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QUANTITATIVE EVALUATION MODEL OF COMPLEX SYSTEM BASED ON FUZZY

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Abstract: The quantitative evaluation model of complex system based on fuzzy that is put forward in this report is a combination of multiple models, including layer analysis method, fuzzy synthesizing evaluation, and T standard score calculation, etc. It is an algorithm model that can be used to evaluate a complex system which is composed of qualitative and quantitative specifications.

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1. Introduction

In the 1970's of the 20-th century, people started to study the evaluation theory of complex systems. AHP layer analysis method (see [4]), fuzzy synthesizing evaluation method (FUZZY) (see [1]), class aggregation analysis method (see [6]), method of value engineering theory (see [3]), BP neuron network evaluation method and many others had been put forward in time sequence. All these methods and theories made the quantitative evaluation of a complex system possible. However, how to scientifically and accurately evaluate a large complex system by reasonably and synthetically applying these models and methods

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has become a question. In the design and development of software system for equipment management and system monitoring and alarming for large petrochemical companies, the author reasonably combined and utilized AHP layer analysis method, FUZZY synthesizing evaluation method and T model method which standardizes the data in statistics to solve the problem of quantitative evaluation of complex system. Here the author names this method as AFT complex system quantitative evaluation algorithm.

2. AFT Algorithm Model

2.1. Establishing of Complex System Specification System

In AFT algorithm, to evaluate a complex system, a committee shall be formed first to setup evaluation specifications for the complex system. The committee is composed of experts in corresponding areas in the evaluated industries and people from evaluation departments. A critical problem in setting up the evaluation specifications is to minimize the correlation among these evaluation specifications.

The specification system of AFT algorithm evaluation is a tree that comprises of many evaluation specifications. Each node of the tree represents an evaluation specification, and each leave node of the tree represents a final evaluation specification, i.e. the specification cannot be detailed any further. The configuration of a tree is shown in Figure 1.

In Figure 1, XX is the unit or system being evaluated; $U_1, U_2, U_3, \cdots, U_s$ refer to that the content to be evaluated is composed of quantity "s" first level specifications; U_{11}, U_{12} and U_{13} indicate that the first level evaluation specification U_1 comprises three second level evaluation specifications; $U_{211}, U_{212}, U_{213}$ and U_{214} illustrate that the second grade evaluation specification U_{21} has four third level evaluation specifications. A tree similar to Figure 1 forms the foundation of AFT evaluation.

2.2. Determination of Weighing Power for Evaluation Specification Tree

The weighing power refers to importance of an evaluation specification in the evaluation system, or the ratio it takes in overall scores. Its quantitative expression is the weight. The AHP method put forward by Professor Saaty, University

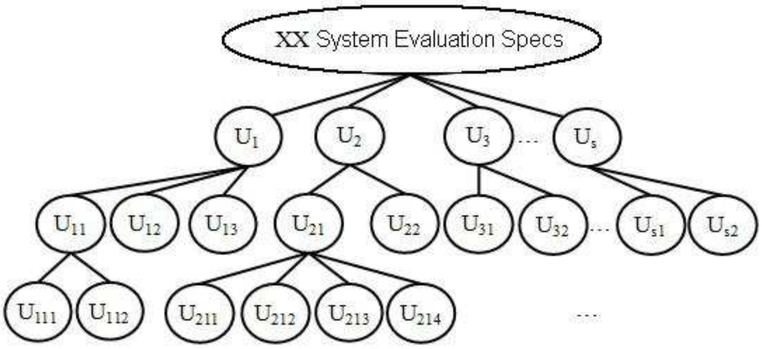

Figure 1: Specification tree for XX system evaluation

of Pittsburg, in the 1970's of 20-th century, can be used to scientifically determine weighing powers of different specifications in a complex system. The steps to determine weighing powers of the nodes (i.e. specifications) in Figure 1 are as the following.

2.2.1. Configuration of Adjustment Matrix

This is to configure a corresponding evaluation matrix for all level evaluation specifications. The evaluation matrix is shown in equation (1):

$$A = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}. \tag{1}$$

Here n is the quantity of evaluation specifications for a certain level. For the first level evaluation specification as shown in Figure 1, $s = 4$ (which indicates that this first level evaluation specification is composed of four items $-U_1, U_2, U_3$ and U_4) corresponds to a 4×4 matrix.

In equation (1), the value of a_{ij} in the matrix represents the degree of importance when U_i is comparing to U_j. Its value is determined with Saaty standard evaluation method. The principle of Saaty's standard evaluation is listed in Table 1.

It can be seen from Table 1 that the value of a_{ij} is reciprocal to that of a_{ji}.

a_{ij}	U_i comparing to U_j	Note
1	U_i is as important as U_j	Contribution of both to the objectives are identical
3	U_i is a little important than U_j	Per experience, U_i has a little advantage over U_j
5	U_i is important than U_j	Per experienceU_i is in advance to U_j
7	U_i is a lot important than U_j	U_i has an obvious advantage over U_j
9	U_i is extremely important than U_j	The importance of U_i over U_j is absolutely the highest
2,4,6,8	Importance of U_i comparing to U_j is between the above	Importance is between the above
Reciprocal of the above	Unimportance of U_i comparing to U_j is in conformance to the above	Importance of U_j to U_i conforms to the above

Table 1: Principle of Saaty's standard evaluation

2.2.2. Calculation of Weighing Power in the Evaluation Matrix and Inspection of Consistence

If the evaluation matrix A given by the decision maker is in consistence, the one-to-one comparison that the decision maker makes to all factors can be deliverable, i.e. when making the one-to-one comparison, the decision maker's evaluation thinking is consistent. However, since human's evaluation thinking is hard to keep absolutely consistent, the evaluation matrix A obtained by one-to-one comparison may not satisfy the consistence condition of matrix, therefore the evaluation matrix must be adjusted.

To facilitate the inspection of consistence using computer, square root method is used in processing the weighing power. Follow the steps below for calculation:

Step 1. Calculate the geometric mean.

Use equation (2) to calculate the geometric mean of each row in the evaluation matrix:

$$\bar{\omega}_i = \sqrt[n]{\prod_{j=1}^{n} a_{ij}}. \tag{2}$$

Step 2. Unifying.

The vector obtained in equation (2) will be unified using equation (3):

$$\omega_i = \frac{\bar{\omega}_i}{\sum_{k=1}^{n} \bar{\omega}_k}. \tag{3}$$

Step 3. Using equation (4) to calculate the characteristic root:

$$\lambda_{max} = \sum_{i=1}^{n} \frac{(A\omega)_i}{n\omega_i}, \tag{4}$$

where $(A\omega)_i$ is the number i element of vector $(A\omega)$.

Step 4. Inspection of consistence.

Calculate CI as per equation (5):

$$CI = \frac{\lambda_{max} - n}{n-1}, \tag{5}$$

where n is the order number of the evaluation matrix. For the evaluation matrix with orders from 1 to 10, Saaty provided a random consistence specification RI, whose value is listed in Table 2.

Order n	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

Table 2: Random consistence specification RI for orders 1 to 10

With the order number n of the evaluation matrix we can get the RI from Table 2, then calculate the random consistence ratio CR:

$$CR = \frac{CI}{RI}. \tag{6}$$

As per Saaty's evaluation method, for evaluation matrix with the order $n \geq 3$, when $CR \leq 0.1$, the evaluation matrix meets the requirement for consistence. Otherwise, re-evaluation is needed and a new evaluation matrix should be created. If the consistence passes the inspection, ω_i is the obtained characteristic vector.

2.3. Evaluation of Qualitative Specifications in the Evaluation Specification System

This is to evaluate the quantitative specifications using FUZZY synthesizing evaluation method. See the following for steps for algorithm:

Step 1. Setting up sets of evaluation elements. According to the levels of evaluation specification tree (depth of the tree), evaluation elements can be divided into sets of evaluation elements with multiple levels.

First level set of elements can be divided into number of s sub-sets as per Figure 1, being noted as $U_1, U_2, \cdots, U_s$. The sets of elements divided shall meet the following two requirements:

$$U_1 \cup U_2 \cup \cdots \cup U_s = U, \quad U_i \cap U_j \neq \phi \quad (i, j = 1, 2, 3, \cdots, s). \tag{7}$$

The first equation in equation (7) denotes that overall evaluation element is a merge set of number of "s" sub-element sets. While in the second equation, the divider set of any two sub-element sets will be a null set, such that it can be ensured that there is no evaluation element to be duplicated calculated.

The element U_i in the first level evaluation specification can be divided into second level elements:

$$U_i = \{U_{i1}, U_{i2}, \cdots, U_{in_i}\} \quad (i = 1, 2, 3, \cdots, s), \tag{8}$$

where n_i is the quantity of elements that form U_i. Equation (8) represents that each evaluation element (each sub-set) comprises a few different evaluation items.

Step 2. FUZZY synthesizing evaluation to each sub-set U_i.

The FUZZY synthesizing evaluation of sub-set U_i starts from the lowest level in the set of evaluation specifications that is setup in Step 1. Suppose there are three levels of evaluation specifications, start first from the third level to do FUZZY synthesizing evaluation, then the second level and at last the first level.

If U_{ij} is the number j evaluation specification in a second level element, its evaluation set is $V = \{v_1, v_2, \cdots, v_m\}$, distribution of weighing power is $A_{ij} = \{a_{ij1}, a_{ij2}, \cdots, a_{ijn_i}\}$, in which shall fulfill the requirement $\sum_{k=1}^{n_i} a_{ijk} = 1$.

To evaluate U_{ij} as a single element, the FUZZY relation matrix R_{ij} corresponding to U_{ij} is:

$$R_{ij} = \begin{vmatrix} r_{ij11} & r_{ij12} & \cdots & r_{ij1m} \\ r_{ij21} & r_{ij22} & \cdots & r_{ij2m} \\ \cdots & \cdots & \cdots & \cdots \\ r_{ijn1} & r_{ijn2} & \cdots & r_{ijnm} \end{vmatrix}. \tag{9}$$

Here r_{ijpq} $(p = 1, 2, \ldots, n; q = 1, 2, \ldots, m)$ represents the degree of attachment when U_{ij} is evaluated as V_k, and n is the quantity of three level specifications that U_{ij} comprises.

Here is the FUZZY synthesizing evaluation to U_{ij}:

$$B_{ij} = A_{ij} \bullet R_{ij} = (b_{ij1}, b_{ij2}, \cdots, b_{ijm}). \tag{10}$$

After all the B_{ij} are calculated, an evaluation matrix to the upper level evaluation specification can be formed:

$$R_i = \begin{vmatrix} B_{i1} \\ B_{i2} \\ \cdots \\ B_{is} \end{vmatrix} = \begin{vmatrix} b_{i11} & b_{i12} & \cdots & b_{i1m} \\ b_{i21} & b_{i22} & \cdots & b_{i2m} \\ \cdots & \cdots & \cdots & \cdots \\ b_{is1} & b_{is2} & \cdots & b_{ism} \end{vmatrix}. \tag{11}$$

Suppose R_i is corresponding to the distribution of weighing power $A_i = (a_{i1}, a_{i2},$
$\cdots, a_{is})$, then synthesizing evaluation of second level is:

$$B_i = A_i \bullet R_i. \tag{12}$$

After $B_1, B_2, \cdots, B_s$ are all calculated, the FUZZY evaluation process comes to an end.

2.4. Evaluation of Quantitative Specifications in the Evaluation Specification System

Some quantitative specifications in the evaluation specifications are processed using T standard score calculation model. The advantage of doing this is making the evaluation specifications that are affected by many different elements in different evaluation units comparable.

The T standard score calculation model is processed as the following:

Step 1. Transfer the value of quantitative specification to Z standard score using the equation below:

$$Z_i = \frac{(X_i - \bar{X}_i)}{S_i}, \tag{13}$$

where Z_i is the standard score of number i quantitative specification in each unit. X_i is the initial score of number i specification. S_i is the standard deviation of this specification.

Step 2. Calculate $\bar{Z}$ according to Z_i using equation (14):

$$\bar{Z} = \frac{\sum_{i=1}^{h} Z_i \bullet \omega_i}{\sum_{i=1}^{h} \omega_i}. \tag{14}$$

Here $\bar{Z}$ is the average standard score of all specifications calculated considering the weighing power of specifications. ω_i is the value of weighing power of each specification. h is the number of evaluation specifications.

Step 3. Transfer Z standard score to T standard score:

$$T = K \bullet \bar{Z} + C. \tag{15}$$

Here K and C are constants, in which K is an integer no less than S and $C \geq 4K$. For example, when $K = 10$, then $C = 60$. Such values represent a general average level, and the T standard score obtained therefore reduces bi-polar differentials.

2.5. Calculation of Synthesizing Evaluation Results

To facilitate comparison, the synthesizing evaluation matrix obtained by FUZZY synthesizing evaluation can be transferred to values with 100 score scale. For example, if the evaluation specification setting up committee set A as 95, B as 85, C as 75, D as 65, and F as 45, it can be denoted with a vector:

$$M = (95, 85, 75, 65, 45). \tag{16}$$

Then calculate the distribution value P_i of each level specification U_i:

$$p_i = \sum_{j=1}^{5} B_{ij} \bullet M_i. \tag{17}$$

At last, calculate the overall evaluation score (denoted with P) according to the value of first level weighing power (denoted with A_i) obtained with AHP method:

$$p = \sum_{i=1}^{s} A_i \bullet p_i. \tag{18}$$

In equation (18), s represents the final result of first level evaluation specification. Until now the system evaluation has finished.

3. Summary

In AFT complex system quantitative evaluation algorithm, to obtain the synthesized evaluation score of a system to be evaluated, first calculate the weighing power of each specification in the specification system using AHP method, then obtain its qualitative specification score using FUZZY method. After that calculate the overall score of each quantitative specification using T standard score model, and at last calculate the overall score of each specification which is summed up with weighing power. The synthesized evaluation of complex system is hereby completed.

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