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STRING COMPACTIFICATIONS AND THE REGGE TRAJECTORIES FOR RESONANCES OF THE STRONG INTERACTIONS

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Abstract: An examination of the physical phenomena of vacuum polarization provides support for the effect of the properties of the medium on the propagation of charged particles and light. The implications for the elementary particle physics and relativistic quantum mechanics are discussed. The relation with dark matter has implications for J vs. M^2 plot of astronomical objects and the w-parameter.

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1. Introduction

Strong interaction resonances have long been known to form asymptotically linear J vs. M^2 trajectories. The approximately linear relation has been extended to Planck and macroscopic scales. The coefficient $\alpha' = \frac{J}{M^2}$ in four dimensions and the radius of the curvature of the six-dimensional compactified space shall be related to the string compactification. While the values of α' at the Planck scale and astronomical scales do not differ more than three orders of magnitude, the difference between these coefficients and the value of α' at hadronic scales can be explained if the string tension T, defined as $\frac{c^3}{2\pi\alpha'}$, decreases with respect to the compactification radius as R^{-2}. This functional dependence of $T(R)$ defines an additional constraint on phenomenological superstring compactifications.

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2. Numerical Value of the Slope of Hadron Trajectories

In superstring theory,

$$\alpha'_{grav} = \frac{c^3}{2\pi T_{string}} = \frac{1}{2\pi \times 3 \times 10^{-3} g^2 \frac{c^4}{G}} \doteq 0.1979\ G/c \tag{2.1}$$

for Planck scale physics [1], [2], with g being defined to be $16\sqrt{\pi}$ [3], whereas the slope of the Regge trajectory of the excitations of a hadron is

$$\alpha' \approx 1.29 \times 10^{38}\ G/c \tag{2.2}$$

From equation (2.2), if ℓ_{hadron} is set equal to $b \times 10^{-15}\ cm$, and α' scales as R^2 where R is the microscopic length scale,

$$(0.1979\ G/c)\left(\frac{b}{1.6161} \times 10^{18}\right)^2 = 1.29 \times 10^{38}\ G/c \tag{2.3}$$

and $b \simeq 41.262$, yielding a length scale of $4.1262 \times 10^{-14} cm$, comparable to the distance between the quarks in the hadron. Units with $c = 1$ shall be used hereafter.

While the asymptotic form of the hadron Regge trajectories is generally linear, nonlinearity can be represented either with a square-root or quadratic term, based on the ratio of the initial and asymptotic slopes. The nucleon resonances can be described by the curve $\alpha^N(\sqrt{s}) = \alpha_{0N} + \alpha_{1N}\sqrt{s} + \alpha'_N s$, $\alpha_{0N} = -0.3913$, $\alpha_{1N} \simeq 0.1518\ GeV^{-1}$, $\alpha'_N \simeq (0.8469 \pm 0.2360)\ GeV^{-2}$, based on the estimated mass of the $J = \frac{3}{2}$ resonance [4]. For the Δ trajectory, with the mean $M_{\frac{15}{2}} = 2.92\ GeV$ of the two $J = \frac{15}{2}$ measurements $2.85\ GeV$ and $2.99\ GeV$ used [4], and the data point $(J, M^2) = \left(\frac{19}{2}, M^2_{\frac{19}{2}}\right)$ omitted, in the plot, the average slope of $0.8661\ GeV^{-2}$, and for the Λ resonances, $\alpha^\Lambda(\sqrt{s}) = \alpha_{0\Lambda} + \alpha_{1\Lambda}\sqrt{s} + \alpha' s$, $\alpha_{0\Lambda} = -0.7098$, $\alpha_{1\Lambda} \simeq 0.1217\ GeV^{-2}$, $\alpha'_\Lambda \simeq 0.8623\ GeV^{-4}$. The average of the three gradients is $0.8584 \pm 0.1241\ GeV^{-2}$.

The Σ trajectory has an average slope of $1.100 \pm 0.0462\ GeV{-2}$, although it is increasing such that a quadratic term of the form $0.01374\ GeV{-4}s^2$ would be necessary to fit the curve. For the Σ resonances, the value of α' is $0.06387 \pm 0.0038\ GeV^{-2}$, and the average of the slopes of all five baryon trajectories is $0.8664 \pm 0.0846\ GeV^{-2}$.

The meson Regge trajectories include the π resonances, with the average slope $0.8252 \pm 0.0210\ GeV^{-2}$, the ω resonances with gradient $0.9232 \pm 0.0048\ GeV{-2}$ and the f resonances, with $\alpha'_f = 0.7731 \pm 0.0151\ GeV^{-2}$.

There are two ρ trajectories with slopes $0.8159 \pm 0.0222\ GeV^{-2}$ and $1.110 \pm 0.1122\ GeV^{-2}$ and two a-meson plots with gradients 0.9441 0.1224 GeV^{-2} and $1.2614 \pm 0.1202\ GeV^{-2}$. The slopes are $0.9340 \pm 0.0916\ GeV^{-2}$, $0.8357 \pm 0.0130\ GeV^{-2}$ and $0.8385 \pm 0.0120\ GeV^{-2}$ if positive parity states are included in the K, K^* and ϕ plots respectively and $0.8878 \pm 0.062\ GeV^{-2}$, $0.8208 \pm 0.0156\ GeV^{-2}$ and $0.8340 \pm 0.0064\ GeV^{-2}$ if the positive parity states are omitted. The average slope of eight of the trajectories, excluding the second set of ρ and a-meson resonances and the positive-parity states in the a, K and ϕ resonacnes, is $0.864392837\ GeV^{-2}$. If additionally, the slope at maximum J in the ρ plot and the slopes at minimum J in the K, K^* and ϕ trajectories are used, the average for all of the meson trajectories is $0.8830 \pm 0.0490\ GeV^{-2}$. Again, without the second set of ρ and a-meson resonacnes, a lower gradient is obtained, $0.8530125 \pm 0.0336875\ GeV^{-2}$.

The widths of the N and Δ resonances have been predicted accurately by the dispersion relations [5], [6]

$$\begin{aligned} Re\ \alpha(\sqrt{s}) &= \alpha(0) + \frac{\sqrt{s}}{\pi}\ P.V. \int_0^\infty \frac{Im\ \alpha^+(s')}{\sqrt{s'}(\sqrt{s'}-\sqrt{s})} d\sqrt{s'} \\ &\quad - \frac{\sqrt{s}}{\pi}\ P.V. \int_0^\infty \frac{Im\ \alpha^-(s')}{\sqrt{s'}(\sqrt{s'}+\sqrt{s})} d\sqrt{s'}, \\ Im\ \alpha^\pm(s) &= s^\delta \sum_n c_n \left(\frac{s-s_n}{s}\right)^{Re\ \alpha^\pm(s_n)} \cdot \vartheta(s-s_n), \\ \alpha'_R(\sqrt{s}) &= \frac{d\ Re\ \alpha(\sqrt{s})}{d\sqrt{s}} = \sum_n c_n \mathcal{D}_n(\sqrt{s}) \end{aligned} \tag{2.4}$$

as

$$\Gamma = 2\frac{Im\ \alpha(M^2)}{|\alpha'_R(M)|} \tag{2.5}$$

for resonances of mass M, and close agreement with experimental values is required of any phenomenological theory of the Regge trajectories.

3. String Configurations and the Strong Interactions

One quadratic fit of the pomeron resonance plot is

$$\begin{aligned} \alpha_{pom}(t) &= 1.10 \pm 0.25t + \alpha''_{pom} t^2 \\ \alpha''_{pom} &= 0.079 \pm 0.012\ GeV^{-4} \end{aligned} \tag{3.1}$$

where t is the center-of-mass squared energy in the cross-channel [5] [7].

Recent data has suggested the relation

$$\alpha_{pom}(t) = \alpha_{pom}(0-) + \alpha'_{pom} t \tag{3.2}$$
$$\alpha'_{pom}(0) = 1 + \delta \qquad \Delta > 0$$

With the value of $\delta = 0.064$ and the plot in Figure 6 of reference [8], it can be deduced that the points $(0, 1.064)$ and $(5, 2.340361445 \pm 0.012048192)$ lie on the trajectory, implying a linear slope of

$$\alpha'_{pom} = 0.255272289 \pm 0.002409638554 \tag{3.3}$$

The pomeron has been identified with a bound state of gluons. If a bound state of N gluons is interpreted to be a configuration of rotating strings which connect them at cusps separated by an angle of $\frac{2\pi}{N}$ [9], the relation between the angular momentum and the square of the energy is asymptotically linear with the slope

$$\alpha'_{pom} = \frac{1}{4\left(1 - \frac{1}{N}\right)} \alpha'_{hadron} \tag{3.4}$$

The cusps would represent the maximum of the wavefunction of the gluons and these can be viewed as the nearly definite location of the quanta of the fields.

For non-strange baryons, $\alpha'_{hadron} \approx 0.8554\ GeV^{-2}$ and $\alpha'_{pom} = 0.25752\ GeV^{-2}$, $0.25547619\ GeV^{-2}$, $0.25503667\ GeV^{-2}$ and $0.24257142\ GeV^{-2}$, when $N = 6$, $\langle N \rangle = 6.25$, $N = 7$ and $N = 8$ respectively. with an average slope of $0.8530125\ GeV^{-2}$ for the meson trajectories, excluding the second set of ρ and a-meson resonances, α'_{pom} would be $0.25590375\ GeV^{-2}$ when $N = 6$, $0.25382767\ GeV^{-2}$ if $\langle N \rangle = 6.25$, $0.248795312\ GeV^{-2}$ for $N = 7$ and $0.243717857\ GeV^{-2}$ when $N = 8$. If both the second set of ρ and a-meson resonances and the postive-parity states in the a, K and ϕ trajectories are omitted, $\alpha'_{hadron} = 0.86392837\ GeV^{-2}$ and α'_{pom} equals $0.2579178511\ GeV^{-2}$, $0.257121538\ GeV^{-2}$, $0.251979107\ GeV^{-2}$ and $0.246836677\ GeV^{-2}$ when $N = 6$, $\langle N \rangle = 6.25$, $N = 7$ and $N = 8$ respectively. With the estimate of $0.8830\ GeV^{-2}$ for all of the meson resonances, the pomeron slopes for $N = 6$, $\langle N \rangle = 6.25$, $N = 7$ and $N = 8$ would be sequentially $0.2649\ GeV^{-2}$, $0.26279619\ GeV^{-2}$, $0.257541667\ GeV^{-2}$ and $0.252285714\ GeV^{-2}$.

These results tend to support the reduction of the number of intermediate vector bosons for strong interactions fromm 8 for an SU(3) gauge theory to an average effective number 6.25 for a phenomenological theory obtained by projection from S^7 to $S^3 \times S^3$ [10], although the presence of eight gluons in a pomeron bound state is not altogether excluded if α'_{hadron} can be set equal to a value greater than 0.885019278 GeV^{-2}. The effective value of 6.25 would follow from the probability of $\frac{1}{4}$ for the seventh intermediate vector boson, corresponding to a nonlinear vector field on $S^3 \times S^3$, transforming independently of the other six projected vector fields, to appear in the forward light cone of a scattering event.

The configuration occurs as a solution to the string equations of motion derived from the action

$$\int \mathcal{L}_{st} d\sigma d\tau = \frac{1}{2\pi\alpha'} \int [(X_{tau} \cdot X_{,\sigma})^2 - X_{,\tau}^2 X_{,\sigma}^2]^{\frac{1}{2}} d\sigma d\tau \tag{3.5}$$

The flat space coordinates of the rotating string $X^\mu = (\tau, r, \theta + \omega\tau, z, 0, ...)$ with derivatives $X^\mu_{,\tau} = (1, 0, \omega, 0, 0, ...)$ and $X^\mu_{,\sigma} = (0, 1, \phi, \zeta, 0, ...)$, where σ is identified with r. With a metric of signature $(+ - - - ...)$, it follows that

$$\begin{aligned} X_{,\tau} \cdot X_{,\sigma} &= -r^2 \omega \phi \\ X_{,\tau}\ cdot X_{,\tau} &= 1 - r^2 \omega^2 \\ X_{,\sigma} \cdot X_{,\sigma} &= -(1 + r^2\phi^2 + \zeta^2) \\ (X_{,\tau} \cdot X_{,\sigma})^2 - X_{,\tau}^2 X_{,\sigma}^2 &= r^4\omega^2\phi^2 + (1 - \omega^2 r^2)(1 + r^2\phi^2 + \zeta^2) \end{aligned} \tag{3.6}$$

Then

$$\begin{aligned} \mathcal{E}(\zeta) = g^{\tau\tau} \frac{\partial \mathcal{L}_{st}}{\partial X^\tau_{,\tau}} &= -\frac{\partial \mathcal{L}_{st}}{\partial X^\tau_{,\tau}} \\ &= \frac{1}{2\pi\alpha'} [(X_{,\tau} \cdot X_{,\sigma})^2 - X_{,\tau}^2 X_{,\sigma}^2]^{-\frac{1}{2}} \frac{\partial}{\partial X^\tau_{,\tau}} [X^\tau_{,\tau} X^\tau_{,\sigma} - X^r_{,\tau} X^r_{,\sigma} - r^2 X^\theta_{,\tau} X^\theta_{,\sigma} \\ &\quad - X^z_{,\tau} X^z_{,\sigma})^2 - (X^\tau_{,\tau} X^\tau_{,\tau} - X^r_{,\tau} X^r_{,\tau} - r^2 X^\theta_{,\tau} X^\theta_{,\tau} X^z_{,\tau} X^z_{,\tau}) \\ &\quad \cdot (X^\tau_{,\sigma} X^\tau_{,\sigma} - X^r_{,\sigma} X^r_{,\sigma} - r^2 X^\theta_{,\sigma} X^\theta_{,\sigma} - X^z_{,\sigma} X^z_{,\sigma})] \\ &= -\frac{1}{2\pi\alpha'} X^\tau_{,\tau} X^2_{,\sigma} [(X_{,\tau} \cdot X_{,\sigma})^2 - X_{,\tau}^2 X_{,\sigma}^2]^{-\frac{1}{2}} \\ &= \frac{1}{2\pi\alpha'} \frac{1 + \phi^2 r^2 + \zeta^2}{\omega^2 r^4 \phi^2 + (1 - \omega^2 r^2)(1 + \phi^2 r^2 + \zeta^2)} \end{aligned} \tag{3.7}$$

The rotating string also has positive energy when a negative sign is placed in front of the integral with signature $(- + + + ...)$, and for this metric, the energy density is equal also to $\vec{X}_{,\tau} \cdot \frac{\partial \mathcal{L}_{st}}{\partial \vec{X}_{,\tau}} - \mathcal{L}_{st}$ [9]. It is preferable to use the action which has a positive constant of proportionality with respect to the area since the surfaces of large area A should be damped by a factor of $e^{-\kappa A}$ in a convergent sum over the string worldsheets.

A two-parameter set of solutions to the equations of motion of the rotating string are

$$\zeta = \frac{A\lambda r}{[\lambda^2 r^2(1 - A^2 - \omega^2 r^2) - A^2(1 - \omega^2 r^2)]^{\frac{1}{2}}} \tag{3.8}$$
$$\phi = \frac{A(1 - \omega^2 r^2)}{r[\lambda^2 r^2(1 - A^2 - \omega^2 r^2) - A^2(1 - \omega^2 r^2)]^{\frac{1}{2}}}$$

Amongst these solutions is the configuration in the $\lambda = 0$ limit with $\zeta = 0$ and $B\lambda - A\omega =$
λA, such that $\phi = \frac{B(1-\omega^2 r^2)^{\frac{1}{2}}}{r(\omega^2 r^2 - B^2)^{\frac{1}{2}}}$. The energy density of this configuration would be

$$\mathcal{E}(\zeta = 0) = \frac{1}{2\pi\alpha'} \frac{\omega r(1 - B^2)}{(\omega^2 r^2 - B^2)^{\frac{1}{2}}(1 - \omega^2 r^2)^{\frac{1}{2}}} \tag{3.9}$$

Integration of the energy and angular momentum densities over a string configuration with N cusps defined by the equation

$$\theta - constant = \frac{1}{2} arc\ sin \left(\frac{(1 + b)^2 \omega^2 r^2 - 2B^2}{(1 - B^2)\omega^2 r^2} \right) + \frac{1}{2} arcsin \left(\frac{1 + B^2 - 2\omega^2 r^2}{1 - B^2} \right) \tag{3.10}$$

gives the relation $J\frac{\alpha'}{4\left(1-\frac{1}{N}\right)}$. The additional dependence on N arises from the location and length of each arc in the disk, $0 \leq \omega r \leq 1$, $-\pi \leq \theta \leq \pi$.

The derivative of the energy density $\mathcal{E}(\zeta)$ with respect to ζ is

$$\frac{d\mathcal{E}(\zeta)}{d\zeta} = \frac{1}{2\pi\alpha'} \frac{[2r^4\omega^2 + (1 - \omega^2 r^2)(1 + r^2\phi^2 + \zeta^2)]\zeta}{r^4\omega^2\phi^2 + (1 - \omega^2 r^2)(1 + r^2\phi^2 + \zeta^2)} \tag{3.11}$$

and $\frac{d\mathcal{E}(\zeta)}{d\zeta} > 0$ if $\zeta > 0$, which is valid when $A,\ \lambda > 0$. Although $\zeta < 0$ in the negative z-direction, this segment would be the mirror image of a string in the positive z-direction, and it is sufficient to reverse the orientation of the z-axis to describe its motion. Since ζ can be chosen to be be positive, the configuration is stable against perturbations in the positive z-direction.

The inclusion of quantum effects in the model of the bound state representing the pomeron would lead to corrections to the trajectory. This can be done by evaluating to leading order the partition function of the string. The boundary contributions to the partition function of a bound state of gluons are identical to those of a rotating string in the limit of the masses of the quarks and anti-quarkis at the ends tending to zero and would vanish. Since the energy of the the configuration is equal to the sum of the energies and angular momenta of each smooth segment, and each arc can be mapped to a straight line in the dis, the shift in the energy is determined by the change in the operator arising in the interior partition function of a straight rotating string [11]

$$Z_I(\omega) = \int e^{iT(L_{cl}(\omega)+L_{fluc}(\omega))}. \tag{3.12}$$

For the straight string, the evaluation of the determinants of differential operators derived from the Lagrangian gives

$$\begin{aligned} Z_I(\omega) &= Det^{-\frac{1}{2}}(-\nabla^2)Det^{-\frac{1}{2}}(-\nabla^2 + 2\omega^2 sec^2(\omega w)) \qquad (3.13)\\ &= Det^{-1}(-\nabla^2)Det^{-\frac{1}{2}}\left[\frac{-\nabla^2 + 2\omega^2 sec^2(\omega w))}{-\nabla^2}\right] \\ x &= \frac{1}{\omega} arcsin(\omega r) \end{aligned}$$

and yields a Lüscher term [11], such that the classical relation for light masses at both ends $J_{cl} = \alpha' E_{cl}^2$ received a quantum correction

$$J = \alpha' E^2 + \frac{7}{12} + \mathcal{O}\left(\frac{1}{2\pi\alpha' E^2}\right) \tag{3.14}$$

The new interior partition function would be given by the determinants of operators evaluated after the inverse of the transformation. The effective Lagrangian, energy and momentum then can be derived. Since it has been shown that $\alpha' \to \frac{\alpha'}{4\left(1-\frac{1}{N}\right)}$, it is sufficient to consider the intercept.

The transformation is given by $(r, \theta) \to (r', \theta')$, where the radial coordinate is reduced by the factor

$$K = \frac{1}{2}\cdot 2\int_{r=\frac{B}{\omega}}^{1} \frac{B(1-\omega^2 r^2)^{\frac{1}{2}}}{r(\omega^2 r^2 - B)^{\frac{1}{2}}} = \int_{r=\frac{B}{\omega}}^{1} \frac{B(1-\omega^2 r^2)^{\frac{1}{2}}}{r(\omega^2 r^2 - B)^{\frac{1}{2}}} dr \tag{3.15}$$

and the angular coordinate is shifted to

$$\begin{aligned} \theta' &= \theta + \frac{1}{2} arc\ sin\left[\frac{(1+B)^2\omega^2 r'^2 - 2B^2}{(1-B^2)\omega^2 r'^2}\right] + \frac{1}{2} arc\ sin\left[\frac{1+B^2 - 2\omega^2 r'^2}{1-B^2}\right] \qquad (3.16)\\ &= \theta + \frac{1}{2} arc\ sin\left[\frac{(1+B)^2\omega^2 K^2 r^2 - 2B^2}{(1-B^2)K^2\omega^2 r^2}\right] + \frac{1}{2} arc\ sin\left[\frac{1+B^2 - 2K^2\omega^2 r^2}{1-B^2}\right] \end{aligned}$$

Then

$$z' = ze^{\frac{i}{2}\left[arc\ sin\left[\frac{(1+B)^2\omega^2 z\bar{z}-2B^2}{(1-B^2)\omega^2 z\bar{z}}\right]+arc\ sin\left(\frac{1+B^2-2\omega^2 z\bar{z}}{1-B^2}\right)\right]} \tag{3.17}$$

and

$$\begin{aligned}
\frac{\partial z'}{\partial z} &= K\left[1+i\frac{B^2}{(1-B^2)K^2\omega^2 z\bar{z}}-i\frac{K^2\omega^2 z\bar{z}}{1-B^2}\right] \\
&\quad e^{\frac{i}{2}\left[arc\ sin\left[\frac{(1+B)^2\omega^2 z\bar{z}-2B^2}{(1-B^2)\omega^2 z\bar{z}}\right]+arc\ sin\left(\frac{1+B^2-2\omega^2 z\bar{z}}{1-B^2}\right)\right]} \\
\frac{\partial \bar{z}'}{\partial z} &= -i\left[\frac{B^2}{(1-B^2)K^2\omega^2 z^2}-\frac{K^2\omega^2\bar{z}^2}{1-B^2}\right] \\
&\quad e^{-\frac{i}{2}\left[arc\ sin\left[\frac{(1+B)^2\omega^2 z\bar{z}-2B^2}{(1-B^2)\omega^2 z\bar{z}}\right]+arc\ sin\left(\frac{1+B^2-2\omega^2 z\bar{z}}{1-B^2}\right)\right]} \\
\frac{\partial z'}{\partial \bar{z}} &= i\left[\frac{B^2}{(1-B^2)K^2\omega^2 z^2}-\frac{K^2\omega^2\bar{z}^2}{1-B^2}\right] \\
&\quad e^{\frac{i}{2}\left[arc\ sin\left[\frac{(1+B)^2\omega^2 z\bar{z}-2B^2}{(1-B^2)\omega^2 z\bar{z}}\right]+arc\ sin\left(\frac{1+B^2-2\omega^2 z\bar{z}}{1-B^2}\right)\right]} \\
\frac{\partial \bar{z}'}{\partial \bar{z}} &= K\left[1-i\frac{B^2}{(1-B^2)K^2\omega^2 z\bar{z}}+i\frac{K^2\omega^2 z\bar{z}}{1-B^2}\right] \\
&\quad e^{-\frac{i}{2}\left[arc\ sin\left[\frac{(1+B)^2\omega^2 z\bar{z}-2B^2}{(1-B^2)\omega^2 z\bar{z}}\right]+arc\ sin\left(\frac{1+B^2-2\omega^2 z\bar{z}}{1-B^2}\right)\right]}
\end{aligned} \tag{3.18}$$

The Jacobian of the transformation $(z,\bar{z}) \to (z',\bar{z}')$ is equal to K^2, and it follows that a factor of K^{-2} multiplies the operators in the partition function

$$Z_I(\omega) \to Det^{-1}(-K^{-2}\nabla^2)Det^{-\frac{1}{2}}\left[\frac{(-K^2\nabla^2+2K-2\omega^2 sec^2(\omega^2 x))}{-K^2\nabla^2}\right]. \tag{3.19}$$

The contribution of the boundary would be identical to that of the ends of the straight string, with the masses of the quark and anti-quark tending to zero, based on the time-component of the gluon Lagrangian, and it therefore vanishes. The relation between the energy and momentum is then

$$J = \frac{\alpha'}{4\left(1-\frac{1}{N}\right)}E^2 + \frac{K^{-1}}{12} + \frac{1}{2} + \mathcal{O}\left(\frac{2\left(1-\frac{1}{N}\right)}{2\pi\alpha' E^2}\right) \tag{3.20}$$

Another solution can be derived by letting $B \to 1 - B$. Then $K^{-1} \to \frac{1}{1-(1-B)^2}$ and pomeron trajectory would be

$$J = \frac{\alpha'}{4\left(1 - \frac{1}{N}\right)E^2} + \frac{N}{1 - \frac{4}{N^2}}\frac{1}{12} + \frac{1}{2} + \mathcal{O}\left(\frac{2\left(1 - \frac{1}{N}\right)}{2\pi\alpha' E^2}\right) \quad (3.21)$$

The intercepts are then 1.0625, 1.0825, $1.131\overline{85}$ and $1.2\overline{11}$ for $N = 6$, $\langle N \rangle = 6.25$, $N = 7$ and $N = 8$ respectively. The value for $\langle N \rangle = 6.25$ is nearly equal to the average of the pomeron slopes in references [5], [8], which is higher than the intercept of the linear trajectory because of a nonlinearity in the pomeron trajectory [12].

When $N = 6$, after the substitution $B \to 1 - B$, $1 - B \to 1 - \frac{2}{N} = \frac{2}{3}$ and the angle subtended by the arc would be $\frac{2\pi}{3}$. This value represent a bound state configuration consisting of two separate three-gluon states, which would be consistent with the exchange of colour-singlet states, since the vector indices of the gluon with respect to $SU(2) \times SU(2)$ can be saturated with the restriction to the subalgebra of $SU(3)$ of $d_{abc}A_aA_bA_c$. The exchange of colour-single rather than two-gluon states has been found to be necessary for the derivation of the correction $\epsilon \simeq 0.067$ [13] to the first-order estimate of the intercept of the pomeron trajectory. A theoretical basis for the leading-order estimate of the intercept has been provided in the Landshoff-Nachtmann model [14].

The results represent a significant improvement over lattice calculations [15], [16]. By Casimir scaling [17], $\frac{\sigma_D}{\sigma_F} = \frac{C_D}{C_F}$, where $\sigma = \frac{1}{2\pi\alpha'}$. It follows that, for $SU(3)$, $\frac{\sigma_D}{\sigma_F} = \frac{3}{\frac{4}{3}} = \frac{9}{4}$. With the value $\alpha'_F = 0.9\ GeV^{-2}$, $\alpha'_D = 0.4\ GeV^{-2}$.

The model of the pomeron as a bound state of N gluons, where N may be greater than 2 is consistent with the glueball spectrum up to 3 [18]. Indeed, the addition of the angular momentum vectors of an $N = 6$ bound state could yield spins ranging from 0 to 6, unless the configuration is composed of two separate three-gluon states, such that the spins would have values between 0 and 3 if a single 3-gluon state is exchanged in the scattering process.

Mesons are best described by a string connecting a quark with an anti-quark, whereas baryons at large distances can be interpreted to be configurations of three quarks connected by strings. While the quark-diquark, linear and three-string configurations consist open strings, the trangle model is closed. However, the linear slope of the trajectory of the triangle configuration is $\frac{3}{8}\alpha'_{trajectory}$ [19], and this model can be eliminated.

The bound states of the intermediate vector bosons of the strong nuclear force are closed string configurations which produce the correct slopes for the Regge trajectories of pomerons. While almost all of the closed string configurations in the path integral for superstring theory at Planck scales are smooth [20], the cusps in the string configuration representing pomerons are a consequence of the transition to a more classical domain, with the unification of the elementary particle interactions with gravity primarily based on closed superstring theory, which would follow from an intepretation of open strings as Wilson lines, arising as observables $exp\left(i\int A_\mu dx^\mu\right)$ and connecting either quarks with anti-quarks, three quarks or three anti-quarks in the hadrons.

4. The Functional Dependence of the String Tension on the Compactification Radius

The Regge trajectories can be obtained from the theory of Type IIB strings in $AdS^5 \times S^5$. For closed strings, the quantum-corrected relations [7] are

$$J \approx \frac{1}{2}\alpha' E^2 + \frac{\pi}{2}\left(\frac{D-2}{12} + 1\right) \tag{4.1}$$

in flat space and

$$\begin{aligned} J = &\frac{1}{2}\alpha'_{eff}E^2 - \alpha'_{eff}z_0 E + \frac{1}{2}\alpha'_{eff}z_0^2 \\ &z_0\pi\left(\frac{3}{2}m_0 - 4\ell\right) \end{aligned}$$

in curved space, where $\alpha'_{eff} = \alpha' g_{00}$, $\ell = \frac{3\epsilon^{\frac{4}{3}}}{2^{\frac{4}{3}}}a_0^2(g, M^2\alpha'^2)$, with $m_0^2 = \underline{3^{\frac{2}{3}}a_1 a_0}$ $\frac{\epsilon^{\frac{4}{3}}}{(g_s M\alpha')^2}$ for the Klebanov-Strassler background [21] and $m_0^2 = \frac{4}{9}\frac{1}{g_s N\alpha'}$ for the Maldacena-Nunez background [22], with ϵ being a deformation parameter, $a_0 \approx 0.71805$, $a_1 = \frac{2^{\frac{2}{3}}3^{\frac{2}{3}}}{18}$, m as a coefficient in the expression for a three-form field strength and the factor g_s as a coefficient in the definition of a two-form for the first space-time and the coefficient of the metric of a five-dimensional submanifold with topology in the second space-time. The factor of $\frac{1}{2}$ in equation (4.2) and the prediction for the pomeron slope has been derived for background geometries of closed superstring theory. The analysis might be extended to Regge trajectories of baryons and mesons using an open superstring model. The constraint on g_{00} to obtain confinement in the dual gauge theory, $f(0) \neq 0$, $\partial_r f(0) = 0$, where $f(r(x))^2 = g_{00}(r(x))g_{x_\parallel x_\parallel}$ [23], with $\{x_\parallel\}$ representing the three-dimensional planar coordinates of the five-dimensional non-compact space, leads to the required functional of α' if $g_{00} \sim \frac{1}{R^2}$. This dependence on R would follows from the equality of the curvature in the closed timelike direction of AdS_5 and the sphere S^5, with the remaining coordinates describing the non-compact four-dimensional space-time.

The dependence of α'_{eff} on the compactification radius follows from the duality of the Type IIB superstring theory in a curved background such as $AdS_5 \times S^5$ and $N = 4$ supersymmetric $SU(N)$ gauge theory in the limit $\frac{1}{2\pi\alpha'} \to \infty$ [24], with the 'tHooft coupling $\lambda = Ng^2_{YM} = \frac{R^4}{\alpha'^2}$ fixed [25]. This coupling arises in the bosonic part of the $AdS^5 \times S^5$ action

$$I = \frac{\sqrt{\lambda}}{4\pi} \int d^2\xi \sqrt{h} h^{ab} \left[g^{(AdS_5)}_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + g^{S^5}_{\alpha\beta} \partial_a y^\alpha \partial_b y^\beta \right] \tag{4.2}$$

if the radii of curvature of AdS_5 and S^5 are equal.

The factor of R^2 also arises in the scaling of the σ-model action

$$\begin{aligned} I_{\sigma-model} = \frac{1}{2\pi\alpha'} \int_\Sigma d^2 z \sqrt{h} \Big[\frac{1}{2} h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu - \frac{1}{2} i \bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu \qquad (4.3) \\ + \bar{\chi}_a \rho^\alpha \rho^a \psi^\mu \partial_\alpha X_\mu + \frac{1}{4} \bar{\psi}^\mu \psi_\mu \bar{\chi}_a \rho^a \rho^b \chi_b \Big] \end{aligned}$$

If $X^\mu \to R \to R_0 X^\mu$, $\psi \to \frac{R}{R_0}\psi^\mu$, $\chi^a \to \chi^a$, the action is multiplied by a factor of $\frac{R^2}{R_0^2}$. In addition, if $\alpha' \to \frac{R^2}{R_0^2}\alpha'$, S is invariant. This transformation commutes with diffeomorphisms, Weyl rescalings and supersymmetry. It follows that the ratio of the slopes of the plots of the resonances will hold to all orders of perturbation theory. The occurrence of the factor R^2 does not lead to constraints on the string compactification.

Let κ^2 be the coupling in the expression of the string partition function

$$\begin{aligned} Z &= \sum_g Z_g \\ &= \sum_g \kappa^{2g-2} \int \mathcal{D}(X, \chi, \psi) \int_{sM_g} D[h] D[X] D[\chi] D[\psi] e^{-S(h,X,\chi,\psi)} \end{aligned} \tag{4.4}$$

If the compactified space has d dimensions and radius of curvature R, the integral over the coordinates $X_{10-d}, ..., X_9$ produces the factor

$$\tau_1(R)(det' \ \tau_2(R)\triangle_0)_g^{-\frac{d}{2}} \tag{4.5}$$

where $\tau_1(R)$ and $\tau_2(R)$ are geometrical factors and $\triangle_0$ is the scalar Laplacian on a genus-g surface.

The relation between the gauge and string coupling in open superstring theory, with closed string bound states, is $g_{gauge}^2 \propto \kappa_P^2 \alpha'$, and g_{gauge} increases as R. For the heterotic string, the relation $g_{gauge}^2 \sim \frac{\kappa_P^2}{\alpha'}$, implies that g_{gauge} decreases as R^{-1}. The extent to which this gauge coupling decreases depends on the scale at which the couplings of the electromagnetic, weak and strong nuclear forces are unified. Below grand unified energies, when the three couplings are distinguished and change according to renormalization group equations, the functional dependence on R is no longer relevant. The effective string coupling, coinciding with the unified gauge coupling, equals $\frac{1}{24.3502855011684}$, whereas the strong coupling, at scales above 1 GeV, is typically larger, while the electromagnetic and weak coupling decrease with the energy. Since the scale of the strong interactions is approximately 10^{19} times that of Planck scales, this difference cannot be incorporated into factors of R and R^{-1}, the relation between the gauge coupling and string tension, required by supersymmetry, would not be valid at energies below the unification scale. It follows that supersymmetry must be broken at lower energies, which includes scales above 1 TeV.

5. The Proportionality of the Angular Momentum and Square Mass at Astronomical Scales

It has been observed that the orbital angular momentum and square mass of objects at astronomical scales are proportional [1], [2]. Since $\vec{J} = \vec{L} + \vec{S}$, the plot of J vs. M^2 is an almost linear trajectory $J = \alpha_{0\ astron.} + \alpha'_{astron.} M^2$, with $\alpha_{0\ astron.} == 10^{-15.1 \pm 0.9} gm^{-1} cm^2 s^{-1}$ [1], [2].

The absence of the nuclear forces at macroscopic scales implies that the slope of the J vs. M^2 plot would be weighted average of $\alpha'_{grav.}$ for electrically neutral matter and $\alpha'_{elec.}$ for electrically charged matter which is interacting according to a unified theory of quantum gravity and the elementary particle interactions. For the electromagnetic interactions, $\frac{g_{elec}^2}{4\pi} = \frac{1}{137}$ and $g_{elec} = 0.091725$, giving a slope of $\alpha'_{elec.} = 5.48 \times 10^2 G/c$ if the heterotic string formula is used with $\kappa^2 = 8\pi G$. The fraction of matter that must be gravitationally interacting to obtain an average slope of $4 \times 10^2 G/c$ is 27.0587759%. The remaining portion must be be electrically neutral, and yet electromagnetically interacting, and should consist of particle-antiparticle pairs, can be identified with Ω_Λ in a phenomenologically realistic cosmological model [26], which can be generated by a potential that produces a cosmological term, with modulus or scalar fields in the heterotic string theory [27], [28].

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