CHAOTIC GENERAL TREE AND IT’S CHAINS

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Abstract: In this paper, we will define the Chaotic general tree. The adjacent, incidence, area, volume, edge area, edge volume, area volume matrices will be introduced. The chains of the chaotic general tree will be identified.

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1. Introduction

In mathematics and physics, chaos theory deals with the behavior of certain nonlinear dynamical systems that under certain conditions exhibit a phenomenon known as chaos, which is characterized by a sensitivity to initial conditions. As a result of this sensitivity, the behavior of the system that exhibit chaos appears to be random, even though the model of the system is deterministic in the sense that it is well defined and contains no random parameters.

2. Definitions and Background

Chaotic graph. (see [3]) A “Chaotic graph” is a geometric graph that carries many physical characters.

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General Tree. (see [4]) It is a tree which consists of vertices V, edges E, areas A, and volume L, where V, E, A, and L are different in dimensions, such that the connection is direct, i.e. the one dimension joined the three dimension directly, Figure 1(a), or gradually, i.e. the one dimension joined with two dimension and the two dimension joined with three dimension Figure 1(b).

In the original tree, the vertices join between edges of the same dimension, but for the general tree the vertices joined between different dimensions.

Example. Consider the following general tree in Figure 2, where $v^0$, $v^1$, $v^2$, $v^3$ are of 0-dimension, and $e^0$, $e^1$, $e^2$ are of 1-dimension, and $a^0$ is of 2-dimension, and $l^0$ is of 3-dimension.

We will describe the above general tree by the matrices adjacent, incidence, area, volume, edge area, edge volume, and area volume, such that

\[
A(G) = \begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}.
\]

Here the “adjacent matrix” of $G$ denoted by $A(G)$ is the $n \times n$ matrix in which the entry in row $I$ and column $J$ is the number of edges joining the vertices $I$ and $J$.

\[
E(G) = \begin{bmatrix}
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0
\end{bmatrix}.
\]
Here the “edge matrix” of $G$ denoted by $E(G_h)$ is the $n \times n$ matrix in which $m_{ij} = 1$ if $e_i$ is incident with $e_j$ and $m_{ij} = 0$ otherwise, $R(G) = [0]$, where the “area matrix” of $G$ denoted by $R(G_h)$ is the $n \times n$ matrix in which $m_{ij} = 1$ if $a_i$ is incident with $a_j$ and $m_{ij} = 0$ otherwise, $V(G) = [0]$. The “volume matrix” of $G$ denoted by $V(G_h)$ is the $n \times n$ matrix in which $m_{ij} = 1$ if $l_i$ is incident with $l_j$ and $m_{ij} = 0$ otherwise.

$$I(G) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Here the “incidence matrix” of $G$ denoted by $I(G)$ is the $n \times m$ binary matrix, $m_{ij} = 1$ if $v_i$ is incident with $e_j$ and $m_{ij} = 0$ otherwise.

$$M(G) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$ 

Here the “area matrix” of $G$ denoted by $M(G)$ is the $n \times m$ binary matrix, where $m_{ij} = 1$ if $v_i$ is incident with $a_i$ and $m_{ij} = 0$ otherwise, and

$$N(G) = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$
The “volume matrix” of $G$ denoted by $N(G)$ is the $n \times m$ binary matrix, where $m_{ij} = 1$ if $v_i$ is incident with $L_i$ and $m_{ij} = 0$ otherwise.

$$H(G) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$  

Here the “edge area matrix” of $G$ denoted by $H(G)$ is the $n \times m$ binary matrix, where $m_{ij} = 1$ if $e_i$ is incident with $a_i$ and $m_{ij} = 0$, otherwise.

$$J(G) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$  

Here the “edge volume matrix” of $G$ denoted by $J(G)$ is the $n \times m$ binary matrix, where $m_{ij} = 1$ if $e_i$ is incident with $l_i$ and $m_{ij} = 0$, otherwise.

$$U(G) = [0].$$  

Here the “area volume matrix” of $G$ denoted by $U(G)$ is the $n \times m$ binary matrix, where $m_{ij} = 1$ if $a_i$ is incident with $l_i$ and $m_{ij} = 0$, otherwise.

Let $G$ be an oriented graph with edges $e^1, e^2, ..., e^r$. A 1-chain on $G$ is a formal sum

$$\lambda_1 e^1 + \lambda_2 e^2 + ... + \lambda_r e^r,$$

where each $\lambda_i$ is an integer. When writing down 1-chains we omit any edges which have coefficient 0. The 1-chain in which all coefficients are zero is denoted 0.

We define the sum of the 1-chains $\sum \lambda_i e^i$ and $\sum \lambda'_i e^i$ to be

$$\sum \lambda_i e^i + \sum \lambda'_i e^i = \sum \left( \lambda_i + \lambda'_i \right) e^i,$$

which also is a 1-chain.

A 0-chain on $G$ is a formal sum $\lambda_1 \nu^1 + \lambda_2 \nu^2 + ... + \lambda_m \nu^m$ where $\nu^1, \nu^2, ..., \nu^m$ are the vertices of $G$.

3. Main Results

The chaotic circle. It is a circle that carries many physical characters.
The chaotic sphere. It is a sphere that carries many physical characters.

The chaotic cylinder. It is a cylinder that carries many physical characters.
Let $G$ be an oriented graph with areas $a_1, a_2, \ldots, a_n$. A 2-chain on $G$ is a formal sum

$$\lambda_1 a_1 + \lambda_2 a_2 + \ldots + \lambda_n a^n,$$

where each $\lambda_i$ is an integer.

Let $G$ be an oriented graph with volumes $l_1, l_2, \ldots, l^j$. A $3$-chain on $G$ is a formal sum $\lambda_1 l_1 + \lambda_2 l_2 + \ldots + \lambda_j l^j$ where each $\lambda_i$ is an integer.

Let $G$ be an oriented graph. Ann-chain on $G$ is a formal sum

$$\lambda_1 x^1 + \lambda_2 x^2 + \ldots + \lambda_s x^s,$$

where each $\lambda_s$ is an integer and $x$ is of $n$-dimension.

We define the sum of the $n$-chains $\sum \lambda_i x^i$ and $\sum \lambda'_j x^i$ to be

$$\sum \lambda_i x^i + \sum \lambda'_j x^i = \sum (\lambda_i + \lambda'_j) x^i,$$

which is also an $n$-chain.

4. Chaotic Chains

Let $G$ be a graph with vertices $V = \{v^1, v^2, \ldots, v^j\}$, edges $E = \{e^1, e^2, \ldots, e^j\}$, areas $A = \{a^1, a^2, \ldots, a^n\}$, and volumes $L = \{l^1, l^2, \ldots, l^n\}$ carries infinite number of chaotics.

The $0$-chaotic chain on $G$ is system of a formal sum

$$\left\{ \begin{array}{l}
\lambda^1_{1h} v^1_{1h} + \lambda^1_{2h} v^2_{1h} + \ldots + \lambda^1_{nh} v^n_{1h} \\
\lambda^2_{1h} v^1_{2h} + \lambda^2_{2h} v^2_{2h} + \ldots + \lambda^2_{nh} v^n_{2h} \\
\vdots \\
\lambda^\infty_{1h} v^1_{\infty h} + \lambda^\infty_{2h} v^2_{\infty h} + \ldots + \lambda^\infty_{nh} v^n_{\infty h}
\end{array} \right\},$$

where each $\lambda^r_i$ is an integer.

A $1$-chaotic chain on $G$ is a system of a formal sum

$$\left\{ \begin{array}{l}
\lambda^1_{1h} e^1_{1h} + \lambda^1_{1h} e^2_{1h} + \ldots + \lambda^1_{jh} e^j_{1h} \\
\lambda^2_{1h} e^1_{2h} + \lambda^2_{1h} e^2_{2h} + \ldots + \lambda^2_{jh} e^j_{2h} \\
\vdots \\
\lambda^\infty_{1h} e^1_{\infty h} + \lambda^\infty_{2h} e^2_{\infty h} + \ldots + \lambda^\infty_{jh} e^j_{\infty h}
\end{array} \right\},$$

where each $\lambda^r_j$ is an integer.
A 2-chaotic chain on $G$ is a system of a formal sum

$$
\begin{align*}
\lambda_1^1 a_1^{1h} + \lambda_2^2 a_2^{1h} + \ldots + \lambda_m^m a_m^{1h} \\
\lambda_1^2 a_1^{2h} + \lambda_2^2 a_2^{2h} + \ldots + \lambda_m^m a_m^{2h} \\
\vdots \\
\lambda_1^\infty a_1^{\infty h} + \lambda_2^\infty a_2^{\infty h} + \ldots + \lambda_m^\infty a_m^{\infty h}
\end{align*}
$$

such that each $\lambda_i^h$ is an integer and the upper bar denote that the chaotics are up and the lower bar denote that the chaotics are down the area.

A 3-chaotic chain on $G$ is a system of a formal sum

$$
\begin{align*}
\lambda_1^1 a_1^{1h} + \lambda_2^2 a_2^{1h} + \ldots + \lambda_n^n a_n^{1h} \\
\lambda_1^2 a_1^{2h} + \lambda_2^2 a_2^{2h} + \ldots + \lambda_n^n a_n^{2h} \\
\vdots \\
\lambda_1^\infty a_1^{\infty h} + \lambda_2^\infty a_2^{\infty h} + \ldots + \lambda_n^\infty a_n^{\infty h}
\end{align*}
$$

such that each $\lambda_i^h$ is an integer and the upper bar denote that the chaotics are outside the volume and the lower bar denote that the chaotics are inside the volume.

**Chaotic General Tree.** A “chaotic general tree” is a general tree that carries many physical characteristics.

Here the chaotic adjacent matrix is

$$
A_h(G_h) = \begin{bmatrix}
0_{1012\ldots \infty h} & 1_{012\ldots \infty h} & 0_{012\ldots \infty h} & 0_{012\ldots \infty h} \\
1_{012\ldots \infty h} & 0_{012\ldots \infty h} & 1_{012\ldots \infty h} & 1_{012\ldots \infty h} \\
0_{012\ldots \infty h} & 1_{012\ldots \infty h} & 0_{012\ldots \infty h} & 0_{012\ldots \infty h} \\
0_{012\ldots \infty h} & 1_{012\ldots \infty h} & 0_{012\ldots \infty h} & 0_{012\ldots \infty h}
\end{bmatrix},
$$

the chaotic edge matrix

$$
E_h(G_h) = \begin{bmatrix}
0_{012\ldots \infty h} & 1_{012\ldots \infty h} & 1_{012\ldots \infty h} \\
1_{012\ldots \infty h} & 0_{012\ldots \infty h} & 1_{012\ldots \infty h} \\
1_{012\ldots \infty h} & 1_{012\ldots \infty h} & 0_{012\ldots \infty h}
\end{bmatrix},
$$

the chaotic area matrix

$$
R_h(G_h) = [0_{012\ldots \infty h}],
$$

the chaotic volume matrix

$$
V_h(G_h) = [0_{012\ldots \infty h}],
$$
the chaotic incidence matrix

$$I_h(G_h) = \begin{bmatrix} 1 & 0 & 1 & 2 & \cdots & \infty & h & 0 & 0 \end{bmatrix}$$

the chaotic area matrix

$$M_h(G_h) = \begin{bmatrix} 0_{012\ldots\infty h} & 0_{012\ldots\infty h} \end{bmatrix}$$

the chaotic volume matrix

$$N_h(G_h) = \begin{bmatrix} 0_{012\ldots\infty h} \end{bmatrix}$$
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the chaotic edges area matrix

\[
H_h(G_h) = \begin{bmatrix}
0_{012\ldots\infty h} \\
0_{012\ldots\infty h} \\
1_{012\ldots\infty h}
\end{bmatrix},
\]

the chaotic edge volume matrix

\[
J_h(G_h) = \begin{bmatrix}
0_{012\ldots\infty h} \\
1_{012\ldots\infty h} \\
0_{012\ldots\infty h}
\end{bmatrix},
\]

and the chaotic area volume matrix

\[
U_h(G_h) = [0_{012\ldots\infty h}],
\]

where the lower suffix \(012\ldots\infty h\) refers to the existence of the chaotics, also its 0-chaotic chain are

\[
\begin{aligned}
\lambda_1^{1h} v_0^{1h} + \lambda_2^{1h} v_1^{1h} + \lambda_3^{1h} v_2^{1h} + \lambda_4^{1h} v_3^{1h} \\
\lambda_1^{2h} v_0^{2h} + \lambda_2^{2h} v_1^{2h} + \lambda_3^{2h} v_2^{2h} + \lambda_4^{2h} v_3^{2h} \\
\vdots \\
\lambda_1^{\infty h} v_0^{\infty h} + \lambda_2^{\infty h} v_1^{\infty h} + \lambda_3^{\infty h} v_2^{\infty h} + \lambda_4^{\infty h} v_3^{\infty h}
\end{aligned}
\]

the 1-chaotic chains are

\[
\begin{aligned}
\lambda_1^{1h} e_0^{1h} + \lambda_2^{1h} e_1^{1h} + \lambda_3^{1h} e_2^{1h} \\
\lambda_1^{2h} e_0^{2h} + \lambda_2^{2h} e_1^{2h} + \lambda_3^{2h} e_2^{2h} \\
\vdots \\
\lambda_1^{\infty h} e_0^{\infty h} + \lambda_2^{\infty h} e_1^{\infty h} + \lambda_3^{\infty h} e_2^{\infty h}
\end{aligned}
\]

the 2-chaotic chains are

\[
\begin{aligned}
\lambda_1^{1h} a_0^{1h} \\
\lambda_1^{2h} a_0^{2h} \\
\vdots \\
\lambda_1^{\infty h} a_0^{\infty h}
\end{aligned}
\]

\[
\begin{aligned}
\lambda_1^{1h} a_0^{1h} \\
\lambda_1^{2h} a_0^{2h} \\
\vdots \\
\lambda_1^{\infty h} a_0^{\infty h}
\end{aligned}
\]
and the 3-chaotic chains are
\[
\begin{cases}
\lambda_1 v_0 + \lambda_2 v_1 + \lambda_3 v_2 + \lambda_4 v_3 \\
\lambda_1 e_0 + \lambda_2 e_1 + \lambda_3 e_2 \\
\lambda_1 a_0 + \lambda_2 a_1
\end{cases}
\]

**Theorem.** The tree in one dimension induces an infinite number of chaotic trees.

**Corollary 1.** When the chaotic tree is on chaotic, then it is the fuzzy tree.

**Corollary 2.** When the tree is a 2-dimensional tree, then the chaotic trees are of 2-chains, one of them up and the other is down.

**Proof.** Let $G$ be a chaotic general tree, where $v^0, v^1, v^2, v^3$ are of 0-dimension, and $e^0, e^1, e^2$ are of 1-dimension, and $a^0, a^1$ are of 2-dimension.

It is 0-chain: $\lambda_1 v^0 + \lambda_2 v^1 + \lambda_3 v^2 + \lambda_4 v^3$;
1-chain: $\lambda_1 e^0 + \lambda_2 e^1 + \lambda_3 e^2$.
2-chain: $\lambda_1 a^0 + \lambda_2 a^1$.
And it is 0-chaotic chain are:

$$\begin{align*}
\lambda_1^1 \nu_{1h}^0 + \lambda_2^1 \nu_{1h}^1 + \lambda_3^1 \nu_{1h}^2 + \lambda_4^1 \nu_{1h}^3 \\
\lambda_1^2 \nu_{2h}^0 + \lambda_2^2 \nu_{2h}^1 + \lambda_3^2 \nu_{2h}^2 + \lambda_4^2 \nu_{2h}^3 \\
\lambda_1^{\infty h} \nu_{\infty h}^0 + \lambda_2^{\infty h} \nu_{\infty h}^1 + \lambda_3^{\infty h} \nu_{\infty h}^2 + \lambda_4^{\infty h} \nu_{\infty h}^3
\end{align*}$$

The 1-chaotic chain are:

$$\begin{align*}
\lambda_1^1 e_{1h}^0 + \lambda_2^1 e_{1h}^1 + \lambda_3^1 e_{1h}^2 \\
\lambda_1^2 e_{2h}^0 + \lambda_2^2 e_{2h}^1 + \lambda_3^2 e_{2h}^2 \\
\lambda_1^{\infty h} e_{\infty h}^0 + \lambda_2^{\infty h} e_{\infty h}^1 + \lambda_3^{\infty h} e_{\infty h}^2
\end{align*}$$

The 2-chaotic chains are

$$\begin{align*}
\lambda_1^{1h} d_{1h}^0 + \lambda_2^{1h} d_{1h}^1 \\
\lambda_1^{2h} d_{2h}^0 + \lambda_2^{2h} d_{2h}^1 \\
\lambda_1^{\infty h} d_{\infty h}^0 + \lambda_2^{\infty h} d_{\infty h}^1
\end{align*}$$

References


