as Narayan et al [14], dealt the same problem for equatorial orbit. In the present paper we have considered effects of the Earth magnetic field and the oblateness of the Earth on the three dimen-

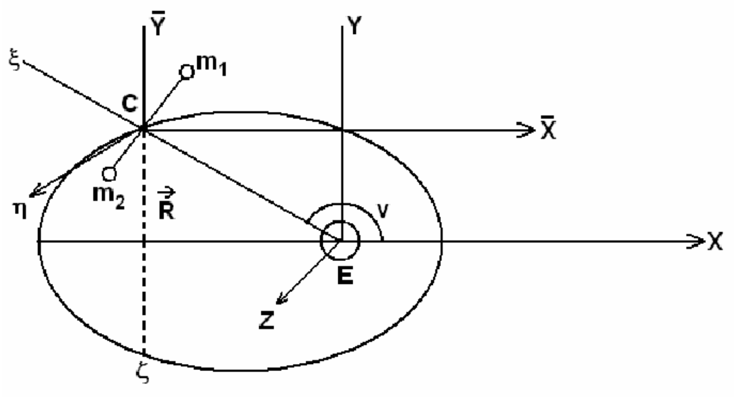


Figure 1: Rotating frame of reference

sional motions and stability on the basis of studying the free and the constraint motion of cable connected satellites system in low altitude equatorial orbit. The perturbing force due to the Earth's magnetic field results from the interaction between space craft's residual magnetic field and the geomagnetic field. The perturbing force is arising due to magnetic moments, eddy current and hysteresis, of these the spacecraft magnetic moment is usually the dominant source of disturbing effects. In the present paper we have analyzed the condition of free and constrained motion, when the system of two satellites, connected by a light, flexible and inextensible cable in a low altitude equatorial orbit under the combined influence of the Earth oblateness and the magnetic field of the Earth, which is facilitating in prediction of behavior of the system. We have used *MATLAB 6.1* version software to analyze and simulate the behavior of the system.

2. Equation of Motion

The combined effects of the Earth's magnetic field and oblateness of the Earth on the motion and and stability of twin satellites system, connected by a light flexible and inextensible cable under central gravitational field of the Earth in the low altitude equatorial orbit, has been discussed. The analysis of free and constraint motion of the system under the influence of the above mentioned perturbing forces has been considered and simulated in the three dimensional plane. The motion and stability of cable connected satellites system under the effects of the Earth's magnetic field, see Das et al [5], Narayan et al [14], in elliptical orbit has been studied. The equation of three dimensional motion of one of the satellites under the rotating frame of reference in Nechvile's coordinate system relative to the centre of mass, which moves along equatorial orbit under the combined influence of the Earth's magnetic field and oblateness of the Earth can be deduced and represented in (2.1).

$$x'' - 2y' - 3\rho x = \lambda_\alpha x + \frac{4Ax}{\rho} - \frac{B}{\rho} \cdot \cos\delta;$$

$$y'' - 2x' = \lambda_\alpha y - \frac{Ay}{\rho} - \frac{B\rho'}{\rho^2} \cdot \cos\delta;$$

$$z'' + z = \lambda_\alpha z - \frac{Az}{\rho} - \frac{B}{\rho}\left[\frac{1}{\mu_E}\left(3p^3\rho^3 - \mu_E\right) \cdot \cos(v - \Omega) - \frac{\rho'}{\rho^2}\sin(v - \Omega)\right] \cdot \sin\delta. \quad (2.1)$$

Here the x-axis is in the direction of the position vector joining the centre of mass and the attracting centre and the y-axis is in along the normal to the position vector in the orbital plane of the centre of the mass in the direction of motion of satellite $m_1$and the z-axis is the normal to the plane containing the motion of the system. Where A is the oblateness due to Earth and B is the magnetic field of the Earth. Moreover

$$A = \frac{3k_2}{\rho^2}; \quad \lambda_\alpha = \frac{p^3\rho^4}{\mu}\left(\frac{m_1 + m_2}{m_1 m_2}\right)\lambda; \quad (2.2)$$

$$B = \left[\frac{m_2}{m_1 + m_2}\right]\left[\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right] - \frac{\mu_E}{\sqrt{\mu\rho}};$$

$$\rho = \frac{R}{p} = \frac{1}{(1 + e\cos v)},$$

where Q_1 and Q_2 are the charge on the satellites of masses m_1 and m_2, where p is the focal parameter, e and v are the eccentricity and true anomaly of the orbit of the centre of mass.

The condition for constraint given by

$$x^2 + y^2 + z^2 \leq \frac{1}{\rho^2}. \quad (2.3)$$

As the general solution of the system of equation of motion (2.1) is beyond our reach. We are interested in analysis of the motion of the system in the case of circular orbit.

3. Analysis of the Motion in the Case of Circular Orbit

In the case of circular orbit of the centre of mass of the system $\rho = 1$, $\rho' = 0$ the equation (2.1) is reduced to the form:

$$\begin{aligned} x'' - 2y' - 3x &= \lambda_\alpha x + 4A_x - B\cos\delta; \\ y'' + 2x' &= \lambda_\alpha y - A_y; \\ z'' + z &= \lambda_\alpha z - Az - B\left[\frac{1}{\mu_E}\left(3p^3 - \mu_E\right)\cdot\cos\left(v - \Omega\right)\right]\cdot\sin\delta, \end{aligned} \tag{3.1}$$

with the condition of constraint

$$x^2 + y^2 + z^2 \leq 1. \tag{3.2}$$

Thus, the small secular affect and long periodic effects of the Earth magnetic field will be studied by averaging the periodic terms in the equation of motion (3.1), integrating with respect to v from 0 to 2π, leading to the following equation:

$$\frac{1}{2\pi}\int_0^{2\pi}\left[\frac{B}{\mu_E}\left(3p^3 - \mu_E\right)\cdot\cos\left(v - \Omega\right)\right]\cdot\sin\delta dv = 0.$$

Therefore, the following averaged equations are describing the long periodic effects and the secular effects of the magnetic field and oblateness on the motion of the system:

$$\begin{aligned} x'' - 2y' - 3x &= \lambda_\alpha x + 4A_x - B\cos\delta; \\ y'' + 2x' &= \lambda_\alpha y - A_y; \\ z'' + z &= \lambda_\alpha z - A_z, \end{aligned} \tag{3.3}$$

when in (3.2) sign of inequality is satisfied, the free motion of the system will take place otherwise the motion will be constrained. Therefore, we have three different types of motion of the system will take place which is given as follows:

(i) Free motion $\lambda_\alpha = 0$;

(ii) Constrained motion $\lambda_\alpha \neq 0$;

(iii) Evolutional motion.

We are interested in the motion and stability of the system in constrained motion. Hence the motion will take place on the unit sphere

$$x^2 + y^2 + z^2 = 1, \quad \text{i.e.} \quad xx' + yy' + zz' = 0. \tag{3.4}$$

Multiplying the first equation of (3.3) by x', the second equation by y', and the third equation by z' respectively, adding and neglecting the small secular and periodic forces, considering only equatorial orbit of the centre of mass and

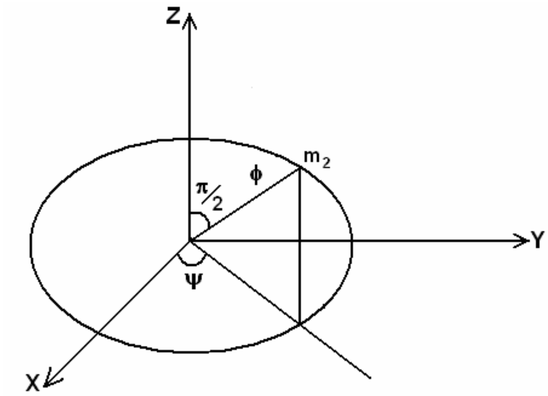


Figure 2: Polar spherical coordinate system

integrating the final equation w.r.t. to v, we get:

$$x'^2 + y'^2 + z'^2 - \left(3x^2 - z^2\right) = -2B\cos\delta x + A\left(4x^2 - y^2 - z^2\right) + h, \qquad (3.5)$$

where h is a constant of integration. The equation (3.5) is the equation of energy with modified potential V is given by:

$$V = -\frac{1}{2}\left(3x^2 - z^2\right) + xB\cos\delta - \frac{A}{2}\left(4x^2 - y^2 - z^2\right). \qquad (3.6)$$

Differentiating (3.4) with respect to v, we get:

$$x'^2 + y'^2 + z'^2 = -\left(xx' + yy' + zz'\right). \qquad (3.7)$$

Again, multiplying the first, the second and the third equation of (3.3) by x, y and z respectively, adding we get:

$$-\lambda_\alpha = \left(x'^2 + y'^2 + z'^2\right) + 2\left(xy' - yx'\right) + \left(3x^2 - z^2\right) - x\cdot B\cos\delta + A\left(4x^2 - y^2 - z^2\right). \qquad (3.8)$$

For further investigation of the system moving in low altitude equatorial orbit we have introduced the polar spherical co-ordinate system:

$$\begin{aligned} x &= \cos\phi\cdot\cos\psi;\\ y &= \cos\phi\cdot\sin\psi;\\ z &= \sin\phi; \end{aligned} \qquad (3.9)$$

on the unit sphere $x^2 + y^2 + z^2 = 1$.

Substituting x, y and z from (3.9) and their derivatives into (3.6) and (3.7) we get the Jacobian integral of the equation of motion of the system of mass m_1 into polar spherical form:

$$\phi'^2 + \psi'^2 \cdot \cos^2 \phi = \cos^2 \phi \left(3 \cos^2 \psi + 1\right) - 2B \cos \delta \cdot \cos \phi \cdot \cos \psi \\ + A \left(5 \cos^2 \phi \cdot \cos^2 \psi - 1\right) - 1 + h. \quad (3.10)$$

Also

$$- \lambda_\alpha = \left(\psi' + 1\right)^2 \cdot \cos^2 \phi + \phi'^2 + 3 \cos^2 \phi \cdot \cos^2 \psi - 1 \\ + A \left[\left(5 \cos^2 \phi - \cos^2 \psi - 1\right) - B \cos \delta \cdot \cos \phi \cdot \cos \psi\right], \quad (3.11)$$

where $\lambda_\alpha = \frac{-p^3 \rho^4}{\mu} \left(\frac{m_1+m_2}{m_1 m_2}\right) \lambda$ and λ is the Lagrange's multiplier. The mathematical implication of the Lagrange's multiplier is that, the constraint will be effective, so long as $\lambda_\alpha (t) \leq 0$ and the moment this condition fails to hold, the constraint is not effective. Thus, we obtain the condition of constrained motion in the form:

$$\left(\psi' + 1\right)^2 \cos^2 \phi + 3 \cos^2 \phi \cdot \cos^2 \psi + \phi'^2 - 1 \\ - B \cos \delta \cdot \cos \phi \cdot \cos \psi + A \left[5 \cos^2 \phi \cdot \sin^2 \phi - 1\right] \geq 0. \quad (3.12)$$

Again, from the equation (3.10), it is clear that the region of possible motion of the particle m_1 on the sphere (3.9) is obtained by the condition:

$$\left(1 + 3 \cos^2 \psi\right) \cos^2 \phi - 2B \cos \delta \cdot \cos \phi \cdot \cos \psi \\ + A \left[\left(5 \cos^2 \phi \cdot \cos^2 \psi - 1\right) - 1\right] + h \geq 0. \quad (3.13)$$

As the term $\left(1 + 3 \cos^2 \psi\right) \cos^2 \phi$ of (3.13) is positive, this condition will certainly hold if

$$h + A \left(5 \cos^2 \phi \cdot \cos^2 \psi - 1\right) - 2B \cos \delta \cdot \cos \phi \cdot \cos \psi \geq 0,$$

$$h \geq 2B \cos \delta \cdot \cos \phi \cdot \cos \psi + 1 - A \left(5 \cos^2 \phi \cdot \cos^2 \psi - 1\right) \quad (3.14)$$

are satisfied.

This condition does not impose any restriction on the value of ϕ and ψ hence the motion will take place around the sphere $x^2 + y^2 + z^2 = 1$. Hence

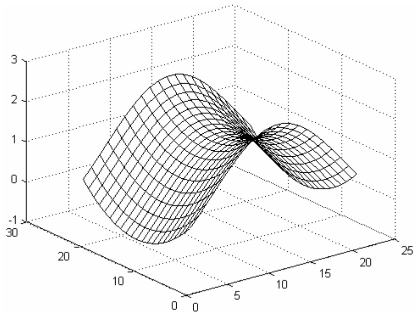
$$h \geq 2B \cos \delta + 1 - 4A. \quad (3.15)$$

Let us consider the case when $h < 0$. From (3.13) it follows that

$$\mid h \mid \leq \left(1 + 3 \cos^2 \psi\right) \cos^2 \phi \\ - 2B \cos \delta \cdot \cos \psi + A \left[\left(5 \cos^2 \phi \cdot \cos^2 \psi - 1\right) - 1\right], \quad (3.16)$$

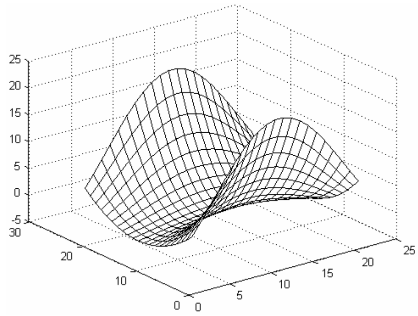
$$\max \mid h \mid \leq \max \left(1 + 3 \cos^2 \psi\right) \cos^2 \phi - 2B \cos \delta \cdot \cos \psi \cdot \cos \phi \\ + A \left[\left(5 \cos^2 \phi \cdot \cos^2 \psi - 1\right) - 1\right] = 4 - 2B \cos \delta + 4A - 1,$$

$$h = 3 - 2B \cos \delta + 4A. \quad (3.17)$$



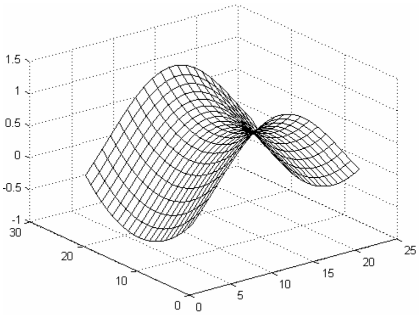
$\phi = 20^0$, $\psi = 20^0$

Figure 3: Classification of evolutional motion



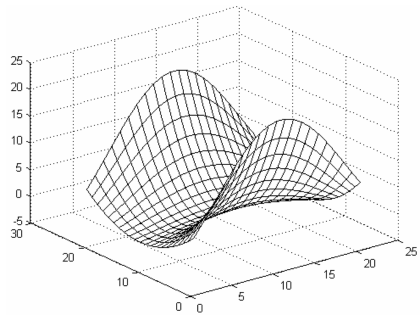
$\phi = -20^0$, $\psi = 20^0$

Figure 4: Classification of evolutional motion



$\phi = 40^0$, $\psi = -40^0$

Figure 5: Classification of evolutional motion



$\phi = -40^0$, $\psi = 40^0$

Figure 6: Classification of evolutional motion

Further analysis of this condition shows that for

$$-3 + 2B\cos\delta - 4A < h < 4A - 1 - 2B\cos\delta, \tag{3.18}$$

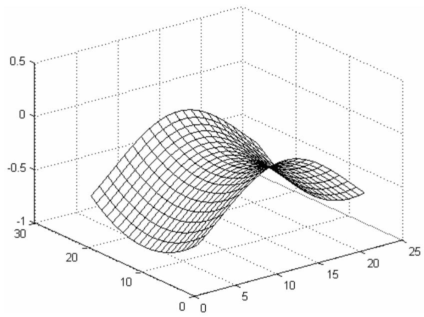
and

$$-1 + 2B\cos\delta - 4A < h < 0. \tag{3.19}$$

The motion of the particle m_1 takes place all around the sphere (3.9). Now (3.13) can be written as:

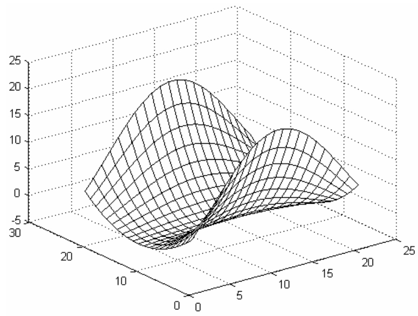
$$\left(1 + 3\cos^2\psi\right)\cos^2\phi - 2B\cos\delta\cdot\cos\phi\cdot\cos\psi + 5A\cos^2\phi\cdot\cos^2\psi - A- \mid h \mid \geq 0,$$

$$(3 + 5A)\cos^2\phi\cdot\cos^2\psi - 2A\cos\delta\cdot\cos\phi\cdot\cos\psi + \cos^2\phi - A- \mid h \mid \geq 0.$$



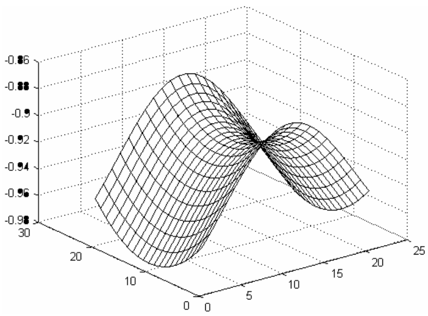
$\phi = 60^0,\ \psi = -60^0$

Figure 7: Classification of evolutional motion



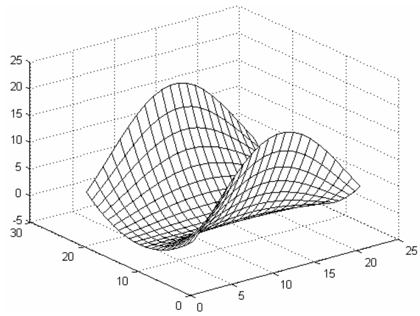
$\phi = -60^0,\ \psi = 60^0$

Figure 8: Classification of evolutional motion



$\phi = 80^0,\ \psi = -80^0$

Figure 9: Classification of evolutional motion



$\phi = -80^0,\ \psi = 80^0$

Figure 10: Classification of evolutional motion

This inequality is quadratic in $\cos\psi$ and hence

$$\cos\psi = \frac{B\cos\delta \pm \sqrt{B^2\cos^2\delta - (3+5A)\left(\cos^2\phi - A - \mid h \mid\right)}}{2(3+5A)\cos\phi}, \qquad (3.20)$$

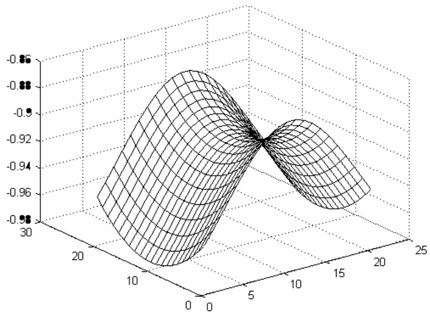
for $\phi = 0, \cos^2\phi = 1(\max)$,

$$\cos\psi_{max} = \frac{B\cos\delta + \sqrt{B^2\cos^2\delta - (3+5A)\left(1 - A - \mid h \mid\right)}}{(3+5A)},$$

for $\phi = \pi, \cos^2\phi = 1$,

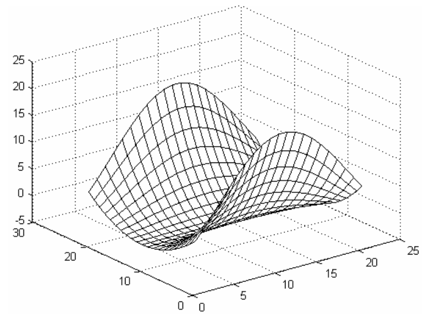
$$\cos\psi_{min} = \frac{B\cos\delta - \sqrt{B^2\cos^2\delta - (3+5A)\left(1 - A - h\right)}}{(3+5A)}.$$

The two values of ψ given by (3.20) show that particle m_2 will oscillate



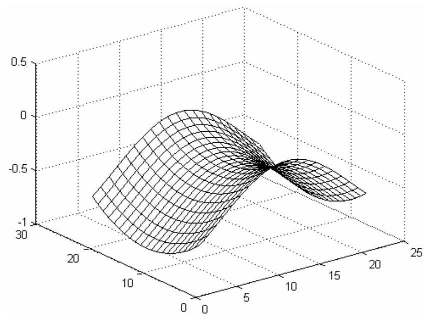
$\phi = 100^0$, $\psi = -100^0$

Figure 11: Classification of evolutional motion



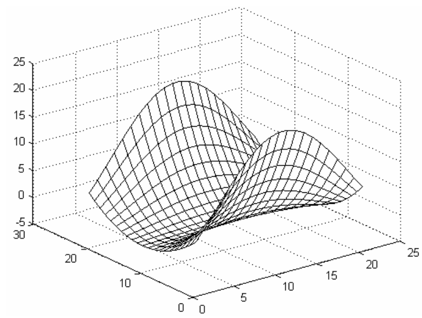
$\phi = -100^0$, $\psi = 100^0$

Figure 12: Classification of evolutional motion



$\phi = 120^0$, $\psi = -120^0$

Figure 13: Classification of evolutional motion



$\phi = -120^0$, $\psi = 120^0$

Figure 14: Classification of evolutional motion

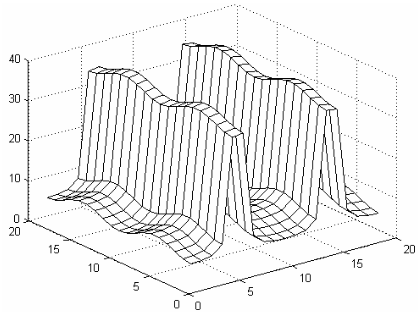
between:

$$\psi = \cos^{-1}\left[\frac{B\cos\delta + \sqrt{B^2\cos^2\delta - (3+5A)\,(1-A- \mid h \mid)}}{(3+5A)}\right]$$

and

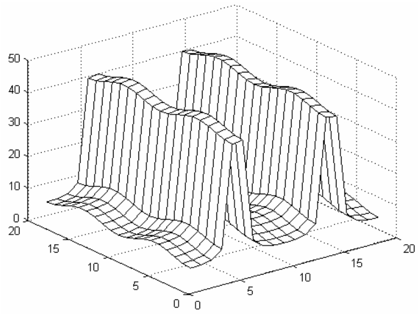
$$\psi = \cos^{-1}\left[\frac{B\cos\delta - \sqrt{B^2\cos^2\delta - 3\,(1-A- \mid h \mid)}}{(3+5A)}\right]$$

will move on the sphere for value of ψ for some fixed value of Jacobian constrain h.



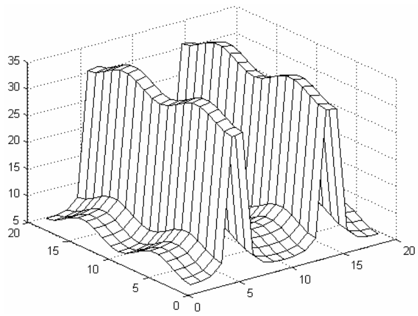
$\psi' = 3,\ h_s = 0.1$

Figure 15: Classification of non-evolutional motion



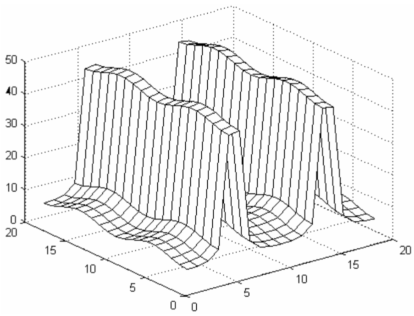
$\psi' = 3,\ h_s = -0.1$

Figure 16: Classification of non-evolutional motion



$\psi' = 3,\ h_s = 0.2$

Figure 17: Classification of non-evolutional motion



$\psi' = 3,\ h_s = -0.2$

Figure 18: Classification of non-evolutional motion

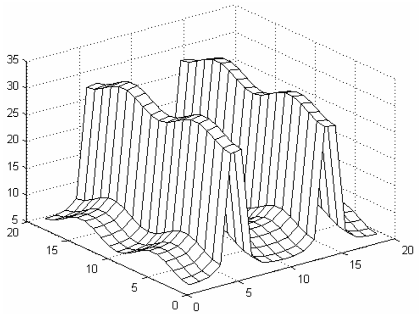
4. Condition of Constraint Motion

According to equation (3.12), the condition for the motion of the particle m_1 under the effective constraint is given by:

$$\left(\psi' + 1\right)^2 \cos^2\phi + 3\cos^2\phi \cdot \cos^2\psi + \phi'^2 - 1 \\ - B\cos\delta \cdot \cos\phi \cdot \cos\psi + A\left[5\cos^2\phi \cdot \cos^2\psi - 1\right] \geq 0. \quad (4.1)$$

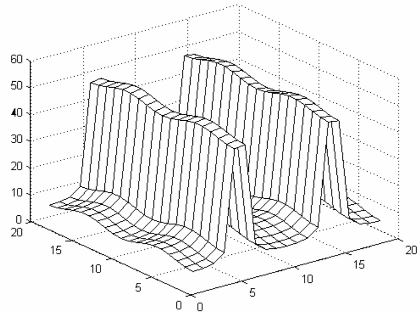
Since $(\psi' + 1)^2 \cos^2\phi, \phi'^2$ and $3\cos^2\phi \cdot \cos^2\psi$ are always positive, then this is obviously satisfied if

$$-1 \geq -B\cos\delta \cdot \cos\phi - A. \quad (4.2)$$



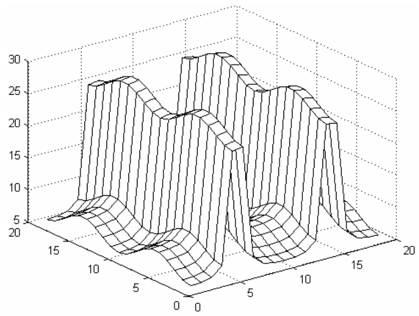
$\psi' = 3,\ h_s = 0.3$

Figure 19: Classification of non-evolutional motion



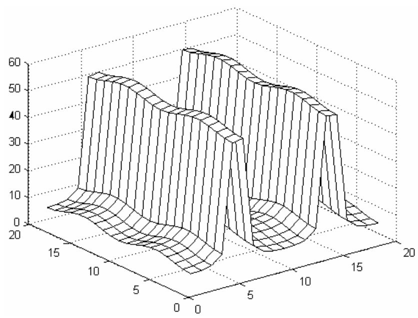
$\psi' = 3,\ h_s = -0.3$

Figure 20: Classification of non-evolutional motion



$\psi' = 3,\ h_s = 0.4$

Figure 21: Classification of non-evolutional motion



$\psi' = 3,\ h_s = -0.4$

Figure 22: Classification of non-evolutional motion

From the inequality (4.1), it follows that the evolutional motion occurs if

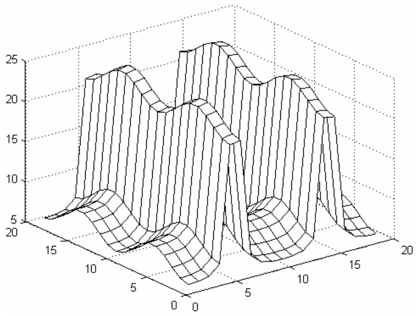
$$-1 \geq -B\cos\delta \cdot \cos\phi \cdot \cos\psi + A. \tag{4.3}$$

Hence the inequality (3.12) can be put in form:

$$\big(\psi' + 2\psi' + 1\big)\cos^2\phi + 3\cos^2\phi \cdot \cos^2\psi + \phi'^2 - 1 \\ + 5A\cos^2\phi \cdot \cos^2\psi - A - B\cos\delta \cdot \cos\phi \cdot \cos\psi \geq 0,$$

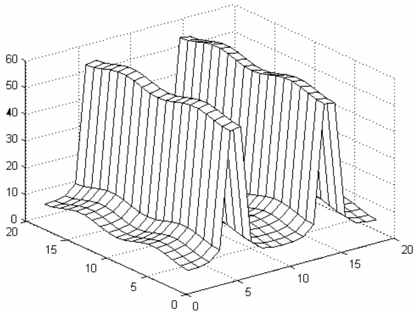
or

$$\big(\psi' + 2\psi' + 3\cos^2\psi + 5\cos^2\psi + 1\big)\cos^2\phi \\ \geq \phi'^2 + 1 + A + B\cos\delta \cdot \cos\phi \cdot \cos\psi. \tag{4.4}$$



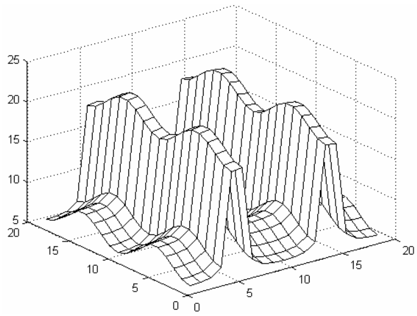
$\psi' = 3,\ h_s = 0.5$

Figure 23: Classification of non-evolutional motion



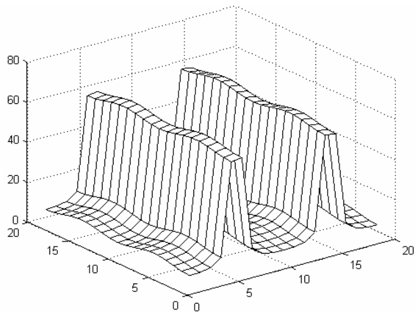
$\psi' = 3,\ h_s = -0.5$

Figure 24: Classification of non-evolutional motion



$\psi' = 3,\ h_s = 0.6$

Figure 25: Classification of non-evolutional motion



$\psi' = 3,\ h_s = -0.6$

Figure 26: Classification of non-evolutional motion

The evolutional motion will take place if,

$$\psi'^2 + 2\psi' + (3+5A)\cos^2\psi$$
$$\leq \tan^2\phi - \phi'^2\sec^2\phi + A\cdot\sec^2\phi + B\cos\delta\cdot\sec\phi\cdot\cos\psi. \quad (4.5)$$

Putting $\phi = 0$, therefore $\phi' = 0$ in the integral (3.10), we get:

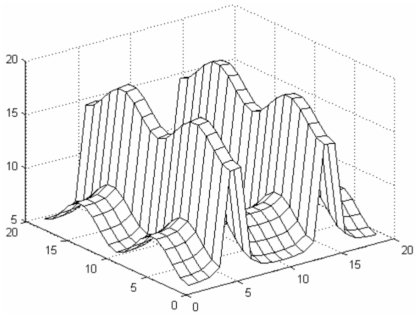
$$\psi'^2 = (3+5A)\cos^2\psi + 2B\cos\delta\cdot\cos\psi + h,$$

where

$$h_s = h - A. \quad (4.6)$$

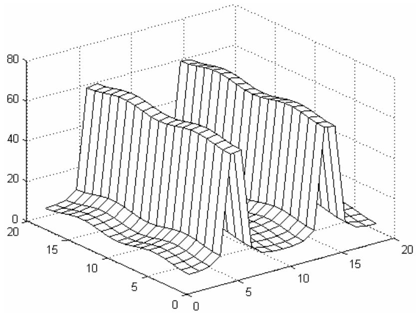
The Jacobian integral takes the form:

$$\psi'^2 - (3+5A)\cos^2\psi$$



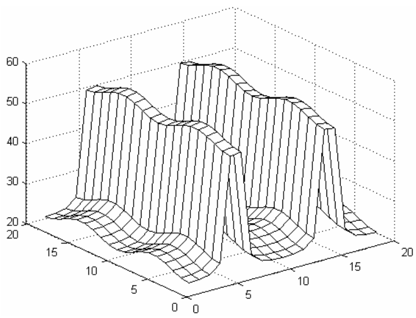
$\psi' = 3,\ h_s = 0.7$

Figure 27: Classification of non-evolutional motion



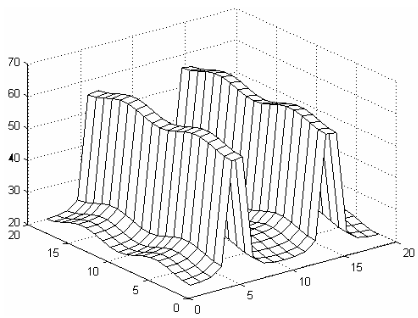
$\psi' = 3,\ h_s = -0.7$

Figure 28: Classification of non-evolutional motion



$\psi' = 5,\ h_s = 0.1$

Figure 29: Classification of non-evolutional motion



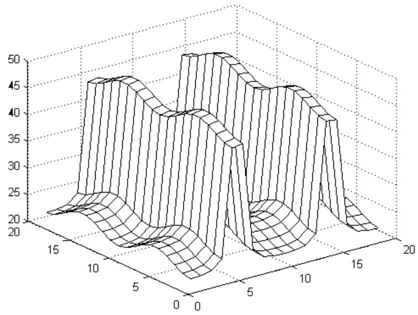
$\psi' = 5,\ h_s = -0.1$

Figure 30: Classification of non-evolutional motion

$$= h_s \sec^2\phi - \tan^2\phi - \phi'^2 \sec^2\phi - 2B\cos\delta\cdot\cos\psi\cdot\sec\phi. \quad (4.7)$$

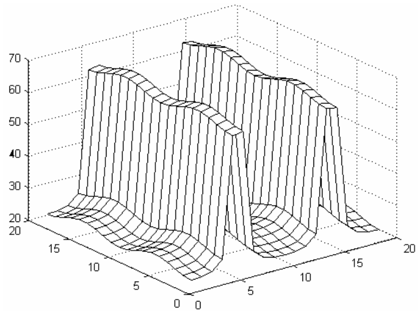
The inequality (4.4) represents a hyper surface in four dimensional phase space (ψ, ψ', ϕ and ϕ'). If the moving particle comes inside this surface in the phase space $(\phi, \psi, \phi', \psi')$ the cable becomes lose, and the motion of the particle in this case will be free from constraints. The equation (4.7) also represents a hyper surface in the phase space $(\phi, \psi, \phi', \psi')$ on which the particle is moving for some fixed value of h_s. Thus all the real curves where there two hyper surfaces intersect in the four dimensional sphere are the region of evolutional motion.

In order to get the intersection of these two hyper surfaces, equation (4.5)



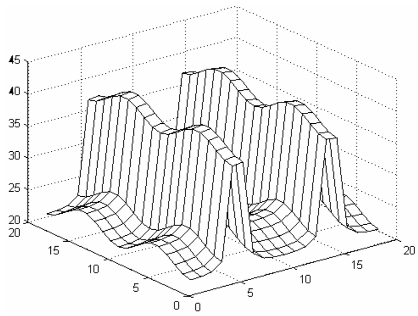
$\psi' = 5,\ h_s = 0.3$

Figure 31: Classification of non-evolutional motion



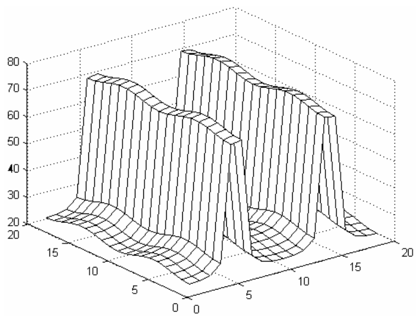
$\psi' = 5,\ h_s = -0.3$

Figure 32: Classification of non-evolutional motion



$\psi' = 5,\ h_s = 0.5$

Figure 33: Classification of non-evolutional motion



$\psi' = 5,\ h_s = -0.5$

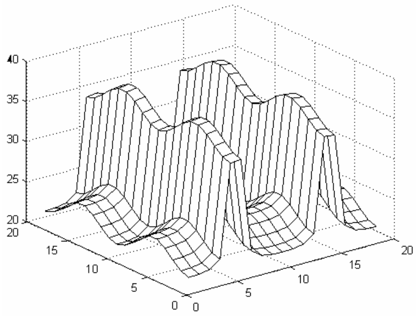
Figure 34: Classification of non-evolutional motion

and equation (4.7) are added to yield following.

$$\psi'^2 + \psi' - \sec^2\phi\left(\frac{h_s}{2} - \phi'^2 - \frac{B}{2}\cos\delta\cdot\cos\phi + A\right) < 0. \tag{4.8}$$

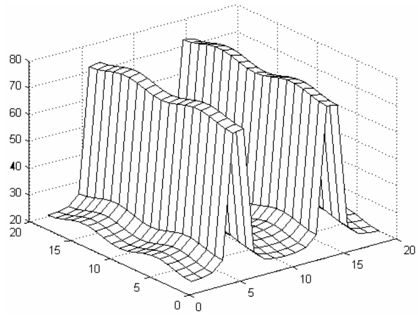
It follows from the inequality (4.8) that the evolutional motion will take place if the value of ψ'satisfying this condition will certainly make this motion evolutional between the positive and negative real roots of the equation, but the theory of quadratic expression gives that the value of ψ' holds if

$$\psi'^2 + \psi' - \sec^2\phi\left(\frac{h_s}{2} - \phi'^2 - \frac{B}{2}\cos\delta\cdot\cos\phi + A\right) = 0. \tag{4.9}$$



$\psi' = 5,\ h_s = 0.6$

Figure 35: Classification of non-evolutional motion



$\psi' = 5,\ h_s = -0.6$

Figure 36: Classification of non-evolutional motion

The roots of the equation will be real, if

$$h_s > 2\phi'^2 - \frac{1}{2}\cos^2\phi + B\cos\delta\cdot\cos\phi - 2A.$$

Thus

$$h_s < 2\phi'^2 - \frac{1}{2}\cos^2\phi + B\cos\delta\cdot\cos\phi - 2A. \tag{4.10}$$

Then the curve of intersection of (4.4) and (4.6) will not be real and hence the motion of the system will be non-evolutional. Examining the real roots of equation (4.8) between which the evolution motion may take place, it is not difficult to see the region of evolutional motion for negative values of ψ' is greater than the region for positive value of ψ'. Hence all motions, for which evolution do not take place in the case of negative ψ' will remain non-evolutional for positive value of ψ'as well. Subtracting (4.4) from equation (4.7), we get:

$$2\psi' - 2(3+5A)\cos^2\psi - 2\tan^2\phi - 3B\cos\delta\cdot\sec\phi\cdot\cos\psi$$
$$= h_s\sec^2\phi + A\sec^2\phi < 0. \tag{4.11}$$

The equation corresponding to the above expression is given by

$$2(3+5A)\cos^2\psi - 3B\cos\delta\cdot\sec\phi\cdot\cos\psi + m = 0, \tag{4.12}$$

where $m = 2\psi' - 2\tan^2\phi + h_s\sec^2\phi - A\sec^2\phi$,

$$\cos\psi = \frac{3B\cos\delta\sec\phi \pm \sqrt{9B^2\cos^2\delta\sec^2\phi - 4\cdot 2(3+5A)\cdot m}}{4(3+5A)},$$

$$9B^2\cos^2\delta\sec^2\phi - 4\cdot 2(3+5A)\cdot m \geq 0,$$

$$9B^2\cos^2\delta\sec^2\phi - 8(3+5A)\cdot\left(2\psi' - 2\tan^2\phi + h_s\sec^2\phi - A\sec^2\phi\right) \geq 0,$$

or

$$-2\psi' + 2\tan^2\phi - h_s\sec^2\phi + 2A\sec^2\phi + \frac{3}{8}B^2\cos^2\delta\sec^2\phi \geq 0, \tag{4.13}$$

or

$$1 \mp \sqrt{1 + 4\sec^2\phi\left\{\frac{h_s}{2} - \phi'^2 - \frac{B}{2}\cos\delta\cdot\cos\phi + A\right\}} < \left[1 + 2\tan^2\phi - h_s\sec^2\phi + \frac{3}{8}B^2\cos^2\delta\sec^2\phi\right],$$

or

$$1 + 4\sec^2\phi\left\{\frac{h_s}{2} - \phi'^2 - \frac{B}{2}\cos\delta\cdot\cos\phi + A\right\} < \left[1 + 2\tan^2\phi - h_s\sec^2\phi + A\sec^2\phi + \frac{3}{8}B^2\cos^2\delta\sec^2\phi\right].$$

Dividing throughout by $\sec^4\phi$ on both sides of the expression, we get:

$$\cos^4\phi + 4\left(\frac{h_s}{2} - \phi'^2 - \frac{B}{2}\cos\delta\cdot\cos\phi + A\right)\cos^2\phi < \left(2 - \cos^2\phi - h_s + \frac{3}{8}\cos^2\delta + A\right)^2,$$

$$\left(2 - h_s + \frac{3}{8}\cos^2\delta + A\right)^2 + \cos^4\phi - 2\cos^2\phi\left(2 - h_s + \frac{3}{8}\cos^2\delta + A\right) > \cos^4\phi + 4\left(\frac{h_s}{2} - \phi'^2 - \frac{B}{2}\cos\delta\cos\phi + A\right)\cos^2\phi,$$

$$\left(2 - h_s + \frac{3}{8}\cos^2\delta + A\right)^2 > 2\cos^2\phi\left(2 - h_s + \frac{3}{8}\cos^2\delta + A\right) + 4\cos^4\phi\left(\frac{h_s}{2} - \phi'^2 - \frac{B}{2}\cos\delta\cos\phi + A\right),$$

$$\Rightarrow \left(2 - h_s + \frac{3}{8}\cos^2\delta + A\right)^2 > 4\cos^2\phi\left(1 - \phi'^2 + \frac{3}{16}\cos^2\delta + \frac{3}{2}A - \frac{B}{2}\cos^2\delta\cos\phi\right). \tag{4.14}$$

For any $\psi' > 0$:

$$\Rightarrow \left(h_s - 2 - \frac{3}{8}\cos^2\delta - A\right)^2$$

$$\leq \left(1 - \phi'^2 + \frac{3}{16}\cos^2\delta + \frac{3}{2}A - \frac{B}{2}\cos^2\delta\cos\phi\right) 4\cos^2\phi. \quad (4.15)$$

Considering all the conditions obtained by non-evolutional motion, it follows that the motion of the system will be similar to the motion of dumbell satellite if any of the conditions in (4.2), (4.8), (4.123) and (4.14) holds. These conditions are only kinematical relations which points out that the non-evolution motion depends upon ψ and ψ' not the masses of the system. It means that the angular velocities of the satellites according to the above condition will keep the system like a dumbell satellite.

The computer simulation of (4.6) and

$$\left(\psi' + 1\right)^2 + \cos^2\phi + \phi'^2 + 3\cos^2\phi \cdot \sec^2\psi - 1 + \left[A\left(5\cos^2\phi \cdot \cos^2\psi - 1\right) - B\cos\delta \cdot \cos\phi \cdot \cos\psi\right] = 0, \quad (4.16)$$

which gives the boundary of evolutional and non-evolutional motions have been plotted in Figure 3 to Figure 36, for different values of h_s and for different values of ϕ and ψ, which are mentioned and taking into account of $A = 0.001$, $B = 0.002$ and $\delta = 30^0$ are computed. The surfaces which cross the boundary (4.15) referred the case of evolutional motion and evidently the surface which does not touch the boundary represents the non-evolutional motion of the system. The region where the curves of intersection of the surfaces represented by Figure 3 to Fig 14, which are intersected by the cures of intersection of Figure 15 to Figure 36, are evolutional motion in which the system is moving with loose cable and evidently the curves of intersection which do not touch the surface represents non evolutional motion. Hence considering all the conditions obtained for non evolutional motion it follows that the motion of the system will be similar to the dumbbell satellites.

5. Discussion and Conclusion

The free and constrained motion of a system of cable connected satellites system in low altitude equatorial orbit has been discussed. We have adopted computer simulation technique using *MATLAB 6.1* which has been used to explain in detail about evolutional and non-evolutional motion of cable connected satellite system in equatorial orbit. The surfaces which crossed the boundary referred the case of evolutional motion and evidently the surface which does not touch the boundary represent the non-evolutional motion of the system. The region where the curves of intersection of the surfaces represented by evolutional motion in which the system is moving with loose cable and evidently the curves of intersection which do not touch the surface represent non evolutional motion. Hence considering all the conditions obtained for non evolutional motion it follows that the motion of the system will be similar to the dumbbell satellites. We arrive at the conclusion that the surface in which non-evolutional motion of the system takes place in which gravity gradient stabilization as well as passive attitude stabilization would be possible.

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