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A MODIFIED SIMILARITY MEASURE AND ITS APPLICATIONS

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Abstract: In this paper, a new similarity between two fuzzy numbers is proposed by considering the mean curve of fuzzy numbers and some of the drawbacks found in the traditional methods will be overcome.

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1. Introduction

The concept of fuzzy sets was originally introduced by Zadeh [14] in 1965. Ever since, many applications of fuzzy sets have been developed. It is widely applied in the fuzzy risk analysis, fuzzy decision-making, optimal economic problems, fuzzy control and approximate reasoning problems. However, all applications mentioned above are based on a concept of similarity between two fuzzy numbers. In fact, to measure the similarity of fuzzy numbers is very important in the research topic of fuzzy decision-making [2, 12] and fuzzy risk analysis [3, 7, 9]. Some methods have been proposed to measure and calculate the similarity between fuzzy numbers [3, 7, 10], but there exist some drawbacks in different traditional methods. These drawbacks come from the fact that the pairs of fuzzy numbers have different mean curve. In this paper, a modified method to measure and calculate the degree of similarity between fuzzy numbers is proposed. Furthermore the proposed method in this paper can overcome the drawbacks of the traditional methods.

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2. Preliminaries

First, we recall some basic concepts of fuzzy numbers [8, 11, 13]. Because of the particularity of trapezoidal fuzzy numbers, it is possible to describe uncertain information easily for the problems of fuzzy control, fuzzy risk analysis and fuzzy decision-making. In this paper, we only consider the trapezoidal fuzzy numbers. Now, for two arbitrary fuzzy numbers $\tilde{u}$, $\tilde{\nu} \in F(R)$, we define a metric on $F(R)$ found in Diamond and Kloeden [5]

$$d^2(\tilde{u},\tilde{\nu}) = \frac{1}{2}\left(\int_0^1 (u^-(\alpha) - \nu^-(\alpha))^2 d\alpha + \int_0^1 (u^+(\alpha) - \nu^+(\alpha))^2 d\alpha\right),$$

where $u_\alpha = [u_\alpha^-, u_\alpha^+]$ or $u_\alpha = [u^-(\alpha), u^+(\alpha)]$ stands for the level sets.

Definition 1. If u_α^-, u_α^+ are linear functions, we say that the fuzzy number $\tilde{u}$ is a trapezoidal fuzzy number. That is, $\tilde{u}$: $R \to [0,1]$ is defined by

$$\widetilde{u}\,(x) = \begin{cases} 0\,, & \text{if } x < u_1, \\ \frac{x-u_1}{u_2-u_1}\,, & \text{if } u_1 < x < u_2, \\ 1\,, & \text{if } u_2 < x < u_3, \\ \frac{u_4-x}{u_4-u_3}\,, & \text{if } u_3 < x < u_4, \\ 0\,, & \text{if } x > u_4, \end{cases}$$

and denoted by (u_1, u_2, u_3, u_4). We call the defuzzification of $\widetilde{u}$ with $[u_2, u_3]$, fuzziness $(u_2 - u_1)$-left and $(u_4 - u_3)$-right unsymmertrically.

If $\widetilde{u}$: $(u_1, u_2, u_3, u_4), \widetilde{u}$: $(\nu_1, \nu_2, \nu_3, \nu_4)$ are two trapezoidal fuzzy numbers, then

$$\widetilde{u} + \widetilde{\nu} = (u_1 + \nu_1, u_2 + \nu_2, u_3 + \nu_3, u_4 + \nu_4);$$

$$k\widetilde{u} = \begin{cases} (ku_1, ku_2, ku_3, ku_4)\,, & \text{if } k \geq 0, \\ (ku_4, ku_2, ku_3, ku_1)\,, & \text{if } k < 0. \end{cases}$$

3. A New Similarity Measure between Fuzzy Numbers

3.1. The Mean Curve for Fuzzy Numbers

Let $\widetilde{u}$ be a fuzzy number. The mean curve of the fuzzy number $\widetilde{u}$ is defined as follows:

$$m^{\widetilde{u}}\,(x) = \begin{cases} \alpha\,, & \text{if } x \ = \ \text{mean}\,(u_\alpha), \\ 0\,, & \text{otherwise.} \end{cases}$$

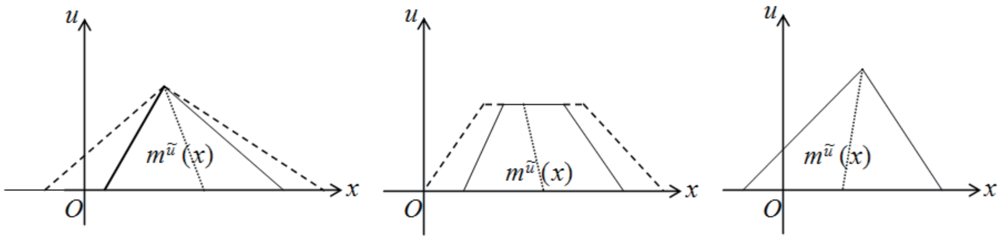

Figure 1: The mean curve for fuzzy numbers

Here $\mathrm{mean}(u_\alpha) = \frac{1}{2}(u^-(\alpha) + u^+(\alpha))$ stands for the mean of u_α.

3.2. Distance Measure for Fuzzy Numbers

In most exiting definitions of distance measures of interval and fuzzy numbers, only the lower and upper bound values are defined (e.g., those used in Bardossy et al [1], Diamond [4], Diamond and Kloeden [5], and Diamond and Tanaka [6]). As an example, consider the crisp number $A : [0, 0]$ and two interval numbers $B : [-2, 3]$, $C : [2, 3]$. By using the distance measure applied in Diamond [4] and Bardossy et al [1], it was found that $d(A, B) = d(A, C) = \frac{1}{2}\sqrt{13}$. Since $A : [0, 0]$ is inside of B but outside of C, the reasonable result should be that $d(A, B) < d(A, C)$. In this section, the mean curve of fuzzy numbers is also taken into consideration in the definition of the metric of two fuzzy numbers.

Definition 2. Let $\widetilde{u}, \widetilde{\nu} \in F(R)$ be two fuzzy numbers. The distance is defined as

$$D^2(\widetilde{u}, \widetilde{\nu}) = \frac{1}{3}\left(\int_0^1 (u^-(\alpha) - \nu^-(\alpha))^2 d\alpha + \int_0^1 (\mathrm{mean}(u_\alpha) - \mathrm{mean}(\nu_\alpha))^2 d\alpha + \int_0^1 (u^+(\alpha) - \nu^+(\alpha))^2 d\alpha\right)$$

Obviously, $D(\widetilde{u}, \widetilde{\nu}) = \sqrt{D^2(\widetilde{u}, \widetilde{\nu})}$ is a distance between two fuzzy numbers $\widetilde{u}, \widetilde{\nu} \in F(R)$.

Example 3. By considering the above example, the crisp number $A : [0, 0]$ and two interval numbers $B : [-2, 3]$, $C : [2, 3]$ and using the distance measure applied in Diamond [9] and Bardossy et al [1], it was found that $d(A, B) = d(A, C) = \frac{1}{2}\sqrt{13}$. However, with our distance measure, $D(A, B) = \frac{1}{3}\sqrt{\frac{53}{4}} <$

$D(A,C) = \sqrt{\frac{77}{4}}$, which is more intuitively appealing.

Theorem 4. *Let $\widetilde{u} = (a_1, a_2, a_3, a_4), \widetilde{\nu} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers. Their a-level sets are of the form*

$u_\alpha = [a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha], \nu_\alpha = [b_1 + (b_2 - b_1)\alpha, b_4 - (b_4 - b_3)\alpha]$.
We have

$$\begin{aligned} D^2(\widetilde{u}, \widetilde{\nu}) &= \frac{1}{48}\left\{[4(c_1 + c_2)^2 + (c_1 + c_2 + c_3 + c_4)^2 + 4(c_3 + c_4)^2]\right. \\ &\quad \left. + \frac{1}{12}[4(c_1 - c_2)^2 + (c_2 - c_1 + c_3 - c_4)^2 + 4(c_3 - c_4)^2]\right\}, \end{aligned}$$

where $c_i = b_i - a_i$, $i = 1, 2, 3, 4$.

3.3. A New Similarity Measure for Fuzzy Numbers

Let $\widetilde{u}$ and $\widetilde{\nu}$ be two fuzzy numbers. Then the degree of similarity $S(\widetilde{u}, \widetilde{\nu})$ between $S(\widetilde{u}, \widetilde{\nu})$ can be calculated as follows:

$$S(\widetilde{u}, \widetilde{\nu}) = \frac{1}{1 + D^2(\widetilde{u}, \widetilde{\nu})},$$

where $D^2(\widetilde{u}, \widetilde{\nu})$ denotes the distance between $\widetilde{u}$ and $\widetilde{\nu}$.

3.4. Comparing the Similarity Measures

1) In [10], the author proposed that a similarity measure $S(\widetilde{u}, \widetilde{\nu})$ between two fuzzy numbers $S(\widetilde{u}, \widetilde{\nu})$ can be calculated as follows:

$$S(\widetilde{u}, \widetilde{\nu}) = \frac{1}{1 + D^2(\widetilde{u}, \widetilde{\nu})},$$

where $d(\widetilde{u}, \widetilde{\nu})) = |P(\widetilde{u}) - P(\widetilde{\nu})|$. $P(\widetilde{u}), P(\widetilde{\nu})$ is the graded mean integral-representation distance of $\widetilde{u}$ and $\widetilde{\nu}$. We can see that the fuzzy numbers $\widetilde{u}$: $(1, 4, 6, 9)$ and $\widetilde{\nu}$: $(1, 5, 5, 9)$ as shown in Figure 2 are quite different. If we apply the similarity measure in [1], i.e., $P(\widetilde{u}) = P(\widetilde{\nu}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6} = 5.0$, and $S(\widetilde{u}, \widetilde{\nu}) = 1$. Obviously, we can calculate the proposed similarity measure between $\widetilde{u} : (1, 4, 6, 9)$ and $\widetilde{\nu} : (1, 5, 5, 9)$. Since $D^2(\widetilde{u}, \widetilde{\nu}) = \frac{5}{6}, S(\widetilde{u}, \widetilde{\nu}) = \frac{6}{11}$.

2) In [3], the authors proposed the degree of similarity between two fuzzy numbers $\widetilde{u} : (a_1, a_2, a_3, a_4)$, $\widetilde{\nu} : (b_1, b_2, b_3, b_4)$ as follows:

$$S(\widetilde{u}, \widetilde{\nu}) = 1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4}.$$

However, we can see that there exist some different fuzzy numbers which have

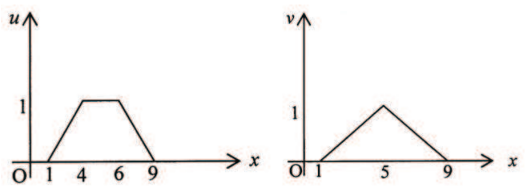

Figure 2

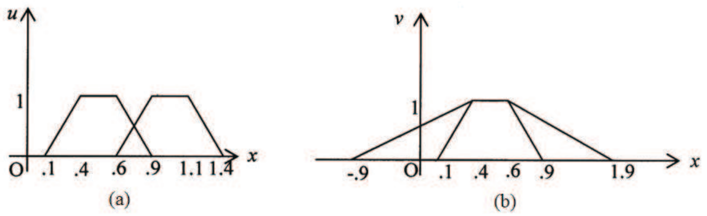

Figure 3

the same degree of similarity. For examples, in Figure 3, two fuzzy numbers in (a): $\widetilde{u_1}$ (0.10.40.60.9), $\widetilde{\nu_1}$ (0.60.91.11.4) and in (b): $\widetilde{u_2}$ (0.10.40.60.9), $\widetilde{\nu_2}$ (−0.90.40.61.9), have the same similarity which are equal to 0.5 by using the method in [3]. However, with our similarity measure, we have the similarity measure of fuzzy numbers in (a): $D^2(\widetilde{u},\widetilde{\nu}) = \frac{1}{4}$, $S(\widetilde{u},\widetilde{\nu}) = \frac{4}{5}$, and similarity measure of fuzzy numbers in (b): $D^2(\widetilde{u},\widetilde{\nu}) = \frac{5}{6}, S(\widetilde{u},\widetilde{\nu}) = \frac{6}{11}$. The results of these calculations support human's judgment.

References

[1] A. Bardossy, R. Hagaman, L. Duckstein, I. Bogardi, Fuzzy least squares regression: theory and application, In: *Fuzzy Regression Analysis* (Eds: J. Kacprzyk, I. Fedrizi), Warsaw and Physica-Verlag, Omniteth Press, Heidelberg (1992), 181-193.

[2] S.M. Chen, A new approach to handling fuzzy decision-making problems, *IEEE Trans, Syst., Man, Cybern.*, **18** (1988), 1012-1016.

[3] S.M. Chen, New methods for subjective mental workload assessment and fuzzy risk analysis, *Cybern. Syst.: Int. J.*, **27** (1996), 449-472.

[4] P. Diamond, Fuzzy least squares, *Inform. Sci.*, **46** (1988), 141-157.

[5] P. Diamond, P. Kloeden, Metric spaces of fuzzy sets, *Fuzzy Sets and Systems*, **35** (1990), 241-249.

[6] P. Diamond, H. Tanaka, Fuzzy regression analysis, In: *Fuzzy Sets in Decision Analysis, Operations Research and Statistics* (Ed. R. Slowinski), Kluwer Academic Publishers, Boston (1998), 349-387.

[7] J. Fan, W. Xie, Some notes on similarity measure and proximity measure, *Fuzzy Sets and Systems*, **101** (1999), 403-412.

[8] R. Goetschel, W. Voxman, Elementary fuzzy calculus, *Fuzzy Sets and Systems*, **18** (1982), 31-43.

[9] L.E. Gleason, Fuzzy set computational processes in risk analysis, *IEEE Trans. Eng. Manag.*, **38** (1991), 177-178.

[10] C.H. Hsieh, S.H. Chen, Similarity of generalized fuzzy numbers with graded mean integration representation, In: *Proc. 8-th Int. Fuzzy Systems Assiciation Worm Congr.*, **2** (1999), 551-555.

[11] O. Kaleva, The Cauchy problem for fuzzy differential equations, *Fuzzy Sets and Systems*, **35** (1990), 389-396.

[12] H.S. Lee, An optimal aggregation method for fuzzy opinions of group decision, *IEEE Int. Conf. Systems, Man, Cybern.*, **3** (1999), 314-319.

[13] A.L. Ralescu, D.A. Ralescu, Probability and fuzziness, *Information Science*, **34** (1984), 85-92.

[14] L.A. Zadeh, Fuzzy sets, *Inform. Control*, **8** (1965), 338-353.