International Journal of Pure and Applied Mathematics
Volume 70 No. 1 2011, 65-75

ON REDUCED AND α-SKEW QUASI-ARMENDARIZ MODULE

Prachi Juyal[1 §], M.R. Khan[2], V.N. Dixit[3]
[1,2,3]Department of Mathematics
Jamia Millia Islamia (Central University)
New Delhi, 110 025, INDIA

Abstract: Let R be an associated ring with identity. A module M_R be an α-weakly rigid module which is generalization of α-compatible module, where α is an monomorphism on R. In this paper we proved the following results. Let M_R be a α-weakly rigid module then the following conditions hold:

(a) (i) If $M[x,\alpha]_{R[x,\alpha]}$ is p.q. Baer then M_R is p.q. Baer, converse is true if M_R is α-reduced.

(ii) If $M[[x,\alpha]]_{R[[x,\alpha]]}$ is p.q. Baer then M_R is p.q. Baer

(b) (i) If $M[x,x^{-1},\alpha]_{R[x,x^{-1},\alpha]}$ is p.q. Baer, Then M_R is p.q. Baer converse is true if M_R is α-reduced

(ii) If $M[[x,x^{-1},\alpha]]_{R[[x,x^{-1},\alpha]]}$ is p.q. Baer, then M_R is p.q. Baer

(c) If M_R is p.q. Baer, then M_R is α-skew quasi Armendariz module of power series type.

(d) If M_R is p.q. Baer, then M_R is α-skew quasi-Armendariz module of Laurent power series type.

AMS Subject Classification: 16D80, 16S36, 16W60
Key Words: α-weakly rigid ring, α-skew quasi-Armendariz module

Received: February 4, 2011

§Correspondence author

1. Introduction

Throughout this paper R denotes an associated ring with identity and M_R will stand for right R-module. Recall that R is called (quasi) Baer ring it the right annihilator of every (right ideal) nonempty subset of R is generated as a right ideal by an idempotent of R. In [14], Kaplansky introduced Baer rings to abstract various prospects of AW*-Algebra and von-Neumann algebras. Quasi-Baer rings, introduced by Clark [9], are used to characterize when finite dimensional algebra with unity over an algebraically closed field is isomorphic to a twisted matrix units semigroup algebra. The definition of quasi-Baer rings is left-right i.e. a ring R is left (quasi) Baer iff R is right (quasi) Baer.

As a generalization of quasi-Baer ring, G.F. Birkenmeier, J.Y. Kim, and J.K. Park in [5], introduced the concept of principally quasi-Baer rings. A ring R is called principally quasi-Baer (or right p.q. Baer) if the right annihilator of principal a right ideal of R generated by an idempotent. The class of p.q. Baer ring includes all Baer rings, quasi Baer rings abelian p.p. rings and biregular rings. A ring R is called reduced ring if it has a non-zero nilpotent elements in [17] Lee-Zhou introduced that a module M_R is called α-reduced module if for any $m \in M, a \in R$ (i) an $ma = 0$ implies $mR \cap Ma = 0$, (ii) $ma = 0 \Leftrightarrow m\alpha(a) = 0$, where α is endomorphism of R. In [1] Annin introduced a module M_R is called α-compatible if $ma = 0 \Leftrightarrow m\alpha(a) = 0$. Every α-reduced module are α-compatible. In [20], A.R. Nasr Isfahani introduced a ring R is called α-weakly rigid if $aRb = 0 \Leftrightarrow a\alpha(Rb) = 0$ where $a, b \in R$ and α is monomorphism of R.

Write $R[x], R[[x]]R[x, x^{-1}]$ and $R[[x, x^{-1}]]$ for the polynomial ring, the power series ring, the Laurent polynomial ring and Laurent power series ring over R, respectively. In [17], Lee-Zhou introduced above notations for module M_R.

$$M[x, \alpha]_{R[x,\alpha]} = \left\{ \sum_{i=0}^{p} m_i x^i | e \geq 0, m_i \in M \right\},$$

$$M[[x, \alpha]]_{R[[x,\alpha]]} = \left\{ \sum_{i=0}^{\infty} m_i x^i | m_i \in M \right\},$$

$$M[x, x^{-1}, \alpha]_{M[x,x^{-1},\alpha]} = \left\{ \sum_{i=-p}^{q} m_i x^i | p \geq 0, q \geq 0, m_i \in M \right\},$$

$$M[[x, x^{-1}, \alpha]]_{M[[x,x^{-1},\alpha]]} = \left\{ \sum_{i=-p}^{\infty} m_i x^j | p \geq 0, m_i \in M \right\}.$$

Each of these is an abelian group under an obvious addition operation. Moreover $M[x,\alpha]_{R[x,\alpha]}$ becomes a module over $R[x,\alpha]$ under following scalar product operation for $M(x)=\sum_{i=0}^{p} m_i x^i \in M[x,\alpha]$ and $f(x)=\sum_{j=0}^{q} f_j x^j \in R[x,\alpha]$

$$m(x)f(x)=\sum_{k=0}^{p+q}\Big(\sum_{i+j} m_i\alpha^i(f_j)\Big)x^k.$$

Similarly, $M[[x,\alpha]]_{R[[x,\alpha]]} M[x,x^{-1}\alpha]_{R[x,x^{-1},\alpha]}$ and $M[[x,x^{-1},\alpha]]_{R[[x,x^{-1}\alpha]]}$ are modules over $R[[x,\alpha]]$, $R[x,x^{-1},\alpha]$ and $R[[x,x^{-1}\alpha]]$ respectively with similar scalar product. The modules $M[x,\alpha]_{R[x,\alpha]}$, $M[[x,\alpha]]_{R[[x,\alpha]]}$, $M[x,x^{-1},\alpha]_{R[x,x^{-1}\alpha]}$ and $M[[x,x^{-1}\alpha]]_{R[[x,x^{-1},\alpha]]}$ are called skew polynomial extension, skew power series extension, skew Laurent polynomial extension and skew Laurent power series extension of M_R respectively.

A module M_R is called.

(1) α-skew quasi-Armendariz, if whenever $m(x)R[x,\alpha]f(x)=0$ for $m(x)=\sum_{i=0}^{p} m_i x^i \in M[x,\alpha]_{R[x,\alpha]}$ and $f(x)=\sum_{j=0}^{q} f_j x^j \in R[x,\alpha]$, then $m_i\alpha^i(Rf_j)=0$ for all i,j.

(2) α-skew quasi-Armendariz of power series type, if whenever $m(x)R[[x,\alpha]]f(x)=0$ for $m(x)=\sum_{i=0}^{\infty} m_i x^i \in M[[x,\alpha]]_{R[[x,\alpha]]}$ and $f(x)=\sum_{j=0}^{\infty} f_j x^j \in R[[x,\alpha]]$ then $m_i\alpha^i(Rf_j)=0$ for all i,j.

Let $\alpha\in \mathrm{Aut}(R)$.

(3) α-skew quasi-Armendariz of Laurent type, if whenever $m(x)R[x,x^{-1},\alpha]f(x)=0$ for $m(x)=\sum_{i=-p}^{q} m_i x^i \in M[x,x^{-1}\alpha]_{R[x,x^{-1},\alpha]}$ and $f(x)=\sum_{j=-s}^{t} f_j x^j \in R[x,x^{-1},\alpha]$. Then $m_i\alpha^i(Rf_j)=0$ for all i and j.

(4) α-skew quasi-Armendariz of Laurent power type if whenever $m(x)R[[x,^{-1}\alpha]]$ $f(x)=0$ for $m(x)=\sum_{i=-p}^{\infty} m_i x^i \in M[[x,x^{-1},\alpha]]_{R[[x,x^{-1},\alpha]]}$ and $f(x)=\sum_{j=-q}^{\infty} f_j x^j \in R[x,x^{-1},\alpha]$ then $m_i\alpha^i(Rf_j)=0\ \forall\ i,j$. In [17], Lee-Zhou introduced Baer module, quasi-Baer and p.p. modules as follows.

(i) M_R is called Baer if, for any subset X of M, $r_R(X)=eR$ where $e^2=e\in R$.

(ii) M_R is called quasi-Baer if for any submodule $X \subseteq M$, $r_R(X) = eR$ where $e^2 = e \in R$.

(iii) M_R is called p.p. module if for any element $m \in M$, $r_R(m) = eR$ where $e^2 = e \in R$.

A module M_R is called principally quasi-Baer (or p.q. Baer) module if $r_R(mR) = eR$ for any $m \in M$ and $e^2 = e \in R$. It is clear that a ring R is right p.q. Baer. iff R_R is p.q. Baer module. Every quasi-Baer module is p.q. baer module. But converse is not true.

In [3], M. Baser and A. Harmanci and in [12], E. Hashemi investigated the results for α-compatible module. Motivated by the results of M. Baser and A. Harmanci in [3] and E. Hashemi in [12] we generalize these results for α-weakly rigid module.

Definition 1.1. A ring R with a monomorphism α is called α-weakly ring. If for any $a, b \in R, aRb = 0 \Leftrightarrow a\alpha(Rb) = 0$

Definition 1.2. A module M_R with a monomorphism α is called α-weakly rigid module if for any $m \in M_R$ and $a \in R$

$$mRa = 0 \Leftrightarrow m\alpha(Ra) = 0$$

Example 1.3. Every α-reduced and α-compatible module are α-weakly rigid module.

Lemma 1.4. *Let R is α-weakly rigid ring then for each $a, b \in R$ and positive integers i and j,*

$$aRb = 0 \quad \Leftrightarrow \quad \alpha^i(a)R\alpha^j(b) = 0$$

Proof. See [20, Lemma 3.1].

Lemma 1.5. *Let M_R is α-weakly rigid module. Then for each $m \in M$ and $a \in R$ and positive integer i,*

$$mRa = 0 \Leftrightarrow m\alpha^i(Ra) = 0 \Leftrightarrow mR\alpha^i(a) = 0$$

Proof. Suppose $mRa = 0 \Rightarrow m\alpha(Ra) = 0$. Since M_R is α-weakly rigid

$$\Rightarrow mR\alpha(a) = 0$$

Now $mR\alpha^i(a) = mR\alpha(\alpha^{i-1}(a)) = 0$

Again suppose

$$mR\alpha^i(a) = 0 \Rightarrow m\alpha^i(Ra) = 0$$

$$\Rightarrow m\alpha(\alpha^{i-1}(Ra)) = 0 \Rightarrow \alpha(R\alpha^{i-1}(a)) = 0$$
$$\Rightarrow mR\alpha^{i-1}(a) = 0$$

Since M_R is α-weakly rigid

$$\Rightarrow m\alpha^{i-1}(Ra) = 0$$
$$\Rightarrow m\alpha(\alpha^{i-2}(Ra)) = 0$$
$$\Rightarrow m\alpha(R\alpha^{i-2}(Ra)) = 0$$
$$\Rightarrow mR\alpha^{i-2}(Ra) = 0$$

Since M_R is α-weakly rigid.

Continuing, we get

$$\Rightarrow mRa = 0$$

Theorem 1.6 ([17, Theorem 1.6]). *The following are equivalent for a module M_R*

(i) *M_R is α-reduced*

(ii) *$M[x,\alpha]_{R[x,\alpha]}$ is reduced*

(iii) *$M[[x,\alpha]]_{R[[x,\alpha]]}$ is reduced*
If α be an automorphism then (i), (ii), (iii) *are equivalent to* (iv) *and* (v).

(iv) *$M[x,x^{-1},\alpha]_{R[x,x^{-1},\alpha]}$ is reduced*

(v) *$M[[x,x^{-1},\alpha]]_{R[[x,x^{-1},\alpha]]}$ is reduced.*

Theorem 1.7. *Let M_R be a module such that for any $m \in R$ and $a \in R, ma = 0$ implies $mRa = 0$. Then M_R is p.q. Baer module if and only if M_R is p.p. module.*

Proof. See [3, Theorem 4]

Theorem 1.8. *Let M_R is α-weakly rigid module and $S = R[[x,\alpha]]$. Then the following results are true.*

(1) (i) *If $M[x,\alpha]_{R[x,\alpha]}$ is p.q. Baer then M_R is p.q. Baer. Converse is true if M_R is α-reduced.*

(ii) *If $M[[x,\alpha]]_{R[[x,\alpha]]}$ is p.q. Baer then M_R is p.q. Baer*

(2) *Let α be an automorphism of R*

(i) *If $M[x, x^{-1}, \alpha]_{R[x,x^{-1},\alpha]}$ is p.q. Baer then M_R is p.q. Baer converse is true if M_R is α-reduced.*

(ii) *If $M[[x, x^{-1}, \alpha]]_{R[[x,x^{-1}.\alpha]]}$ is p.q. Baer then M_R is p.q. Baer.*

(3) *If M_R is p.q. Baer then M_R is α-skew quasi-Armendariz of power series type*

(4) *If M_R is p.q. Baer then M_R is α-skew quasi-Armendariz of Laurent power series type*

Proof. (1) Let $M[x, \alpha]_{R[x,\alpha]}$ is p.q. Baer Module. Now only to show that M_R is p.q. Baer that is $r_R(mR) = e_0R$ for any $m \in M_R$ and $e_0^2 = e_0 \in M_R$. Since $M[x, \alpha]_{R[x,\alpha]}$ is p.q. Baer so there exists an idempotent $(e(x))^2 = e(x) = e_0 + e_1x + \ldots + e_px^p \in R[x, \alpha]$ such that $r_{R[x,\alpha]}(m(x)R[x, \alpha]) = e(x)R[x, \alpha]$ or $r_{R[x,\alpha]}(mR[x, \alpha]) = e(x)R[x, \alpha]$ so $mR[x, \alpha]e(x) = 0 \Rightarrow mRe(x) = 0$. Since M_R is α weakly rigid and by lemma 2.5

$$\begin{aligned}
&\Rightarrow mr(e_0 + e_1x + \ldots e_px^p) = 0\\
&\Rightarrow mre_0 + mre_1x + \ldots + mre_px^p\\
&\Rightarrow mre_0 = 0, mre_1 = 0 \ldots\ mre_p = 0\\
&\Rightarrow mRe_0 = 0\\
&\Rightarrow e_0R \subseteq r_R(mR).
\end{aligned}$$

Again suppose that any element $a \in r_R(mR) \Rightarrow mRa = 0$. Since M_R is α-weakly rigid so $mRa = 0 \Leftrightarrow m\alpha(Ra) = 0$

$$\Leftrightarrow \qquad mR\alpha \quad \forall\ K(a) = 0$$

[From lemma 2.5] $mR[[x, \alpha]]a = mf(x)$ a for any $f(x) = f_0 + f_1x + \ldots + f_px^p \in R[x, \alpha]$

$$\begin{aligned}
\Rightarrow mf(x)a &= m(f_0 + f_1x + f_2x^2 + \ldots + f_px^p)a\\
&= mf_0a + mf_1xa + mf_2x^2a + \ldots + mf_px^pa\\
&= mf_0a + mf_1\alpha(a)x + mf_2\alpha^2(a) + \ldots + +mf_p\alpha^p(a)x^p\\
&= 0 + 0 + 0 + 0
\end{aligned}$$

From above calculation (Since M_R is α-weakly rigid)

$$\Rightarrow mf(x)a = 0 \Rightarrow a \in r_{R[x,\alpha]}(mR[x, \alpha]) = e(x)R[x, \alpha]$$

$$\begin{aligned} &\Rightarrow a = e(x)a\,, \quad \text{where } \; e(x) \in R[x,\alpha] \\ &\Rightarrow a = e_0 a \\ &\Rightarrow a = e_0 a \in e_0 R \\ &\Rightarrow r_R(mR) \subseteq e_0 R \end{aligned}$$

So $r_R(mR) = e_0R$ and therefore M_R is p.q. Baer module.

Conversely, suppose M_R is α-reduced and M_R is p.q. Baer module proof similar to [3, Theorem 7((1) (a))].

(ii) Similar to proof [1(i)]

(2)

(i) Similar to proof [1(i)]. Here we take α-be an automorphism of R.

(ii) Similar to proof [1(i)].

(3) Suppose M_R is p.q. Baer and α-weakly rigid module.

Now to show that M_R is α-skew-quasi Armendariz power series type. For any $m(x) \in \sum\limits_{q=0}^{\infty} m_q x^q \in M[[x,\alpha]]R[[x,\alpha]]$ and $f(x) = (f_0 + f_1x + f_2x^2 + \ldots + \ldots) \in \sum\limits_{p=0}^{\infty} f_p x^p \in R[[x,\alpha]]$ such that $m(x)R[[x,\alpha]]f(x) = 0$. Then $m_q\alpha^q(Rf_p) = 0 \; \forall \; p, q$.

Take $m(x)R[[x,\alpha]]f(x) = 0$

$$\begin{aligned} \Rightarrow \quad & m(x)Rf(x) = 0 \\ \Rightarrow \quad & \left(\sum_{i=0}^{\infty} m_i x^i\right) r \left(\sum_{j=0}^{\infty} f_j x^j\right) = 0 \\ \Rightarrow \quad & \sum_{k=0}^{\infty} \left(\sum_{i+j=k} m_i \alpha^i(rf_j) x^k\right) = 0 \end{aligned}$$

It follows that

$$\sum_{i+j=k} m_i \alpha^i(rf_j) = 0 \tag{1}$$

If $k = 0$,

$$m_0 r f_0 = 0 \text{ implies } \; f_0 \in r_R(m_0R) \tag{2}$$

Since M_R is right p.q. Baer so $f_0 \in r_R(m_0R) = e_0R$. Then $f_0 = e_0f_0$ and $m_0Re_0 = 0$.

If $k = 1$,

$$m_0rf_1 + m_1\alpha(rf_0) = 0. \tag{3}$$

Let $r' \in R$ and $r = r'e_0$. Then

$$m_0r'e_0f_1 + m_1\alpha(r'e_0f_0) = 0$$

from (i) $m_0r'e_0 = 0$. So

$$m_1\alpha(r'e_0f_0) = 0 \text{ implies } \ m_1\alpha(r'f_0) = 0$$

Since R is α-weakly rigid module so

$$m_1r'f_0 = 0 \text{ implies } \ f_0 \in r_R(m_1R) = e_1R$$

Thus $f_0 = e_1f_0$ and $m_1Re_1 = 0$. Now from

(iii) $m_0rf_1 = 0$ implies $f_1 \in r_R(m_0R) = e_0R$.

Thus $f_1 = e_0f_1$ and $m_0Re_0 = 0$.

Now assume that $f_i \in e_iR = r_R(m_iR)$ is true for $i + j = 01, 2, \ldots K^{-1}$ that means $m_iRf_j = 0$ implies $m_i\alpha(Rf_j) = 0$ (Since R is α-weakly rigid) is true for $i + j = 0, 1, 2, \ldots, k - 1$. Now using induction method if take $i + j = k$.

$$\sum_{i+j=k} m_i\alpha^i(rf_j) = 0$$

i.e. $$m_0rf_k + m_1\alpha(rf_{k-1}) + m_2\alpha^2(rf_{k-2}) + m_{k-1}\alpha^{k-1}(rf_1) + m_k\alpha^k(rf_0) = 0$$

Let $r' \in R$ and $r = r'e_0$, put in (iv)

$$m_0r'e_0f_k + m_1\alpha(r'e_0f_{k-1}) + m_{k-1}\alpha^{k-1}(r'e_0f_1) + m_k\alpha^k(r'e_0f_0) = 0$$

Since $m_0r'e_0 = 0$ implies $m_0r'e_0f_k = 0$

$$m_1\alpha(r'f_{k-1}) + m_2\alpha^2(r'f_{k-2}) + \ldots + m_{k-1}\alpha^{k-1}(r'f_1) + m_k\alpha^k(r'f_0) =$$
$$m_1\alpha(r'e_0f_{k-1}) + m_2\alpha^2(r'e_0f_{k-2}) + \ldots + m_k\alpha^{k-1}(r'e_0f_0) + m_k\alpha^k(r'e_0f_0) = 0$$

Again let $s \in R$ and $r' = se_1$ and put in (v)

$$m_1\alpha(se_1f_{k-1}) + \ldots + m_{k-1}\alpha^{k-1}(se_1f_1) + m_k\alpha^k(se_1f_0) = 0$$

Since $m_1 s e_1 = 0$ implies $m_1 s e_1 f_{k-1} = 0$.

Thus $m_1 \alpha(s e_1 f_{k-1}) = 0$ since R is α-weakly rigid ring.

So

$$\begin{aligned} & m_2\alpha^2(se_1f_{k-2}) + \ldots + m_{k-1}\alpha^{k-1}(se_1f_1) + m_k\alpha^k(se_1f_0) \\ = \; & m_2\alpha^2(sf_{k-2}) + \ldots + m_{k-1}\alpha^{k-1}(sf_1) + m_k\alpha^k(sf_0) = 0 \end{aligned}$$

Continuing in this manner, we get

$$m_k\alpha^k(sf_0) = m_k\alpha^k(se_{k-1}f_0) = 0 \quad \text{for any} \quad s \in R.$$

Since R is α-weakly rigid ring. So this implies that

$$m_{k-1}\alpha^{k-1}(sf_1) = 0, m_{k-1}\alpha^{k-2}(sf_2) = 0, m_0 s f_k = 0.$$

Therefore, by induction we get $m_i\alpha^i(Rf_j) = 0$ for all i and j. Hence M_R α-quasi-Armendariz module.

(4) Similar to proof (3).

Corollary 1.9. (see [3, Theorem 7]) *Let M_R is α-compatible module. Then the following are true:*

- (i) (a) *If $M[x,\alpha]_{R[x,\alpha]}$ is p.q. Baer module then M_R is p.q. Baer. Then converse is true if M_R is α-reduced.*
  - (b) *If $M[[x,\alpha]]_{R[[x,\alpha]]}$ is p.q. Baer, then M_R is p.q. Baer.*
- (ii) (a) *If $M[x,x^{-1}\alpha]_{R[x,x^{-1},\alpha]}$ is a p.q. Baer module, then M_R is p.q. Baer module. The converse is true of M_R is α-reduced.*
  - (b) *If $M[[x,\alpha]]_{R[[x,\alpha]]}$ is a p.q. Baer then M_R is p.q. Baer.*

Corollary 1.10 ([12, Theorem 2.3]). *Let M_R be an α-compatible module and $T = R[[x,\alpha]]$.*

- (i) *If $M[[x]]_{R[[x,\alpha]]}$ is p.q. Baer. Then M_R is p.q. Baer*
- (ii) *If M_R is p.q. Baer then M_R is power series wise α-quasi Armendariz.*

References

[1] S. Annin, Associated Primes over skew-polynomial rings, *Comm. Algebra*, **30** (2002), 2511-2528.

[2] E.P. Armendariz, A note on extensions of Baer and p.q Baer ring, *J. Aust. Math. Soc.*, **18** (1974), 470-410.

[3] M. Baser and A. Harmanci, Reduced and p.q. Baer module, *Taiwanese J. Math.*, **11**, No. 1 (2007), 267-275.

[4] M. Baser and M.T. Kosan, On Quasi-Armendariz modules, *Taiwanese J. Math.*, **12** (2008) 573-582.

[5] G.F. Birkenmeier, J.Y. Kim and J.K. Park, Principally quasi-Baer rings, *Comm. Algebra*, **29** (2001), 639-660.

[6] G.F. Birkenmeier, J.Y. Kim and J.K. Park, On polynomial extension of principally quasi-Baer rings, *Kyungpook. Math. J.*, **40** (2000), 247-253.

[7] G.F. Birkenmeier, J.Y. Kim and J.K. Park, Polynomial extension of Baer and quasi-Baer rings, *J. Pure Appl. Algebra*, **159** (2001), 25-42.

[8] G.F. Birkenmeier and F.K. Huang, Annihilator condition on polynomials, *Comm. Algebra*, **29** (2001), 567-580.

[9] W.E. Clark, Twisted matrix units semigroup algebras, *Duke Math. J.*, **34** (1967), 417-424.

[10] J.A. Fraser and W.K. Nicholson, Reduced p.p. ring, *Math. Japonica*, **34** (1989), 715–725.

[11] K.R. Goodearl, *Von Neumann Regular Ring.*

[12] E. Hashemi, A note on p.q. Baer module, *New York J. Math.*, **14** (2008), 403-410.

[13] C.Y. Hong N.K. kim and T.K. Kwak, Ore extensions of Baer and p.p. rings, *J. Pure Appl. Algebra*, **151** (2000), 215-226.

[14] I. Kaplansky, *Rings of Operators*, W.A. Benjamin, New York (1968).

[15] N.K. Kim and Y. Lee, Armendariz rings and Reduced rings, *J. Algebra*, **223** (2000), 477-488.

[16] T.Y. Lam, *Lactures on Modules and Rings*, Springer Verlag, New York (1999).

[17] T.K. Lee and Y. Zhou, *Reduced Modules, Rings Modules, Algebras and Abelian Groups*, 365-377.

[18] T.K. Lee and Y. Zhou, Armendariz and reduced rings, *Comm. Algebra*, **6** (2004), 2287-2299.

[19] Z. Liu, A note on principally quasi-Baer rings, *Comm. Algebra*, **30**, No. 8 (2002), 3885-3890.

[20] A.R. Nasr, Isfahani and A. Moussavi, On weakly rigid ring, Glasgow, *Math. J.*, **51** (2009), 425-440.