EFFECTS OF PHOTOGRAVITATIONAL AND OBLANTENSSS ON THE TRIANGULAR LAGRANGIAN POINTS IN THE ELLIPTICAL RESTRICTED THREE BODY PROBLEM

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Abstract: The effects of the oblateness and the photogravitational of the bigger primary and the oblateness of the smaller primary on the location and the stability of the triangular Lagrangian equilibrium points in the elliptical restricted three body problem have been studied. We have exploited the method of averaging used by Grebenikov, throughout the analysis of the stability of the equilibrium points. We have developed simulation technique using Matlab 6.1 for analysis the stability of the system. It is also observed that for these points, the range of stability decreases as the oblateness and the radiation pressure parameter increases.

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1. Introduction

The present paper is devoted to the analysis of the effects of the photogravitational of the bigger primary and the oblateness of the smaller primary on the location and stability of triangular Lagrangian equilibrium points of the

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planner elliptical restricted three body problem. The elliptical restricted three body problem describes the dynamical system more accurately on account of realistic assumptions of the motion of the primaries are subjected to move along the elliptical orbit. We have attempted to investigate the stability of triangular equilibrium points under the effects of photogravitational and the oblateness of primaries, i.e. both of the primaries are considered to be oblate spheroid and the bigger primary is the source of radiation.

The stability of triangular equilibrium points of the elliptical restricted three body problem has been studied by Danby [1]. He investigated the stability using only the first order linear equations of displacements from the points of equilibrium. The method used is to find the characteristic exponents numerically. Jefferys [2], [3] shows there exist doubly symmetric almost circular periodic solution of one of the primaries is sufficiently small. He further showed that the existence of families of elliptical orbits for any value of eccentricity and critical inclination. The elliptic restricted three body problem has not been fully explored(planer or spheroid) although a number of research papers have been devoted to it. Szebehely [4] presented the differential equations of motions of the Jupiter’s orbit for non zero eccentricity using true anomaly of the primaries as the independent variable and by introducing special set of dimensionless variable describing the position of the third body.

He used several possible regularizing transformations to obtain regularized equations in the term of integral differential equations for the elliptical restricted three body problem. Bennett [5] developed an analytical technique for determination of characteristic exponents and applied it to equilibrium solution in the elliptic restricted of three bodies.

Non-linear stability of the triangular Lagrangian points in the elliptic restricted problem of three bodies has been studied by Gyorgyey [6]. The same problem was dealt by Kumar and Choudhary [7], when both the attracting bodies are radiating. Effects of radiation on the non-linear stability zones of the Lagrangian points has been investigated by Papadakis [8]. Kashan [9], [10] studied the existence of Libration points and their stability in the photogravitational elliptical restricted three body problem. The different aspect of the problem in detail, of elliptical restricted three body problem has been studied, Sandoor Zsolt et al [11], Halan and Rana [12], Alexander D. Burno [13], J.F. Palacios [14], and A.P. Markeev [15]. M.K. Ammar [16] studied the effect of solar radiation pressure on the Lagrangian points in the elliptic restricted three body problem. A. Narayan and C. Ramesh [17], [18], [19] studied the stability of triangular points in the generalised oblate elliptical restricted three body problem. The same problems have also been dealt in resonance and parametric
The present paper deals with the effects of photogravitational and oblateness of the primaries on location and stability of the triangular Lagrangian points in elliptical restricted three body problem. We have obtained the coordinates of the triangular equilibrium points of the problem. For the circular problem primaries are fixed with respect to uniformly rotating axis and hence the Hamiltonian does not involve time explicitly, when the primaries move in the elliptical orbit. The introduction of non-uniformly rotating and pulsating co-ordinate system results again in fixed location of primaries. The Hamiltonian however, does not depend explicitly on independent variable in this case. The method, which on stability of the triangular equilibrium points of planar elliptical restricted three body problem is being explored, is the method of averaging due to Grebenikov [20]. It is also observed that for these points the range of stability decreases as the oblateness parameter increases. The dimensionless variables are introduced by using the distance $r$ between primaries given by:

$$r = \frac{a(1-e^2)}{(1+e \cos v)},$$

where $a$ and $e$ are the semi-major axis and the eccentricity of the elliptical orbit of the either primary around other and $v$ is the true anomaly of one of the primary of mass $m_1$. A coordinate system which rotates with the variable angular velocity $\omega$ is introduced. This angular velocity is given by

$$\frac{d\omega}{dt^*} = \frac{k (m_1 + m_2)^{1/2} (1 + e \cos v)^2}{a^{3/2} (1 - e^2)^{3/2}},$$

where $t^*$ is dimensionless time.

The equation follows from the principal of the conversation of angular momentum in the problem of two bodies formed by the primaries of masses $m_1$ and $m_2$. This principle is expressed as follows:

$$\omega r^2 = \left[a (1 - e^2) k^2 (m_1 + m_2)\right]^{1/2},$$

where $k = k_1 + k_2$, where $k_1$ and $k_2$ are the product of the universal gravitational constants with the mass of primaries.

The force of radiation is given by

$$F = F_g - F_p = \left(1 - \frac{F_p}{F_g}\right) F_g = q f_g,$$
where \( F_g \) is the gravitational attraction force; \( F_p \) is the radiation pressure; \( q \) is the mass reduction factor.

The massive primary is supposed to be oblate spheriod as well as the source of radiation and the smaller one is also considered to be oblate spheroid. The radiation and oblateness of massive primary does not affect the motion of the smaller primary.

2. Equation of Motion in Elliptical Restricted Problem of Three Bodies when Both the Primaries are Oblate Spheroid

Consider the two bodies \( s_1 \) and \( s_2 \) of masses \( m_1 \) and \( m_2 \), \( m_1 > m_2 \) moving in a plane about their centre of mass \( O \) in Keplerian elliptical orbit having eccentricity \( e \). We assume further the bigger mass \( m_1 \) and the smaller mass \( m_2 \) are an oblate spheroid and the bigger mass is also a source of radiation. A third body \( P \) of infinitesimal mass is moving in the plane of motion of \( s_1 \) and \( s_2 \) without influencing their motion, under the gravitational attraction of the finite masses \( s_1 \) and \( s_2 \). The oblateness and radiation of the bigger mass \( s_1 \) and the oblateness of smaller mass \( s_2 \) affects the motion of the infinitesimal body \( P \), but the motion of the smaller mass \( m_2 \), remains unaffected due to large mass.

![Figure 1: Motion of infinitesimal under the oblate primaries](image)

Consider the equation of motion in a fixed co-ordinate system using dimensional quantities and variable. Suppose \((x, y)\) be the co-ordinates of the infinitesimal body \( P \) while \((x_1, y_1)\) and \((x_2, y_2)\) be those of the primaries \( s_1 \) and \( s_2 \) in a fixed co-ordinate system with origin at the centre of mass of the primaries and the line joining the primaries is taken as the \( x \)-axis. The \( y \)-axis is taken as the line passing through \( O \) and perpendicular to the \( x \)-axis. Let \( R_1 \) and \( R_2 \) be the dimensional distance of the third body \( P \) from the the primaries \( s_1 \) and \( s_2 \) respectively. Let \( R_0 \) be the distance between the primaries \( s_1 \) and \( s_2 \).
Let $R_{E1}$ and $R_{E2}$ be the equatorial radii of the oblate primaries $\varepsilon_1^*$ and $\varepsilon_2^*$ be the non-dimensional oblateness parameters. Then the equation of motion of this system can be written as:

$$\frac{d^2x}{dt^*} = -\frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left[ \frac{m_1 k^2 (x - x_1) q}{R_1^3} + \frac{m_2 k^2 (x - x_2)}{R_2^3} + \frac{3m_1 k^2 (x - x_1) q}{2 R_1^3 r_1^2} A_1 + \frac{3m_2 k^2 (x - x_2)}{2 R_2^3 r_2^2} A_2 \right],$$

$$\frac{d^2y}{dt^*} = -\frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left[ \frac{m_1 k^2 (y - y_1) q}{R_1^3} + \frac{m_2 k^2 (y - y_2)}{R_2^3} + \frac{3m_1 k^2 (y - y_1) q}{2 R_1^3 r_1^2} A_1 + \frac{3m_2 k^2 (y - y_2)}{2 R_2^3 r_2^2} A_2 \right], \tag{2.1}$$

where

$$R_1^2 = (x - x_1)^2 + (y - y_1)^2, \quad R_2^2 = (x - x_2)^2 + (y - y_2)^2. \tag{1}$$

where $t^*$ is the dimensional time, $q$ is a parameter characterizing the radiation pressure of the bigger primary $s_1$ and $A_1 = \frac{A_1^*}{R_0^2} = \varepsilon_1^* R_{E1}^2$ and $A_2 = \frac{A_2^*}{R_0^2} = \varepsilon_2^* R_{E2}^2$ are a non-dimensional number characterizing the oblateness effect of these primaries. We shall introduce a rotating co-ordinate system $(\overline{x}, \overline{y})$ by substituting

$$Z = z e^{i\nu}, \tag{2.3}$$

where

$$Z = x + iy, \quad \text{and} \quad z = \overline{x} + i\overline{y}. \tag{2.4}$$

After, using the complex vector $Z$, the equation of motion of the (2.1) takes the form:

$$\frac{d^2Z}{dt^*} = -\frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left[ \frac{m_1 k^2 (z - z_1) q}{R_1^3} + \frac{m_2 k^2 (z - z_2)}{R_2^3} + \frac{3m_1 k^2 (z - z_1) q}{2 R_1^3 r_1^2} A_1 + \frac{3m_2 k^2 (z - z_2)}{2 R_2^3 r_2^2} A_2 \right]. \tag{2.5}$$
Now, differentiating (2.3) twice with respect to dimensional time $t^*$ we get:

\[
\frac{d^2 Z}{dt^*^2} = \left[ \frac{d^2 z}{dt^*^2} e + 2i \frac{dz}{dt^*} \frac{dv}{dt^*} - z \left( \frac{dv}{dt^*} \right)^2 + iz \frac{d^2 v}{dt^*^2} \right] e^{iv}. \tag{2.6}
\]

Using (2.3) and (2.6), the equation of motion (2.5) takes the form:

\[
\frac{d^2 z}{dt^*^2} + 2i \frac{dz}{dt^*} \frac{dv}{dt^*} - z \left( \frac{dv}{dt^*} \right)^2 + iz \frac{d^2 v}{dt^*^2} = - \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left[ \frac{m_1 k^2 (z - z_1)}{R_1^3} + \frac{m_2 k^2 (z - z_2)}{R_2^3} + \frac{3m_1 k^2 (z - z_1)}{2R_1^3 r_1^2} A_1 + \frac{3m_2 k^2 (z - z_2)}{2R_2^3 r_2^2} A_2 \right]. \tag{2.7}
\]

Thus, the equation of motion in the rotating coordinate system are:

\[
\frac{d^2 z}{dt^*^2} + 2i \frac{dz}{dt^*} \frac{dv}{dt^*} = - \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left[ \frac{m_1 k^2 (z - z_1)}{R_1^3} + \frac{m_2 k^2 (z - z_2)}{R_2^3} + \frac{3m_1 k^2 (z - z_1)}{2R_1^3 r_1^2} A_1 + \frac{3m_2 k^2 (z - z_2)}{2R_2^3 r_2^2} A_2 \right] + z \left( \frac{dv}{dt^*} \right)^2 - iz \frac{d^2 v}{dt^*^2}. \tag{2.7}
\]

The second term of the equation (2.7) in the left hand side is the Coriolis acceleration, the first term in the right hand side of the same equation is gravitational effects due to the oblate primaries as well as radiation, the third and the fourth term represent the centrifugal effect and the acceleration normal to the radius vector due to the non-uniform rotation of the system.

The complex position vectors $z_1$ and $z_2$ are the location of the primaries. Since these primaries are permanent on the real axis of $(\tau, \eta)$ system, we have:

\[
Z_1 = X_1 = \frac{-p_1}{(1 + e \cos v)}, \quad Z_2 = X_2 = \frac{p_2}{(1 + e \cos v)}; \tag{2.8}
\]

where $p_1$ and $p_2$ are positive and

\[
\frac{p_1}{p_2} = \frac{a_1}{a_2} = \frac{m_2}{m_1}, \tag{2.9}
\]
where $a_1$ and $a_2$ are the semi-major axes of the elliptic orbits of the massive bodies described around their centre of mass $m_1$ and $m_2$ are the masses of the massive bodies (primaries). The distance between the primaries and the third body are:

\[ R_1 = |Z - Z_1 r| = r |z - \overline{x}_1 r| = l \left[ (x - \overline{x}_1)^2 + \overline{y}^2 r \right]^{\frac{1}{2}}, \]

\[ R_2 = r |Z - Z_2 r| = r |z - \overline{x}_2 r| = l \left[ (x - \overline{x}_2)^2 + \overline{y}^2 r \right]^{\frac{1}{2}}. \]  \hfill (2.10)

Further, we shall introduce dimensionless pulsating co-ordinates system given by:

\[ \rho = \left( \frac{z}{r} \right) = (x + iy), \]  \hfill (2.11)

where

\[ r = \frac{a (1 - e^2)}{1 + e \cos v} \]  \hfill (2.12)

is the distance between the primaries in which $a$ is the semi-major axis of the elliptic orbit of the one primary around the other. From equations (2.11) and (2.12) we have

\[ x = \frac{\overline{x}_1 (1 + e \cos v)}{(1 - e^2) a}, \quad \text{and} \quad y = \frac{\overline{y} (1 + e \cos v)}{(1 - e^2) a}. \]  \hfill (2.13)

Since, these primaries are fixed in the co-ordinate system, we have from equations (2.8) and (2.13) the co-ordinates of these primaries are represented as follows:

\[ x_1 = \frac{\overline{x}_1 (1 + e \cos v)}{a (1 - e^2)} = \frac{-p_1}{a (1 - e^2)} = \frac{-a_1}{a} = -\mu, \]

\[ x_2 = \frac{\overline{x}_2 (1 + e \cos v)}{a (1 - e^2)} = \frac{-p_2}{a (1 - e^2)} = \frac{-a_2}{a} = (1 - \mu), \]

\[ y_1 = 0, \quad y_2 = 0. \]

Here $\mu = \left( \frac{m_2}{m_1 + m_2} \right)$. Therefore, the fixed locations of these primaries in terms of coordinates $(x, y)$ system are represented as $(-\mu, 0)$ and $(1 - \mu, 0)$.

We observe the four different conditions into consideration regarding the problem of two bodies moving in elliptical orbit:

(i) the orbit of $m_1$ relative to $m_2$ with a semi major axis $a$.

(ii) the orbit of $m_2$ relative to $m_1$ with the semi major axis $a$. 
(iii) the orbit of $m_1$ with respect to the centre of mass with semi-major axis $a_1 = a\mu$.

(iv) the orbit of $m_2$ with respect to the centre of mass with semi-major axis $a_2 = a(1 - \mu)$.

We introduce the true anomaly $v$ as independent variable for solving the equation governing the motion of the system. We have from equation (2.11)

$$z = r\rho.$$  

Hence, we have

$$\frac{d^2z}{dt^{2*}} = \frac{d}{dt^*} \left( r \frac{dp}{dt^*} + \rho \frac{dr}{dt^*} \right) = r \left[ \frac{d^2p}{dt^{2*}} + 2 \frac{dr}{dt^*} \cdot \frac{dp}{dt^*} + \rho \frac{d^2r}{dt^{2*}} \right]. \tag{2.14}$$

Introducing $v$ as an independent variable, we have:

$$\frac{d^2z}{dt^{2*}} = r \left[ \frac{d^2r}{dt^{2*}} \cdot \frac{dp}{dv} + \frac{d^2\rho}{dv^2} \left( \frac{dv}{dt^*} \right)^2 \right] + 2 \left( \frac{dr}{dt^*} \cdot \frac{dp}{dv} \right) + \left[ \rho \frac{d^2r}{dt^{2*}} \right], \tag{2.15}$$

and

$$2i \frac{dv}{dt^*} \cdot \frac{d^2z}{dt^{2*}} = 2i \frac{dv}{dt^*} \left( \frac{d^2r}{dt^{2*}} + r \cdot \frac{dp}{dv} \cdot \frac{dv}{dt^*} \right), \tag{2.16}$$

and

$$\frac{m_1 k^2 (Z - Z_1)}{R_1^3} = \frac{m_1 k^2 (\rho - \rho_1) q}{r^2 r_1^3}, \tag{2.17}$$

where

$$r_1^2 = |\rho - \rho_1|^2 = (x + \mu)^2 + y^2 \tag{2.18}$$

since $\rho_1 = x_1 = -\mu$. Similarly

$$\frac{m_2 k^2 (Z - Z_2)}{R_2^3} = \frac{m_2 k^2 (\rho - \rho_2)}{r^2 r_2^3}, \tag{2.19}$$

where

$$r_2^2 = |\rho - \rho_2|^2 = (x - 1 + \mu)^2 + y^2. \tag{2.20}$$

Hence $\rho_2 = x_2 = 1 - \mu$.

Now substituting the values from (2.15), (2.16), (2.17) and (2.19) in (2.7) we get:

$$r \left( \frac{dv}{dt^*} \right)^2 \left[ \frac{d^2p}{dv^2} + 2i \frac{dp}{dv} \right] + \rho \left[ \frac{d^2p}{dt^{2*}} - r \left( \frac{dv}{dt^*} \right)^2 \right]$$
We have \( r = r(v) = \frac{a(1-e^2)}{(1+e\cos v)} \), which is the solution of the two body problem involving the primaries \( s_1 \) and \( s_2 \). The integral of the angular momentum of the two body problem is given by:

\[
\left( \frac{r^2 dv}{dt^*} \right)^2 = a \left( 1 - e^2 \right) k^2 (m_1 + m_2);
\]  

(2.22)

Differentiating (2.22) with respect to \( t^* \), we have:

\[
r \frac{d^2 v}{dt^{2*}} + 2 \frac{dr}{dt^*} \frac{dv}{dt^*} = 0;
\]  

(2.23)

The equation of motion of the two primaries are given by

\[
\frac{d^2 r}{dt^{2*}} - r \left( \frac{dv}{dt^*} \right)^2 = \frac{-k^2 (m_1 + m_2)}{r^2}.
\]  

(2.24)

The equation (2.24) takes the form after using (2.22);

\[
\frac{d^2 r}{dt^{2*}} - r \left( \frac{dv}{dt^*} \right)^2 = \frac{-r^2}{a \left( 1 - e^2 \right)} \cdot \left( \frac{dv}{dt^*} \right)^2.
\]  

(2.25)

Substituting the values of (2.23) and (2.25) in (2.21), we get

\[
\left[ \frac{d^2 v}{dt^{2*}} \right] + \left( \frac{d^2 \rho}{dv^2} + 2 \frac{d \rho}{dv} \right) + \frac{r^2}{a (1 - e^2)} \cdot \left( \frac{dv}{dt^*} \right)^2
\]

\[
= - \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left[ \frac{m_1 k^2 (\rho - \rho_1) q}{r^2 r_1^3} + \frac{m_2 k^2 (\rho - \rho_2)}{r^2 r_2^3} + \frac{3 m_1 k^2 A_1 (\rho - \rho_1) q}{2 r^2 r_1^5}
\]

\[
+ \frac{3 m_2 k^2 A_2 (\rho - \rho_2)}{2 r^2 r_2^5} \right] .
\]
Dividing throughout by \( r \) and using the value from (2.22) in the given equation, we get:

\[
\frac{d^2 \rho}{dv^2} + 2i \frac{d \rho}{dv} = \rho - \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left\{ \frac{m_1 (\rho - \rho_1) q}{m_1 + m_2} \frac{r_1^3}{r_1^3} + \frac{m_2 (\rho - \rho_2)}{m_1 + m_2} \frac{r_2^3}{r_2^3} \right\} + 3A_1 \left( \frac{m_1}{m_1 + m_2} \right) \frac{(\rho - \rho_1) q}{2r_1^5} + \frac{3A_2}{2} \left( \frac{m_2}{m_1 + m_2} \right) \frac{(\rho - \rho_2)}{r_2^5}.
\]

Using \( r = \frac{a(1-e^2)}{(1-e \cos v)} \), \( \mu = \left( \frac{m_2}{m_1 + m_2} \right) \) and \( 1 - \mu = \frac{m_1}{m_1 + m_2} \) the above expression will be reduced to the following form:

\[
\frac{d^2 \rho}{dv^2} + 2i \frac{d \rho}{dv} = \frac{1}{1 + e \cos v} \left\{ \rho - \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left\{ \frac{(1-\mu)(\rho - \rho_1) q + \mu (\rho - \rho_2)}{r_1^3} + \frac{3A_1 (1-\mu)(\rho - \rho_1) q + 3A_2 \mu (\rho - \rho_2)}{2r_1^5} \right\} \right\}.
\]

Replacing \( \rho = x + iy \), \( \rho_1 = x_1 + iy_1 \) and \( \rho_2 = x_2 + iy_2 \) in the above equation and equating the real and imaginary parts, we get:

\[
\frac{d^2 x}{dv^2} - 2 \frac{dy}{dv} = \frac{1}{1 + e \cos v} \left\{ x - \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left\{ \frac{(1-\mu)(x - x_1) q + \mu (x - x_2)}{r_1^3} + \frac{3A_1 (1-\mu)(x - x_1) q + 3A_2 \mu (x - x_2)}{2r_1^5} \right\} \right\},
\]

\[
\frac{d^2 y}{dv^2} + 2 \frac{dx}{dv} = \frac{1}{1 + e \cos v} \left\{ y - \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left\{ \frac{(1-\mu)(y - y_1) q + \mu (y - y_2)}{r_1^3} + \frac{3A_1 (1-\mu)(y - y_1) q + 3A_2 \mu (y - y_2)}{2r_1^5} \right\} \right\}.
\]

Hence the equation of motion shall be reduced to the form by replacing \( x_1 = -\mu \), \( x_2 = (1 - \mu) \), \( y_1 = 0 \) and \( y_2 = 0 \) we get:
\[
\frac{d^2 x}{dv^2} - 2 \frac{dy}{dv} = \frac{1}{(1 + e \cos v)} \left[ x - \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left\{ (1 - \mu) \left( x + \mu \right) \frac{q}{r_1^3} + \mu \left( x - 1 + \mu \right) \frac{1}{r_2^3} \\
+ \frac{3A_1 (1 - \mu) \left( x + \mu \right)}{2r_1^5} + \frac{3A_2 \mu \left( x - 1 + \mu \right)}{2r_2^5} \right\} \right]
\]

and

\[
\frac{d^2 y}{dv^2} + 2 \frac{dx}{dv} = \frac{1}{(1 - e \cos v)} \left[ y - \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left\{ (1 - \mu) q \frac{y}{r_1^3} + \mu y \frac{1}{r_2^3} + \frac{3A_1 (1 - \mu) y q}{2r_1^5} + \frac{3A_2 \mu y}{2r_2^5} \right\} \right].
\]

The differential equation of motion of third body \( P \) in non-dimension body barycentric, pulsating non-uniformly rotating co-ordinate system \((x, y)\) are written in the form:

\[
x'' - 2y' = \frac{1}{1 + e \cos v} \left( \frac{\partial \Omega}{\partial x} \right), \quad y'' - 2x' = \frac{1}{1 + e \cos v} \left( \frac{\partial \Omega}{\partial y} \right), \quad (2.26)
\]

where \( ' \) denotes differentiation with respect to \( v \),

\[
\Omega = \frac{x^2 + y^2}{2} + \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left\{ (1 - \mu) \frac{q}{r_1} + \frac{\mu}{r_2} + \frac{(1 - \mu) q A_1}{2r_1^3} + \frac{\mu A_2}{2r_2^3} \right\}, \quad (2.27)
\]

and

\[
r_1^2 = (x + \mu)^2 + y^2, \quad r_2^2 = (x - 1 + \mu)^2 + y^2. \quad (2.28)
\]

Thus, the equations of motion of a elliptical restricted three body problem in which one of the bigger primaries is a radiating oblate, the smaller primaries is oblate spheroid. We observe that the equation obtained is in a form identical with the form of equation of circular restricted three body problem.

3. Location of Equilibrium Points in the Elliptical Restricted Three Body Problem under the Radiating and Oblate Primaries

The equilibrium points of the system are determined by the equation:

\[
\frac{\partial \Omega}{\partial x} = 0 \quad \text{and} \quad \frac{\partial \Omega}{\partial y} = 0; \quad (3.1)
\]
where \( \Omega \) is represented by the equation (2.27). Differentiating (2.27) partially with respect to \( x \) and \( y \) and equating to zero, we get:

\[
x \left[ 1 - \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left\{ \frac{(1 - \mu) q}{r_1^3} + \frac{\mu q}{r_2^3} + \frac{3A_1 (1 - \mu) q}{2r_1^5} + \frac{3\mu A_2}{2r_2^5} \right\} \right]
- \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left\{ \frac{\mu (1 - \mu) q}{r_1^3} + \frac{\mu (\mu - 1)}{r_2^3}
+ \frac{3A_1 (1 - \mu) \mu q}{2r_1^5} + \frac{3\mu A_2 (\mu - 1)}{2r_2^5} \right\} = 0, \quad (3.2)
\]

\[
y \left[ 1 - \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left\{ \frac{(1 - \mu) q}{r_1^3} + \frac{\mu q}{r_2^3} + \frac{3A_1 (1 - \mu) q}{2r_1^5} + \frac{3A_2 \mu (\mu - 1)}{2r_2^5} \right\} \right]
= 0. \quad (3.3)
\]

From equation (3.2) and equation (3.3) we get:

\[-\frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left[ \frac{\mu (1 - \mu) q}{r_1^3} + \frac{\mu (\mu - 1)}{r_2^3}
+ \frac{3A_1 (1 - \mu) \mu q}{2r_1^5} + \frac{3A_2 \mu (\mu - 1)}{2r_2^5} \right] = 0,
\]

as

\[-\frac{\mu (\mu - 1)}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \neq 0 \quad \text{and} \quad \frac{1}{r_1^3} - \frac{1}{r_2^3} + \frac{3qA_1}{2r_1^3} - \frac{3A_2}{2r_2^3} = 0, \quad (3.4)
\]

where these primaries are not a oblate spheroid, i.e. \( A_1 = 0 \), the solution of the equation (3.2) and (3.3) are \( r_i = 1, i = 1, 2 \).

Therefore, we can assume that the solution of (3.1) and (3.2) are \( r_i = 1 + \varepsilon_i \) where \( 0 < \varepsilon_1 < 1 \) and \( 0 < \varepsilon_2 < 1 \). Here \( \varepsilon_i \) are “very-very” small, \( i = 1, 2 \).

Putting these values of \( r_1 \) and \( r_2 \) in (3.4) we get

\[
x = \frac{1}{2} - \mu - \frac{A_1}{3} - \frac{A_2}{2} + \frac{5A_1 q}{6};
\]

and

\[
y = \pm \frac{\sqrt{3}}{2} \left[ 1 - \frac{2}{9} A_1 + \frac{1}{3} A_2 - \frac{5A_1 q}{9} \right].
\]

Thus the coordinate of the triangular equilibrium points up to the first order terms in the parameter \( A_1, A_2 \) and \( q \) are oblateness.
4. Stability of Equilibrium Points

There are two triangular equilibrium points of the problem in the plane of the finite bodies. In the co-ordinate system \((x, y)\) the three bodies form nearly equilateral triangles.

Since the equilateral points are symmetrical to each other, the nature of motion near the two triangular points is the same. Therefore, it is sufficient to analyse the motion of the equilibrium points having the location \((x_0, y_0)\) given by:

\[
x_0 = \frac{1}{2} - \mu - \frac{A_1}{3} - \frac{A_2}{2} + \frac{5A_1q}{6},
\]

and

\[
y_0 = \pm \frac{\sqrt{3}}{2} \left[ 1 - \frac{2}{9} A_1 + \frac{1}{3} A_2 - \frac{5A_1q}{9} \right].
\]

In order to investigate the stability of the equilibrium points (4.1) in the first approximation, we derive the equation for the variation in the coordinate.

Let \(\xi, \eta\) denote the small displacement in \(x_0, y_0\). Then

\[
x = x_0 + \xi, \quad y = y_0 + \eta.
\]

Differentiating we get

\[
x' = \xi', \quad y' = \eta',
\]

and \(x'' = \xi'', \ y'' = \eta''\).

Now, by applying Taylor’s Theorem and retaining first order terms in the infinitesimals \(\xi\) and \(\eta\), we get:

\[
\Omega_x = \Omega^0_x + \xi \Omega^0_{xx} + \eta \Omega^0_{xy} \quad \text{and} \quad \Omega_y = \Omega^0_y + \xi \Omega^0_{yx} + \eta \Omega^0_{yy}.
\]

Substituting the value from equation (4.2) in equation (2.26), the equation of motion takes the form:

\[
\xi'' - 2\eta' = \phi \left[ (\Omega^0_{xx} \xi + \Omega^0_{xy} \eta) \right],
\]

\[
\eta'' + 2\xi' = \phi \left[ (\Omega^0_{yx} \xi + \Omega^0_{yy} \eta) \right].
\]

where \(\phi = \frac{1}{1 + e \cos v}\).

Differentiating \(\Omega\) with respect to \(x\) and \(y\), we get:

\[
\Omega_x = x - \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left[ \frac{(1 - \mu)(x + \mu) q}{r_1^3} + \frac{\mu(x - 1 + \mu)}{r_2^3} \right].
\]
\[ + \frac{3 (1 - \mu) (x + \mu) A_1 q}{2 r_1^5} + \frac{3 (x - 1 + \mu) A_2}{2 r_2^5} \], \quad (4.4) \]

\[ \Omega_y = y - \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left[ \frac{y (1 - \mu) q}{r_1^3} + \frac{\mu y}{r_2^3} + \frac{3 (1 - \mu) y A_1 q}{2 r_1^3} + \frac{3 \mu A_2 y}{2 r_2^3} \right]. \quad (4.5) \]

Differentiating (4.4) with respect to \( x \) and \( y \) and (4.5) with respect to \( y \), we get:

\[ \Omega_{xx} = 1 - \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left[ \frac{(1 - \mu) q}{r_1^3} - \frac{3 (1 - \mu) (x + \mu)^2 q}{2 r_1^5} + \frac{\mu}{r_2^3} - \frac{3 \mu (x - 1 + \mu)^2}{2 r_2^5} \right. \]
\[ + \left. \frac{3 (1 - \mu) A_1 q}{2 r_1^3} - \frac{15 (1 - \mu) (x + \mu)^2 q A_1}{2 r_1^5} + \frac{33 \mu A_2}{2 r_2^3} - \frac{15 \mu A_2 (x - 1 + \mu)^2}{2 r_2^5} \right], \]

\[ \Omega_{yy} = 1 - \frac{1}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left[ \frac{(1 - \mu) q}{r_1^3} - \frac{3 (1 - \mu) y^2 q}{2 r_1^5} + \frac{\mu}{r_2^5} - \frac{3 \mu y^2}{2 r_2^3} - \frac{3 (1 - \mu) A_1 q}{2 r_1^3} \right. \]
\[ + \left. \frac{3 \mu A_2}{2 r_2^5} - \frac{15 A_1 (1 - \mu) y^2 q}{2 r_1^5} - \frac{15 \mu A_2 y^2}{2 r_2^3} \right], \]

\[ \Omega_{xy} = \frac{3 y}{1 + 3 \left( \frac{A_1 + A_2}{2} \right)} \left[ \frac{(1 - \mu) (x + \mu) q}{r_1^5} - \frac{\mu}{r_2^5} (x - 1 + \mu) \right. \]
\[ - \left. \frac{5 (1 - \mu) (x + \mu) A_1 q}{2 r_1^5} + \frac{5 \mu A_2 (x - 1 + \mu)}{2 r_2^3} \right]. \]

Evaluating \( \Omega_{xx}, \Omega_{xy} \) and \( \Omega_{yy} \) at the equilibrium point \((x_0, y_0)\) given by equation (4.1), we get:

\[ \Omega_{xx}^0 = \frac{1}{2} - \frac{3}{2} A_1 \]

\[ - \frac{3}{4} A_2 + \mu - \frac{17}{8} \mu A_1 + \frac{3}{8} \mu A_2 + \epsilon \left( \frac{1}{2} + \frac{3}{2} A_1 + \frac{3}{4} A_2 - \frac{5}{2} + \frac{3}{8} A_1 - \frac{3}{4} A_2 \right), \]
\[ \Omega_{xy}^0 = \frac{\sqrt{3}}{2} \left[ \left( -1 + 2\mu - \frac{16}{9} A_1 - \frac{1}{3} A_2 - \frac{169 A_1 \mu}{36} + \frac{77 A_2 \mu}{12} \right) 
\right. 
\left. - \epsilon \left( -1 + \mu - \frac{16}{9} A_1 - \frac{1}{3} A_2 - \frac{71 \mu A_1}{36} + \frac{\mu A_2}{3} \right) \right], \]

\[ \Omega_{yy}^0 = \frac{3}{2} - \frac{1}{2} A_1 + \frac{3}{4} A_2 + \frac{25 \mu}{8} A_1 - \frac{45}{4} \mu A_2 
\left. + \epsilon \left( \frac{1}{2} + \frac{1}{2} A_1 - \frac{3}{4} A_2 + \frac{\mu}{2} - \frac{39 \mu A_1}{8} + \frac{3 \mu A_2}{4} \right) \right), \]

when the eccentricity is zero, the variational equations reduce directly to those for the circular restricted three body problem.

5. First Order Stability of the Equilibrium Points

In order to investigate the stability of the equilibrium points, we introduce a new variable given by:

\[ x_1 = \xi, \quad x_2 = \eta, \quad x_3 = \frac{d\xi}{dv}, \quad x_4 = \frac{d\eta}{dv}, \quad (5.1) \]

in the equation of variations (4.3), the system of equation takes the form, after using these variables:

\[ \frac{dx_i}{dv} = P_{i1} x_1 + P_{i2} x_2 + P_{i3} x_3 + P_{i4} x_4; \quad i = 1, 2, 3, ..., \quad (5.2) \]

where \( P_{11} = P_{12} = P_{14} = P_{21} = P_{22} = P_{23} = P_{33} = P_{44} = 0, \) \( P_{13} = 1, \) \( P_{24} = 1, \)

\( P_{34} = 2, \) \( P_{43} = -2, \)

\[ P_{31} = \Omega_{xx}^0 \phi = \left( \frac{1}{2} - \frac{3}{2} A_1 - \frac{3}{4} A_2 + \mu - \frac{17}{8} \mu A_1 + \frac{3}{8} \mu A_2 
\right. 
\left. + \epsilon \left( \frac{1}{2} + \frac{3}{2} A_1 + \frac{3}{4} A_2 - \frac{\mu}{2} + \frac{3 \mu A_1}{8} - \frac{3 \mu A_2}{4} \right) \right) \phi, \]

\[ P_{32} = \Omega_{xy}^0 \phi = \frac{\sqrt{3}}{2} \left[ \left( -1 + 2\mu - \frac{16}{9} A_1 - \frac{1}{3} A_2 - \frac{169 A_1 \mu}{36} + \frac{77 A_2 \mu}{12} \right) \right] \]
The co-efficient in the system of equation (5.2) are periodic function of \( v \) with period \( 2\pi \). We now consider the averaged system:

\[
\frac{dx_i^0}{dv} = P_i^{(0)} x_1^{(0)} + P_{i2}^{(0)} x_2^{(0)} + P_{i3}^{(0)} x_3^{(0)} + P_{i4}^{(0)} x_4^{(0)},
\]

where

\[
P_{i,s}^{(0)} = \frac{1}{2\pi} \int_{0}^{2\pi} P_{is}(v) \, dv, \quad i, s = 1, 2, 3, 4.
\]

From the formula (5.3), we obtain after evaluation:

\[
P_{11}^{(0)} = P_{12}^{(0)} = P_{14}^{(0)} = P_{21}^{(0)} = P_{22}^{(0)} = P_{23}^{(0)} = P_{33}^{(0)} = P_{44}^{(0)} = 0,
\]

\[
P_{13}^{(0)} = 1, \quad P_{24}^{(0)} = 1, \quad P_{34}^{(0)} = 2, \quad P_{43}^{(0)} = -2,
\]

and

\[
P_{31}^{(0)} = \frac{1}{2\pi} \int_{0}^{2\pi} P_{31} \, dv = \frac{1}{8} \left[ (4 - 12A_1 - 6A_2 - 17\mu A_1 + 3\mu A_2 + 8\mu) \right.
\]

\[
+ \epsilon \left( 4 + 12A_1 + 6A_2 - 4\mu + 3\mu A_1 - 6\mu A_2 \right) \left. \right] \cdot \frac{1}{(1 - e^2)^{\frac{1}{4}}},
\]

\[
P_{32}^{(0)} = \frac{1}{2\pi} \int_{0}^{2\pi} P_{32} \, dv = \frac{\sqrt{3}}{72} \left[ (-36 - 64A_1 - 12A_2 + 72\mu - 16A_1\mu + 231\mu A_2) \right.
\]

\[
- \epsilon (-36 + 36\mu - 64A_1 - 12A_2 - 71\mu A_1 + 12\mu A_2) \left. \right] \cdot \frac{1}{(1 - e^2)^{\frac{1}{4}}} = P_{41}^{(0)}.
\]
\[ P_{42} = \frac{1}{2\pi} \int_0^{2\pi} P_{42} dv = \frac{1}{8} \left[ (12 - 4A_1 + 6A_2 + 25\mu A_1 - 90\mu A_2) \right. \\
+ \varepsilon \left( -4 + 4A_1 - 6A_2 + 4\mu - 39\mu A_1 + 6\mu A_2 \right) \left. \right] \frac{1}{(1 - e^2)^{\frac{3}{2}}} \]  \hspace{1cm} (5.6)

The characteristic equation for the system of equations of variations (5.4) is given by:

\[ \lambda^4 - Q\lambda^2 + R = 0, \]  \hspace{1cm} (5.7)

where

\[ Q = P_{31}^{(0)} + P_{42}^{(0)} \]

\[ - 4 \left[ (16 - 16A_1 + 8\mu + 8\mu A_1 - 87\mu A_2) + \varepsilon (16A_1 - 36\mu A_1) \right] \cdot \frac{1}{8 (1 - e^2)^{\frac{3}{2}}} - 4, \]

\[ R = P_{31}^{(0)} \cdot P_{42}^{(0)} - P_{32}^{(0)} \cdot P_{41}^{(0)} = \]

\[ \frac{1}{1728 (1 - e^2)} \left[ (29736A_1 - 52704A_2 - 5784) \mu^2 + (7776 - 5760A_1 - 27108A_2) \mu \\
+ (-8298A_1 - 2160A_2) + \varepsilon \left\{ (6048 - 35352A_1 + 29700A_2) \mu^2 \\
+ (4320A_1 - 33372A_2 - 9936) \mu + (3456 + 14400A_1 + 4320A_2) \right\} \right] \]

From (5.7), we obtain the characteristic roots given by:

\[ \lambda = \pm \left[ \frac{Q}{2} \pm \left( \frac{Q^2 - 4R}{2} \right)^{\frac{1}{2}} \right]^{\frac{1}{2}}. \]  \hspace{1cm} (5.8)

The characteristic equation will be purely imaginary if

\[ Q < 0 \]  \hspace{1cm} (5.9)

and

\[ Q^2 - 4R \geq 0. \]  \hspace{1cm} (5.10)
From inequalities (5.9), it follows that:

\[
[(16 - 16A_1 + 8\mu + 8\mu A_1 - 87\mu A_2) + \epsilon (16A_1 - 36\mu A_1)] \frac{1}{8(1 - e^2)^{\frac{1}{2}}} - 4 \leq 0,
\]

\[
\Rightarrow 0 < e \leq 0.866025 + 0.866025A_1 - 0.4330125\mu A_1 + 2.147019\mu A_2 - 0.2886646\epsilon A_1 + 1.2268\epsilon A_1. \quad (5.11)
\]

It is clear that, when \( A_1 = A_2 = 0 \), we shall get \( 0 \leq e \leq 0.8660254 \).

The schematic behaviour of the system can be studied, when a system is moving under the effects of the two oblate primaries in which one of the bigger oblate primary is also radiating by plotting the transition curve for different values of \( e \) and oblateness parameter of the primaries. Using Matlab 6.1 version, shows that the region of stability decreases with the increase of \( \mu \).

In case the eccentricity \( e \) does not satisfy the inequality (5.11), the characteristics roots will be either real or complex conjugate. In case of complex roots there must be roots with positive real parts leading to instability of the equilibrium points. It is interesting that the inequality (5.11) holds for every value of \( A_1 \) and \( A_2 \) and \( q \). From the inequality (5.10), we obtain:

\[
\frac{1}{1 - e^2} \left[ (14 - 67A_1 + 98A_2 + 73\epsilon A_1 - 69\epsilon - 14\epsilon)\mu^2 + \mu \left( 1 - e^2 \right)^{\frac{1}{2}} \middle\{ (-14 + 8A_1 + 19A_2 - 19\epsilon A_1 + 77\epsilon A_2 + 23\epsilon) - (8 + 8A_1 - 87A_2 - 36A_1\epsilon) \right. \right.
\]

\[
\left. + \left\{ 16 \left( 1 - e^2 \right) - (16 - 16A_1 + 16A_1\epsilon) \left( 1 - e^2 \right)^{\frac{1}{2}} 
\right. \right.
\]

\[
\left. + (21A_1 + 5A_2 + 4 - 8\epsilon - 25A_1\epsilon - 10A_2\epsilon) \right\} \right] \geq 0.
\]

Since \( \mu \leq \frac{1}{2} \), the inequality is satisfied when \( 0 \leq \mu < \mu^* \), where

\[
\mu^* = -\left\{ (-14 + 8A_1 + 19A_2 - 19\epsilon A_1 + 77\epsilon A_2 + 23\epsilon) \right.
\]

\[
- (8 + 8A_1 + 87A_2 - 36A_1\epsilon) \left( 1 - e^2 \right)^{\frac{1}{2}} \right\}.
\]
\[
\pm \left[ \left( -12 - 244A_1 - 2360A_2 - 304A_1\epsilon + 2884A_2\epsilon + 1284\epsilon \right) + \left( 1 - \epsilon^2 \right)^{\frac{1}{2}} \right] \\
\left\{ -768 + 4416A_1 - 766A_2 - 5248\epsilon A_1 + 4416\epsilon A_2 + 896\epsilon \right\} + \left( 1 - \epsilon^2 \right)^{\frac{1}{2}} \\
\frac{(1056 - 2600A_1 + 9012A_2 - 6208A_1\epsilon + 3184A_2\epsilon - 368\epsilon)}{2 \left( 14 - 67A_1 + 98A_2 + 73\epsilon A_1 - 69\epsilon A_2 - 14\epsilon \right)} \right]. 
\]

Thus, the triangular equilibrium solutions are stable if the eccentricity \( e' \) satisfies the condition of (5.11) and the mass ratio \( \mu^* \) obeys the inequality (5.12).

The dependence of \( \mu^* \) on the eccentricity are plotted in graph for different values of oblateness parameter \( A_1 \) and \( A_2 \) and \( \epsilon \). It is observed that if the values of \( A_1 \) and \( A_2 \) are increasing the range of stability is decreasing or it is obvious from Figures 2-11.

![Figure 2: Correlation between mass ratio and eccentricity](image1)

![Figure 3: Correlation between mass ratio and eccentricity](image2)

6. Discussion and Conclusion

The stability of the elliptical restricted three body problem in which bigger primary is oblate as well as radiating and smaller primary is oblate spheriods is studied. The problem is studied under the assumption that the eccentricity of the orbit of the gravitating bodies is small. The oblateness and radiate of the more massive primary does not affect the motion of the smaller primary due to its large mass whereas is affects motion of infinitesimal body. The differential equations governing the motion of the elliptical restricted three body problem...
Figure 4: Correlation between mass ratio and eccentricity

Figure 5: Correlation between mass ratio and eccentricity

Figure 6: Correlation between mass ratio and eccentricity

Figure 7: Correlation between mass ratio and eccentricity

Figure 8: Correlation between mass ratio and eccentricity

Figure 9: Correlation between mass ratio and eccentricity
under the bigger oblate primary which is radiating and smaller primary is oblate spheroid has been derived and the configuration of the triangular equilibrium points are presented. The stability of the triangular equilibrium points of the same problem is also studied in detail. We have exploited the method of averaging due to Grabenikov throughout the analysis of stability of the triangular equilibrium points of the problem. We have also exploited the simulation technique to study the linear stability to the triangular equilibrium points. We thus conclude that the location and stability of the equilibrium points of the elliptical restricted three body problem in which both these primaries are oblate spheroid, as well as the one of bigger is radiating and range of the stability decreases as the oblateness parameters as well as radiating parameter of the primaries increases.

References


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Table 1: Correlation between oblateness of primaries and the mass ratio


Table 2: Continuation of Table 1

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