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C-NUMBERS, Q-NUMBERS AND THE FOUNDATIONS OF RELATIVISTIC QUANTUM MECHANICS

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Abstract: I discuss the definition of the momentum operator in momentum space for the case of the Weyl (zero mass) neutrino, which is known to be the conformal symmetric case. I argue that my interpretation of momentum is correct and arises from the non-compact associated group 0(4,2).

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Let us consider the momentum operator in momentum space for the case of the conformal symmetric Weyl neutrino. This operator is defined to be the one which acting on the momentum space eigenstates (the Dirac spinor space) yield the physical form-momentum as eigenvalue. We thus make a distinction between c-numbers (ordinary coupled numbers) and q-numbers (operators), see [1].

In non-relativistic quantum mechanics, the momentum eigsenstates are defined by the eponymous Dirac delta function, see [3] (states of sharp momentum). However, in the more general relativistic quantum mechanical treatment (the Dirac equation), the momentum operator is a q-number with components that do not commute in the singular solution corresponding to the Weyl theory, see [2]. I note in passing that the group 0(4,2) has an alternation of signs in the metric (++++–). This is what defines it as a non-compact group as opposed to groups like 0(3) with the metric (+++) (no alternation of sign).

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The reader is encouraged to observe that the conformal symmetric massless case is also a state of fixed, negative helicity. The antiparallel momentum in thus tied to the (internal) spin and therefore transforms like the spin under rotation. This is what accounts for the non-commutation of the momentum components in this instance, since rotations do not commute (they commute infinitesimally).

References

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