International Journal of Pure and Applied Mathematics
Volume 70 No. 5 2011, 711-722

CHAOTIC GENERAL TREE AND IT'S CHAINS

M. El-Ghoul[1 §], Sh.A. Mousa[2]

[1]Mathematics Department
Faculty of Science
Tanta University
Tanta, EGYPT

[2]Mathematics Department
Faculty of Girls
AinShams University
EGYPT

Abstract: In this paper, we will define the Chaotic general tree.The adjacent, incidence, area, volume, edge area, edge volume, area volume matrices will be introduced. The chains of the chaotic general tree will be identified.

AMS Subject Classification: 51H10, 57N10
Key Words: chaotic graph, general tree

1. Introduction

In mathematics and physics, chaos theory deals with the behavior of certain nonlinear dynamical systems that under certain conditions exhibit a phenomenon known as chaos, which is characterized by a sensitivity to initial conditions. As a result of this sensitivity, the behavior of the system that exhibit chaos appears to be random, even though the model of the system is dereministic in the sense that it is well defined and contains no random parameters.

2. Definitions and Background

Chaotic graph. (see [3]) A "Chaotic graph" is a geometric graph that carries many physical characters.

Received: February 10, 2011

[§]Correspondence author

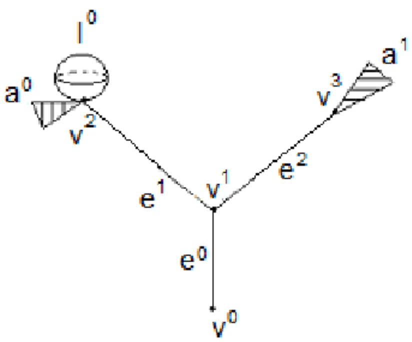

Figure 1(a)

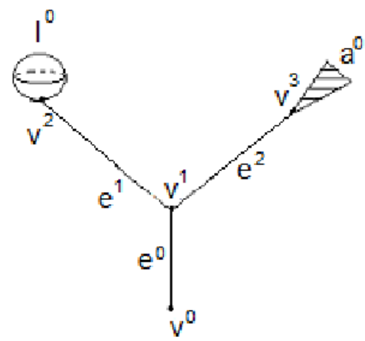

Figure 1(b)

General Tree. (see [4]) It is a tree which consistes of vertices V, edges E, areas A, and volume L, where V, E, A, and L are different in dimensions, such that the connection is direct, i.e. the one dimension joined the three dimension directly, Figure 1(a), or gradually, i.e. the one dimension joined with two dimension and the two dimension joined with three dimension Figure 1(b).

In the original tree, the vertices join between edges of the same dimension, but for the general tree the vertices joined between different dimesions.

Example. Consider the following general tree in Figure 2, where v^0, v^1, v^2, v^3 are of 0-dimension, and e^0, e^1, e^2 are of 1-dimension, and a^0 is of 2-dimension, and l^0 is of 3-dimension.

We will describe the above general tree by the matrices adjacent, incidence, area, volume, edge area, edge volume, and area volume, such that

$$A(G) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Here the "adjacent matrix" of G denoted by $A(G)$ is the $n * n$ matrix in which the entry in row I and column J is the number of edges joining the vertices I and J.

$$E(G) = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

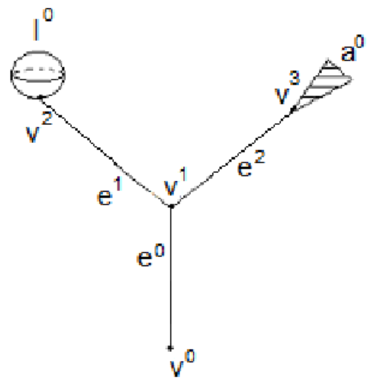

Figure 2

Here the "edge matrix" of G denoted by $E(G_h)$ is the $n \times n$ matrix in which $m_{ij} = 1$ if e_i is incident with e_j and $m_{ij} = 0$ otherwise, $R(G) = [0]$, where the "area matrix" of G denoted by $R(G_h)$ is the $n \times n$ matrix in which $m_{ij} = 1$ if a_i is incident with a_j and $m_{ij} = 0$ otherwise, $V(G) = [0]$. The "volume matrix" of G denoted by $V(G_h)$ is the $n \times n$ matrix in which $m_{ij} = 1$ if l_i is incident with l_j and $m_{ij} = 0$ otherwise

$$I(G) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Here the "incidence matrix" of G denoted by $I(G)$ is the $n \times m$ binary matrix, $m_{ij} = 1$ if v_i is incident with e_j and $m_{ij} = 0$ otherwise.

$$M(G) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

Here the "area matrix" of G denoted by $M(G)$ is the $n \times m$ binary matrix, where $m_{ij} = 1$ if v_i is incident with $a_i$and $m_{ij} = 0$ otherwise, and

$$N(G) = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

The "volume matrix" of G denoted by $N(G)$ is the $n \times m$ binary matrix, where $m_{ij} = 1$ if v_i is incident with L_i and $m_{ij} = 0$ otherwise.

$$H(G) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Here the "edge area matrix" of G denoted by $H(G)$ is the $n \times m$ binary matrix, where $m_{ij} = 1$ if e_i is incident with a_i and $m_{ij} = 0$, otherwise.

$$J(G) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

Here the "edge volume matrix" of G denoted by $J(G)$ is the $n \times m$ binary matrix, where $m_{ij} = 1$ if e_i is incident with l_i and $m_{ij} = 0$, otherwise.

$$U(G) = [0].$$

Here the "area volume matrix" of G denoted by $U(G)$ is the $n \times m$ binary matrix, where $m_{ij} = 1$ if a_i is incident with l_i and $m_{ij} = 0$, otherwise.

Let G be an oriented graph with edges $e^1, e^2 ..., e^r$. A *1-chain* on G is a formal sum

$$\lambda_1 e^1 + \lambda_2 e^2 + ... + \lambda_r e^r,$$

where each λ_i is an integer. When writing down 1-chains we omit any edges which have coefficient 0. The 1-chain in which all coefficients are zero is denoted 0.

We define the *sum* of the 1-chains $\sum \lambda_i e^i$ and $\sum \lambda_i' e^i$ to be

$$\sum \lambda_i e^i + \sum \lambda^{i'} e^i = \sum \left(\lambda_i + \lambda_i'\right) e^i,$$

which also is a 1-chain.

A *0-chain* on G is a formal sum $\lambda_1 \nu^1 + \lambda_2 \nu^2 + ... + \lambda_m \nu^m$ where $\nu^1, \nu^2, ..., \nu^m$ are the vertices of G.

3. Main Results

The chaotic circle. It is a circle that carries many physical characters.

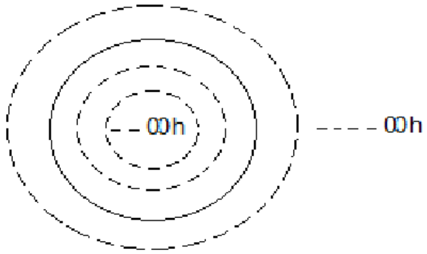

Figure 3

The chaotic sphere. It is a sphere that carries many physical characters.

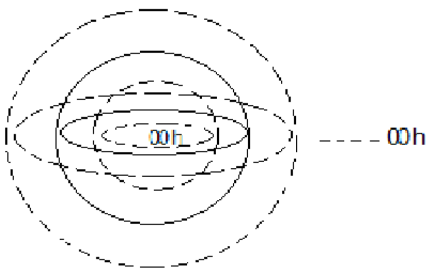

Figure 4

The chaotic cylinder. It is a cylinder that carries many physical characters.

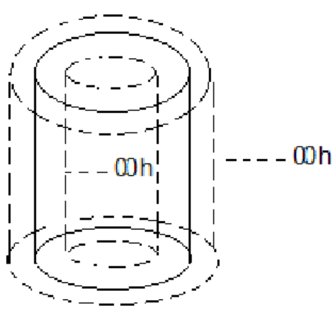

Figure 5

Let G be an oriented graph with areas $a^1, a^2, ..., a^n$. A *2-chain* on G is a formal sum

$$\lambda_1 a^1 + \lambda_2 a^2 + ... + \lambda_n a^n,$$

where each λ_i is an integer.

Let G be an oriented graph with volumes $l^{\,1}, l^{\,2}, ..., l^{\,j}$. A *3-chain* on G is a formal sum $\lambda_1 l^{\,1} + \lambda_2 l^{\,2} + ... + \lambda_j l^{\,j}$ where each λ_i is an integer.

Let G be an oriented graph. An*n-chain* on G is a formal sum

$$\lambda_1 x^1 + \lambda_2 x^2 + ... + \lambda_s x^s,$$

where each λ_s is an integer and x is of n-dimension.

We define the *sum of the n-chains* $\sum \lambda_i x^i$ and $\sum \lambda_j^{'} x^i$to be

$$\sum \lambda_i x^i + \sum \lambda_j^{'} x^i = \sum \left(\lambda_i + \lambda_j^{'}\right) x^i,$$

which is also an n-chain.

4. Chaotic Chains

Let G be a graph with vertices $V = \{v^1, v^2, ...v^i\}$, edges $E = \{e^1, e^2, ...e^j\}$, areas $A = \{a^1, a^2, ...a^m\}$, and volumes $L = \{l^1, l^2, ...l^n\}$ carries infinite number of chaotics.

The *0-chaotic chain* on G is system of a formal sum

$$\left\{\begin{array}{c} \lambda_1^{1h}\nu_{1h}^1 + \lambda_2^{1h}\nu_{1h}^2 + ... + \lambda_i^{1h}\nu_{1h}^i \\ \lambda_1^{2h}\nu_{2h}^1 + \lambda_2^{2h}\nu_{2h}^2 + ... + \lambda_i^{2h}\nu_{2h}^i \\ . \\ . \\ \lambda_1^{\infty h}\nu_{\infty h}^1 + \lambda_2^{\infty h}\nu_{\infty h}^2 + ... + \lambda_i^{\infty h}\nu_{\infty h}^i \end{array}\right\},$$

where each λ_i^{rh} is an integer.

A *1-chaotic chain* on G is a system of a formal sum

$$\left\{\begin{array}{c} \lambda_1^{1h}e_{1h}^1 + \lambda_2^{1h}e_{1h}^2 + ... + \lambda_j^{1h}e_{1h}^j \\ \lambda_1^{2h}e_{2h}^1 + \lambda_2^{2h}e_{2h}^2 + ... + \lambda_j^{2h}e_{2h}^j \\ . \\ . \\ \lambda_1^{\infty h}e_{\infty h}^1 + \lambda_2^{\infty h}e_{\infty h}^2 + ... + \lambda_j^{\infty h}e_{\infty h}^i \end{array}\right\},$$

where each λ_j^{rh} is an integer.

A *2-chaotic chain* on G is a system of a formal sum

$$\left\{\begin{array}{c} \lambda_1^{\overline{1h}} a^1_{\overline{1h}} + \lambda_2^{\overline{1h}} a^2_{\overline{1h}} + ... + \lambda_m^{\overline{1h}} a^m_{\overline{1h}} \\ \lambda_1^{\overline{2h}} a^1_{\overline{2h}} + \lambda_2^{\overline{2h}} a^2_{\overline{2h}} + ... + \lambda_m^{\overline{2h}} a^m_{\overline{2h}} \\ . \\ . \\ \lambda_1^{\overline{\infty h}} a^1_{\overline{\infty h}} + \lambda_2^{\overline{\infty h}} a^2_{\overline{\infty h}} + ... + \lambda_m^{\overline{\infty h}} a^m_{\overline{\infty h}} \end{array}\right\}, \left\{\begin{array}{c} \lambda_1^{\underline{1h}} a^1_{\underline{1h}} + \lambda_2^{\underline{1h}} a^2_{\underline{1h}} + ... + \lambda_m^{\underline{1h}} a^m_{\underline{1h}} \\ \lambda_1^{\underline{2h}} a^1_{\underline{2h}} + \lambda_2^{\underline{2h}} a^2_{\underline{2h}} + ... + \lambda_m^{\underline{2h}} a^m_{\underline{2h}} \\ . \\ . \\ \lambda_1^{\underline{\infty h}} a^1_{\underline{\infty h}} + \lambda_2^{\underline{\infty h}} a^2_{\underline{\infty h}} + ... + \lambda_m^{\underline{\infty h}} a^m_{\underline{\infty h}} \end{array}\right\},$$

such that each λ_m^{rh} is an integer and the upper bar denote that the chaotics are up and the lower bar denote that the chaotics are down the area.

A *3-chaotic chain* on G is a system of a formal sum

$$\left\{\begin{array}{c} \lambda_1^{\overline{1h}} l^1_{\overline{1h}} + \lambda_2^{\overline{1h}} l^2_{\overline{1h}} + ... + \lambda_n^{\overline{1h}} l^n_{\overline{1h}} \\ \lambda_1^{\overline{2h}} l^1_{\overline{2h}} + \lambda_2^{\overline{2h}} l^2_{\overline{2h}} + ... + \lambda_n^{\overline{2h}} l^n_{\overline{2h}} \\ . \\ . \\ \lambda_1^{\overline{\infty h}} l^1_{\overline{\infty h}} + \lambda_2^{\overline{\infty h}} l^2_{\overline{\infty h}} + ... + \lambda_n^{\overline{\infty h}} l^n_{\overline{\infty h}} \end{array}\right\}, \left\{\begin{array}{c} \lambda_1^{\underline{1h}} l^1_{\underline{1h}} + \lambda_2^{\underline{1h}} l^2_{\underline{1h}} + ... + \lambda_n^{\underline{1h}} l^n_{\underline{1h}} \\ \lambda_1^{\underline{2h}} l^1_{\underline{2h}} + \lambda_2^{\underline{2h}} l^2_{\underline{2h}} + ... + \lambda_n^{\underline{2h}} l^n_{\underline{2h}} \\ . \\ . \\ \lambda_1^{\underline{\infty h}} l^1_{\underline{\infty h}} + \lambda_2^{\underline{\infty h}} l^2_{\underline{\infty h}} + ... + \lambda_n^{\underline{\infty h}} l^n_{\underline{\infty h}} \end{array}\right\},$$

such that each λ_n^{rh} is an integer and the upper bar denote that the chaotics are outside the volume and the lower bar denote that the chaotics are inside the volume.

Chaotic General Tree. A "chaotic general tree" is a genaral tree that carries many physical characters.

Here the chaotic adjacent matrix is

$$A_h(G_h) = \begin{bmatrix} 0_{012...\infty h} & 1_{012...\infty h} & 0_{012...\infty h} & 0_{012...\infty h} \\ 1_{012...\infty h} & 0_{012...\infty h} & 1_{012...\infty h} & 1_{012...\infty h} \\ 0_{012...\infty h} & 1_{012...\infty h} & 0_{012...\infty h} & 0_{012...\infty h} \\ 0_{012...\infty h} & 1_{012...\infty h} & 0_{012...\infty h} & 0_{012...\infty h} \end{bmatrix},$$

the chaotic edge matrix

$$E_h(G_h) = \begin{bmatrix} 0_{012...\infty h} & 1_{012...\infty h} & 1_{012...\infty h} \\ 1_{012...\infty h} & 0_{012...\infty h} & 1_{012...\infty h} \\ 1_{012...\infty h} & 1_{012...\infty h} & 0_{012...\infty h} \end{bmatrix},$$

the chaotic area matrix

$$R_h(G_h) = [0_{012...\infty h}],$$

the chaotic volume matrix

$$V_h(G_h) = [0_{012...\infty h}],$$

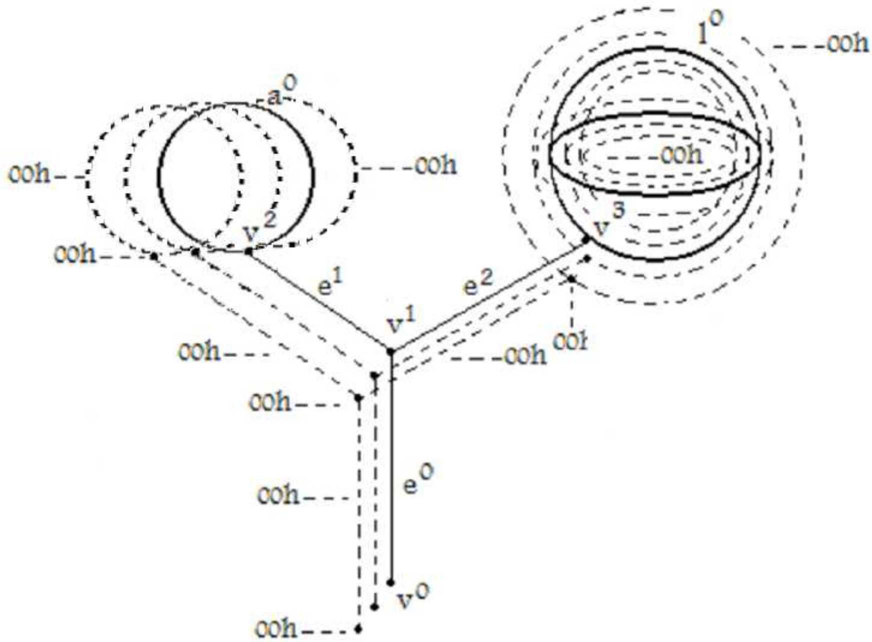

Figure 6

the chaotic incidence matrix

$$I_h(G_h) = \begin{bmatrix} 1_{012\ldots\infty h} & 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} & 1_{012\ldots\infty h} & 1_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 1_{012\ldots\infty h} & 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} & 0_{012\ldots\infty h} & 1_{012\ldots\infty h} \end{bmatrix},$$

the chaotic area matrix

$$M_h(G_h) = \begin{bmatrix} 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} \end{bmatrix},$$

the chaotic volume matrix

$$N_h(G_h) = \begin{bmatrix} 0_{012\ldots\infty h} \\ 0_{012\ldots\infty h} \\ 1_{012\ldots\infty h} \\ 0_{012\ldots\infty h} \end{bmatrix},$$

the chaotic edges area matrix

$$H_h(G_h) = \begin{bmatrix} 0_{012...\infty h} \\ 0_{012...\infty h} \\ 1_{012...\infty h} \end{bmatrix},$$

the chaotic edge volume matrix

$$J_h(G_h) = \begin{bmatrix} 0_{012...\infty h} \\ 1_{012...\infty h} \\ 0_{012...\infty h} \end{bmatrix},$$

and the chaotic area volume matrix

$$U_h(G_h) = [0_{012...\infty h}],$$

where the lower suffix ($012...\infty h$) refers to the existance of the chaotics, also its 0-chaotic chain are

$$\left\{ \begin{array}{c} \lambda_1^{1h}\nu_{1h}^0 + \lambda_2^{1h}\nu_{1h}^1 + \lambda_3^{1h}\nu_{1h}^2 + \lambda_4^{1h}\nu_{1h}^3 \\ \lambda_1^{2h}\nu_{2h}^0 + \lambda_2^{2h}\nu_{2h}^1 + \lambda_3^{2h}\nu_{2h}^2 + \lambda_4^{2h}\nu_{2h}^3 \\ . \\ . \\ \lambda_1^{\infty h}\nu_{\infty h}^0 + \lambda_2^{\infty h}\nu_{\infty h}^1 + \lambda_3^{\infty h}\nu_{\infty h}^2 + \lambda_4^{\infty h}\nu_{\infty h}^3 \end{array} \right\},$$

the 1-chaotic chains are

$$\left\{ \begin{array}{c} \lambda_1^{1h}e_{1h}^0 + \lambda_2^{1h}e_{1h}^1 + \lambda_3^{1h}e_{1h}^2 \\ \lambda_1^{2h}e_{2h}^0 + \lambda_2^{2h}e_{2h}^1 + \lambda_3^{2h}e_{2h}^2 \\ . \\ . \\ \lambda_1^{\infty h}e_{\infty h}^0 + \lambda_2^{\infty h}e_{\infty h}^1 + \lambda_3^{\infty h}e_{\infty h}^2 \end{array} \right\},$$

the 2-chaotic chains are

$$\left\{ \begin{array}{c} \lambda_1^{\overline{1h}}a_{\overline{1h}}^0 \\ \lambda_1^{\overline{2h}}a_{\overline{2h}}^0 \\ . \\ . \\ \lambda_1^{\overline{\infty h}}a_{\overline{\infty h}}^0 \end{array} \right\}, \left\{ \begin{array}{c} \lambda_1^{\underline{1h}}a_{\underline{1h}}^0 \\ \lambda_1^{\underline{2h}}a_{\underline{2h}}^0 \\ . \\ . \\ \lambda_1^{\underline{\infty h}}a_{\underline{\infty h}}^0 \end{array} \right\}$$

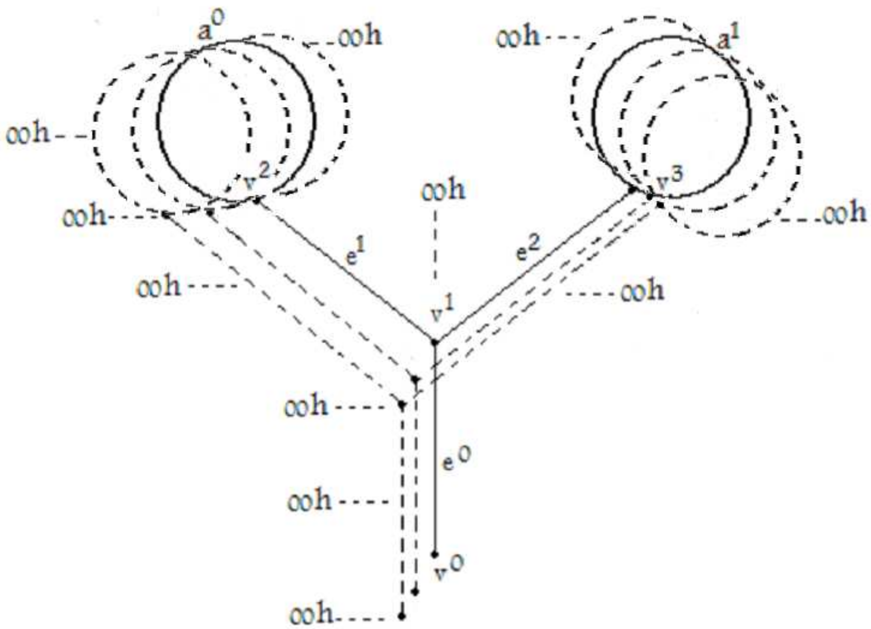

Figure 7

and the 3-chaotic chains are

$$\left\{\begin{array}{c} \lambda_1^{\overline{1h}} l^0_{\overline{1h}} \\ \lambda_1^{\overline{2h}} l^0_{\overline{2h}} \\ . \\ . \\ \lambda_1^{\overline{\infty h}} l^0_{\overline{\infty h}} \end{array}\right\}, \left\{\begin{array}{c} \lambda_1^{\underline{1h}} l^0_{\underline{1h}} \\ \lambda_1^{\underline{2h}} l^0_{\underline{2h}} \\ . \\ . \\ \lambda_1^{\underline{\infty h}} l^0_{\underline{\infty h}} \end{array}\right\}.$$

Theorem. *The tree in one dimension induces an infinite number of chaotic trees.*

Corollary 1. *When the chaotic tree is on chaotic, then it is the fuzzy tree.*

Corollary 2. *When the tree is a 2-dimensional tree, then the chaotic trees are of 2-chains, one of them up and the other is down.*

Proof. Let G be a chaotic general tree, where v^0, v^1, v^2, v^3 are of 0-dimension, and e^0, e^1, e^2 are of 1-dimension, and a^0, a^1 are of 2-dimension.

It is 0-chain: $\lambda_1 \nu^0 + \lambda_2 \nu^1 + \lambda_3 \nu^2 + \lambda_4 \nu^3$;

1-chain: $\lambda_1 e^0 + \lambda_2 e^1 + \lambda_3 e^2$;

2-chain: $\lambda_1 a^0 + \lambda_2 a^1$.
And it is 0-chaotic chain are:

$$\left\{\begin{array}{c} \lambda_1^{1h}\nu_{1h}^0 + \lambda_2^{1h}\nu_{1h}^1 + \lambda_2^{1h}\nu_{1h}^2 + \lambda_3^{1h}\nu_{1h}^3 \\ \lambda_1^{2h}\nu_{2h}^0 + \lambda_2^{2h}\nu_{2h}^1 + \lambda_3^{2h}\nu_{2h}^2 + \lambda_3^{2h}\nu_{2h}^3 \\ . \\ . \\ \lambda_1^{\infty h}\nu_{\infty h}^0 + \lambda_2^{\infty h}\nu_{\infty h}^1 + \lambda_3^{\infty h}\nu_{\infty h}^2 + \lambda_3^{\infty h}\nu_{\infty h}^3 \end{array}\right\}.$$

The 1-chaotic chain are:

$$\left\{\begin{array}{c} \lambda_1^{1h}e_{1h}^0 + \lambda_2^{1h}e_{1h}^1 + \lambda_2^{1h}e_{1h}^2 \\ \lambda_1^{2h}e_{2h}^0 + \lambda_2^{2h}e_{2h}^1 + \lambda_3^{2h}e_{2h}^2 \\ . \\ . \\ \lambda_1^{\infty h}e_{\infty h}^0 + \lambda_2^{\infty h}e_{\infty h}^1 + \lambda_3^{\infty h}e_{\infty h}^2 \end{array}\right\}.$$

The 2-chaotic chains are

$$\left\{\begin{array}{ccc} \lambda_1^{\overline{1h}}a_{\overline{1h}}^0 & + & \lambda_2^{\overline{1h}}a_{\overline{1h}}^1 \\ \lambda_1^{\overline{2h}}a_{\overline{2h}}^0 & + & \lambda_2^{\overline{2h}}a_{\overline{2h}}^1 \\ & . & \\ & . & \\ \lambda_1^{\overline{\infty h}}a_{\overline{\infty h}}^0 & + & \lambda_2^{\overline{\infty h}}a_{\overline{\infty h}}^1 \end{array}\right\}, \left\{\begin{array}{ccc} \lambda_1^{\underline{1h}}a_{\underline{1h}}^0 & + & \lambda_2^{\underline{1h}}a_{\underline{1h}}^1 \\ \lambda_1^{\underline{2h}}a_{\underline{2h}}^0 & + & \lambda_2^{\underline{2h}}a_{\underline{2h}}^1 \\ & . & \\ & . & \\ \lambda_1^{\underline{\infty h}}a_{\underline{\infty h}}^0 & + & \lambda_2^{\underline{\infty h}}a_{\underline{\infty h}}^1 \end{array}\right\}.$$

References

[1] M. El-Ghoul, Folding of fuzzy graphs and fuzzy spheres, *Fuzzy Sets and Systems*, **58** (1993), 355-363.

[2] M. El-Ghoul, The most general set and chaos graph, *Chaos, Solitons and Fractals*, **13** (2002), 833-838.

[3] M. El-Ghoul, A. El-Ahmady, T. Homoda, On chaotic graphs and applications in physics and biology, *Chaos, Solitons and Fractals*, **27** (2006), 159-173.

[4] M. El-Ghoul, Sh. A. Mousa, Some geometric transformations on new trees, *Journal of Mathematics Research*, Canada (2006).

[5] A.T. White, *Graphs, Groups and Surfaces*, Amesterdam, North-Holland Publishing Company (1973).

[6] P.J. Giblin, *Graphs, Surfaces and Homology. An Introduction to Algebraic Topology*, Chapman and Hall Ltd, London (1977).

[7] W.S. Massey, *Algebraic Topology. An Introduction*, Harcourt, Brace and World Inc., New York, USA (1976).