International Journal of Pure and Applied Mathematics
Volume 70 No. 5 2011, 735-754

STABILITY OF TRIANGULAR EQUILIBRIUM POINTS IN ELLIPTICAL RESTRICTED THREE BODY PROBLEM UNDER THE EFFECTS OF PHOTOGRAVITATIONAL AND OBLATENESS OF PRIMARIES

A. Narayan[1 §], C. Ramesh Kumar[2]

[1]Department of Mathematics
Bhilai Institute of Technology
Durg, C.G., INDIA

[2]Department of Mathematics
Rungta College of Engineering and Technology
Bhilai, C.G., INDIA

Abstract: The effects of the photogravitational and oblateness of the bigger primariy and oblateness of smaller primary to study the stability of the triangular equilibrium points in the elliptical restricted three body problem have been discussed. We have exploited an analytical method for determining of characteristic exponent based on the Floquet's theory to the variational equations of motion.The stability of the triangular points under the effects of the photogravitational and oblateness of bigger primary and oblateness of smaller primariy have been studied using simulation technique by drawing transition curves. It is observed that the region within the transition curves, the system are unstable, whereas the region outside these curves are stable.

AMS Subject Classification: 70F15, 70F07
Key Words: photogravitational, elliptical restricted three body problem, stability, Lagrangian points, oblateness

1. Introduction

The present paper is devoted to the analysis of the effects of the photogravitational

Received: February 12, 2011

§Correspondence author

and oblateness of the bigger primary and oblateness of smaller primary on the stability of triangular equilibrium points of the planar elliptical restricted three body problem. The elliptical restricted three body problem describes the dynamical system more accuretely on account of realistic assumptions of the motion of the primaries are subjected to move along the elliptical orbit. We have attempted to investigate the stability of triangular equilibrium points under the effects of the photogravitational and oblateness of the bigger primary and oblateness of smaller primary. The stability of triangular equilibrium points of the elliptical restricted three body has been studied, Danby [1]. Jefferys [2], [3] ,showed, their exists doubly symmetric, almost circular periodic solution of one of the primaries is sufficiently small. He further showed that the existence of families of elliptical orbits for any value of eccentricity and critical inclination. The elliptical restricted three body problem has not been fully explored (planar or spheroid),although a number of research paper have been devoted to it. Non-linear stability of triangular Lagrangian points in the elliptical restricted problem of three bodies has been studied. Gyorgrey [4], the same problem has been dealt by Kumar and Choudhary [5], when the attracting bodies are radiating.

Khasan [6,7], studied the existence of Liberation points and their stability in the photogravitational elliptical restricted three body problem. Selaru and Cucu Dumitrescu [8] performed an analytical investigation concerning the structure of asymptotic perturbative approximation for small amplitude motions, if the third point mass in the neighbouhood of a Lagarangian equilateral libration position in the planar, elliptical restricted three bodies. After a sequence of canonical transformations, they formulated the Hamiltonian governing the motion of the negligible mass body, using the eccentric anomaly of the primaries elliptical Keplerian orbit as the independent variable; they studied the linearized system of differential equations of motion obtained from expanding the Hamiltonian around a Lagrangian solution. Also, they developed their theory and calculations of an asymptotic solution up to the first order in the orbital eccentricity of the primaries taken as the perturbing parameter. Selaru and Cucu-Dumitrescu [9] presented existence of these considerations to a second order theory. Floria [17] under takes an approximate of the elliptical restricted integration three body problem by means of perturbation technique based on Lie series devlopement, which leads to an approximate solution of the differential system of canonical equations of motion derived from the chosen Hamiltonian function.The study of the spatial restricted three body problem in case when the small particles are far from the primaries in circular and elliptic case. It has been investigated in detail the transition between the circular and elliptical problem using double averaging method. Palacian, J.F. [18]. Zsoft Sandoor [13] applied the method of the short time Lypunov indicators to the planar circular to the planar elliptic restricted three body problem in order to study the structure of the phase space in some selected regions.Narayan A and Ramesh C [23], [24], [25] studied the stability of triangular points in the generalised oblate elliptical restricted three body problem. The same problems have also been dealt in resonance and parametric resonance cases.

Halan and Rana [15], Alexander D. Burno [19], Markeev A.P. [11] Ammar M.K. [20] and Robert G.E. [10], have studied different aspect of the problem of elliptical restricted three body problem.

The present paper deals with the effects of photogravitational and oblateness of the bigger primary and oblateness of smaller primary by exploiting an analytical technique developed by Bennett [22]. The method, which on stability of the triangular equilibrium points of the planar elliptical restricted three body problem is being exploited to discuss the stability of elliptical restricted three body problem is based upon Floquet's theory for the determination characteristic exponents for a system with periodic coefficients.

The present paper comprises of three sections. The first section of the paper deals with the variational equation of motion of the system in matrix form. In the second section, we have derived the characteristic exponents of the system up to the order of $e^2$and in the third section of the paper derived an expression obtained for the transition curves.

The transition curves have been presented through simulation techniques, which shows the region of stability as well as instability. The effect of oblateness of primaries is playing an important role in the stability of infinitesimal which is obvious from the transition curves traced.

2. Variational Equation of Motion

The differential equations of motion for the elliptical restricted three body problem under the oblate primaries in barycentric, pulsating and non-dimensional coordinates are:

$$\begin{aligned} x'' - 2y' &= \phi\Omega_x, \\ y'' + 2x' &= \phi\Omega_y, \end{aligned} \tag{2.1}$$

where,

$$\Omega = \frac{x^2+y^2}{2} + \frac{1}{1+3\left(\frac{A_1+A_2}{2}\right)}\left[\frac{(1-\mu)\cdot q}{r_1} + \frac{\mu}{r_2} + \frac{(1-\mu)\,A_1 q}{2r_1^3} + \frac{\mu A_2}{2r_2^3}\right],$$

$$r_1^2 = (x+\mu)^2 + y^2,$$
$$r_2^2 = (x-1+\mu)^2 + y^2 \tag{2.2}$$

and

$$\phi = \left(\frac{1}{1+e\cos v}\right).$$

Here Ω_x denotes the partial differentiation of Ω with respect x and Ω_y denotes the differentiation of Ω partially with respect y. Where $A_1$and A_2 are the oblateness parameters of primaries. q is the source of radiation of bigger primary.

The co-ordinates of the triangular equilibrium points $L_4$and $L_5$are determined as follows:

$$x = \frac{1}{2} - \mu - \frac{A_1}{3} - \frac{A_2}{2} + \frac{5A_1 q}{6},$$
$$y = \pm\frac{\sqrt{3}}{2}\left[1 - \frac{2A_1}{9} + \frac{A_2}{3} - \frac{5A_1 q}{9}\right]. \tag{2.3}$$

There are two triangular equilibrium points of the problem in the plane of finite bodies. In this coordinate system $(x, y)$the three bodies form nearly equilateral triangles. Since the equilibrium points are symmetrical to each other, the nature of the motion near the two triangular points are the same. therefore, it is sufficient to analyze the motion of the equilibrium points having the location (x_0, y_0)given by:

$$x_0 = \frac{1}{2} - \mu - \frac{A_1}{3} - \frac{A_2}{2} + \frac{5A_1 q}{6},$$
$$y_0 = \pm\frac{\sqrt{3}}{2}\left[1 - \frac{2A_1}{9} + \frac{A_2}{3} - \frac{5A_1 q}{9}\right]. \tag{2.4}$$

In order to investigate the stability of the equilibrium point (2.4)in the first approximation, we derive the equations for the variations in the coordinates.

Let ξ, η denotes the small displacement in x_0, y_0. Then

$$x = x_0 + \xi \quad \text{and} \quad y = y_0 + \eta. \tag{2.5}$$

Differentiating these, we get;

$$x' = \xi';\ y' = \eta';\ x'' = \xi'';\ y'' = \eta'',$$

where

$$\Omega_x = \Omega_x(x, y) = \Omega_x(x_0 + \xi,\ y_0 + \eta). \tag{2.6}$$

Expanding equation (2.6) by Taylor's theorem and retaining only the first order term in the infinitesimal ξ and η, we get:

$$\Omega_x = \Omega_x^0 + \xi\Omega_{xx}^0 + \eta\Omega_{xy}^0;$$

$$\Omega_y = \Omega_y^0 + \xi\Omega_{yx}^0 + \eta\Omega_{yy}^0, \tag{2.7}$$

where Ω_x^0 and Ω_y^0 are the values of $\Omega_x$and Ω_y at the equilibrium point (x_0, y_0)given by (2.4).

At the equilibrium point (x_0, y_0)we have:

$$\Omega_x^0 = 0 = \Omega_y^0. \tag{2.8}$$

Hence, the set of equation (2.1) with the help (2.7) and (2.8) reduced to the form:

$$\xi'' - 2\eta' = \phi\left(\Omega_{xx}^0\xi + \Omega_{xy}^0\eta\right),$$

$$\eta'' + 2\xi = \phi\left(\Omega_{xy}^0\xi + \Omega_{yy}^0\eta\right), \tag{2.9}$$

differentiating partially Ω with respect to x and y and equating Ω_{xx}, Ω_{xy} and Ω_{yy} at the equilibrium point (x_0, y_0) given by (2.4), we get:

$$\begin{aligned}
\Omega_{xx}^0 &= \frac{1}{2} - \frac{3A_1}{2} - \frac{3A_2}{4} + \mu - \frac{17\mu A_1}{8} + \frac{3\mu A_2}{8} \\
&+ \in \left(\frac{1}{2} + \frac{3A_1}{2} + \frac{3A_2}{4} - \frac{\mu}{2} + \frac{3\mu A_1}{8} + \frac{3\mu A_2}{4}\right), \\
\Omega_{xy}^0 &= \frac{\sqrt{3}}{2}\left[\left(-1 + 2\mu - \frac{16A_1}{9} - \frac{A_2}{9} - \frac{169\mu A_1}{36} + \frac{77\mu A_2}{12}\right)\right. \\
&\left. - \in \left(-1 + \mu - \frac{16A_1}{9} - \frac{A_2}{3} - \frac{71\mu A_1}{36} + \frac{A_2\mu}{3}\right)\right], \\
\Omega_{yy}^0 &= \left(\frac{3}{2} - \frac{A_1}{2} + \frac{3A_2}{4} + \frac{25A_1\mu}{8} - \frac{45\mu A_2}{4}\right) \\
&+ \in \left(-\frac{1}{2} + \frac{A_1}{2} - \frac{3A_2}{4} + \frac{\mu}{2} - \frac{39\mu A_1}{8} + \frac{3\mu A_2}{4}\right).
\end{aligned} \tag{2.10}$$

Here $q = 1 - \varepsilon$, $0 < \epsilon < 1$. Transforming the variational equation of motion of elliptical restricted three body problem under oblate primaries in matrix form represented as follows:

$$X' = P(X), \tag{2.11}$$

where

$$X = \left\{ \begin{array}{c} \xi \\ \eta \\ \xi' \\ \eta' \end{array} \right\}, \; X' = \left\{ \begin{array}{c} \xi' \\ \xi'' \\ \eta' \\ \eta'' \end{array} \right\} \text{ and } p\,(v, e) = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \phi\Omega_{xx} & \phi\Omega_{xy} & 0 & 2 \\ \phi\Omega_{yx} & \phi\Omega_{yy} & -2 & 0 \end{array} \right]. \tag{2.12}$$

3. Determination of Characteristic Exponents

In order to solve variational equation of motion of the system, we are exploiting the Floquet's theory of determining characteristic exponents in the system with periodic coefficients.

We seek the solution of the system of equation (2.11) in the form:

$$x_k = y_k e^{\lambda_k v}, \tag{3.1}$$

where y_k is periodic with period 2π and λ_k are the characteristic exponents of (2.12) . Dropping the suffix in (3.1), we get:

$$x = ye^{\lambda v}$$

Differentiating with respect to v, we get:

$$x' = \left(y' + \lambda y\right) e^{\lambda v}, \tag{3.2}$$

using the equation (3.2), the variational equation of motion takes the form:

$$y' = (P - \lambda I\,)\, y, \tag{3.3}$$

where I is the unit matrix of the same order as that of y.

Now, using the expression which are mentioned below:

$$y = y^{(0)} + ey^{(1)} + e^2 y^{(2)} + ...,$$

$$\lambda = \lambda_0 + e\lambda_1 + e^2\lambda_2 + ..., \tag{3.4}$$

and the corresponding matrix P is expanded as follows:

$$P\,(v, e) = p^{(0)} + ep^{(1)} + e^2 p^{(2)} + ..., \tag{3.5}$$

where

$$p^{(0)} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \Omega^0_{xx} & \Omega^0_{xy} & 0 & 2 \\ \Omega^0_{yx} & \Omega^0_{yy} & -2 & 0 \end{bmatrix} \tag{3.6}$$

and

$$p^{(m)} = (-\cos v)^m \, c\,, \quad m = 1, 2, 3..., \tag{3.7}$$

where

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \Omega^0_{xx} & \Omega^0_{xy} & 0 & 0 \\ \Omega^0_{yx} & \Omega^0_{yy} & 0 & 0 \end{bmatrix}. \tag{3.8}$$

substituting the values of y, y', λ and p from (3.4), (3.5) in (3.3) we get:

$$y'^{(0)} + ey'^{(1)} + e^2 y'^{(2)} ...$$
$$= \left\{ \left\{ \left(P_0^{(0)} + eP^{(1)} + e^2 P^{(2)} + ...\right) - I \left(\lambda_0 + e\lambda_1 + e^2\lambda_2 + ...\right) \right\} \right.$$
$$\left. \left(y^{(0)} + ey^{(1)} + e^2 y^{(2)} + ... \right) \right\}. \tag{3.9}$$

Equating the coefficients of terms with the same power of e from (3.9) both sides and using (3.4), we get:

$$y'^0 + \left[I\lambda_0 - P^{(0)} \right] y^{(0)} = 0,$$

$$y'^1 + \left[I\lambda_0 - P^{(0)} \right] y^{(1)} = \left[-c\cos v - \lambda_1 I \right] y^{(0)},$$

$$y'^2 + \left[I\lambda_0 - P^{(0)} \right] y^{(2)} = \left[-c\cos v - I\lambda_1 \right] y^{(1)} + \left(c\cos^2 v - I\lambda_2 \right) y^{(0)},$$

$$y'^{(n)} + \left[I\lambda_0 - P^{(0)} \right] y^{(n)} = \sum_{m=1}^{n} c \left[(-\cos v)^m - I\lambda_m \right] y^{(n-m)}. \tag{3.10}$$

If a constant vector is assumed for the zeroth order solution, then for the nth order equation of non-homogenous terms have frequencies upto and including $\frac{n}{2\pi}$.

We take the particular solution as:

$$y^{(n)} = \sum_{k=-n}^{k=+n} a^{(n,k)} e^{ikv}, \quad n = 1, 2, 3..., \tag{3.11}$$

where

$$a^{(n,k)} = \begin{bmatrix} a_1^{(n,k)} \\ a_2^{(n,k)} \\ a_3^{(n,k)} \\ a_4^{(n,k)} \\ \vdots \\ \vdots \end{bmatrix}. \tag{3.12}$$

From the equation (3.11), we have

$$y^{(0)} = a^{(0,0)},$$

$$y'^{(0)} = 0,$$

$$y^{(1)} = a^{(0,-1)}e^{-iv} + a^{(0,0)} + a^{(1,1)}e^{iv},$$

$$y'^{(1)} = -ia^{(0,-1)}e^{-iv} + ia^{(1,1)}e^{iv},$$

$$y^{(2)} = a^{(2,-2)}e^{-2iv} + a^{(2,-1)}e^{-iv} + a^{(2,0)} + a^{(2,1)}e^{iv} + a^{(2,2)}e^{2iv},$$

$$y'^{(2)} = -2ia^{(2,-2)}e^{-2iv} - ia^{(2,-1)}e^{-iv} + ia^{(2,1)}e^{iv} + 2ia^{(2,2)}e^{2iv}.$$

Substituting these values in the set of equation (3.10), we obtain a system of equations necessary for the determination λ of up to the order of given e^2 as follows:

$$\left[I\lambda_0 - P^{(0)}\right] a^{(0,0)} = 0, \tag{3.13}$$

$$\left[I\lambda_0 - P^{(0)}\right] a^{(1,0)} = -\lambda_1 a^{(0,0)},$$

$$\left[(I\lambda_0 + i) - P^{(0)}\right] a^{(1,1)} = -\frac{1}{2}c \cdot a^{(0,0)},$$

$$\left[(I\lambda_0 - i) - P^{(0)}\right] a^{(1,-1)} = \frac{1}{2}c \cdot a^{(0,0)},$$

$$\left[I\lambda_0 - P^{(0)}\right] a^{(2,0)}$$

$$= -\lambda_1 a^{(1,0)} + \left[\left(\frac{1}{2}c - I\lambda_2\right)\right] a^{(0,0)} - \frac{1}{2}c\left(a^{(1,1)} + a^{(1,-1)}\right). \tag{3.14}$$

From the equation (3.13), it is evident that for the existence of $a^{(0,0)}$, it is necessary that

$$\det\left(I\lambda_0 - P^{(0)}\right) = 0, \tag{3.15}$$

i.e.

$$\begin{bmatrix} \lambda_0 & 0 & -1 & 0 \\ 0 & \lambda_0 & 0 & -1 \\ -\Omega^0_{xx} & -\Omega^0_{xy} & \lambda_0 & -2 \\ -\Omega^0_{yx} & -\Omega^0_{yy} & 2 & \lambda_0 \end{bmatrix} = 0.$$

From the above relation, we get

$$\lambda_0^4 + \left(4 - \Omega^0_{xx} - \Omega_{yy}\right)\lambda_0^2 + \Omega^0_{xx}\Omega^0_{yy} - \left(\Omega_{xy}\right)^2 = 0. \tag{3.16}$$

Substituting the value from the set of equation (2.10) in equation (3.16), we get:

$$\lambda_0^4 - Q\lambda_0^2 + R = 0, \tag{3.17}$$

where

$$Q = \Omega^0_{xx} + \Omega^0_{yy} - 4 = -2 - 2A_1 + \mu + \mu A_1 - \frac{87}{8}\mu A_2 + \in \left(2A_1 - \frac{9}{2}\mu A_1\right)$$

and

$$R = \Omega^0_{xx}\cdot\Omega^0_{yy} - \left(\Omega^0_{xy}\right)^2 = -\frac{3}{2} - \frac{21}{2}A_1 - \frac{9}{4}A_2 + \frac{21}{2}\mu - 9\mu^2 - \frac{21}{4}\mu A_1 - \frac{183}{4}\mu A_2$$

$$+ \in \left(5 + 19A_1 + \frac{9}{2}A_2 - \frac{29}{2}\mu + \frac{19}{2}\mu^2 + \frac{27}{4}\mu A_1 - \frac{645}{16}\mu A_2\right) \tag{3.18}$$

The relation for the exponent in the elliptical restricted three body problems can be obtained using the equation (3.17):

$$\lambda_0^2 = \frac{Q \pm \sqrt{Q^2 - 4R}}{2}$$

$$= \frac{1}{2}\Bigg[\left(-2 - 2A_1 + \mu + \mu A_1 - \frac{87}{8}\mu A_2\right) + \in \left(2A_1 - \frac{9}{2}\mu A_1\right) \pm \Bigg\{\Bigg(10 + 50A_1$$

$$+9A_2 - 46\mu + 36\mu^2 + 21\mu A_1 + \frac{453}{2}\mu A_2\Bigg) - 4 \in \Bigg(5 + 21A_1 + \frac{9}{2}A_2$$

$$-\frac{29}{2}\mu + \frac{9}{4}\mu A_1 - \frac{645}{16}\mu A_2\Bigg)\Bigg\}^{\frac{1}{2}}. \tag{3.19}$$

From the frist equation of the system of equation (3.14), we observe that it is necessary that the determinant of the coefficient on the left with any column replaced by the non-homogenous terms on the right be zero. That is represented as follows:

$$det\left[I\lambda_0 - P^{(0)}\right] + \lambda_1 a^{(0,0)} = 0. \tag{3.20}$$

But λ enters as a factor in all elements of the replaced column, therefore

$$\lambda_1 det\left[I\lambda_0 - P^{(0)}\right] a^{(0,0)} = 0 \tag{3.21}$$

Since the determinant of the equation (3.20) is not zero, in general, we conclude therefore that

$$\lambda_1 = 0. \tag{3.22}$$

Again from the second and the third equation of (3.14), we have the solutions for $a^{(1,1)}$ and $a^{(1,-1)}$ are

$$a^{(1,1)} = -\frac{1}{2}\left[I\left(\lambda_0 + i\right) - P^{(0)}\right]^{-1} ca^{(0,0)}$$

$$a^{(1,-1)} = -\frac{1}{2}\left[I\left(\lambda_0 - i\right) - P^{(0)}\right]^{-1} ca^{(0,0)}. \tag{3.23}$$

Substituting this value of $a^{(1,1)}$and $a^{(1,-1)}$from (3.23)in the last equation of (3.14), we get:

$$\left(I\lambda_0 - P^{(0)}\right) a^{(2,0)} = \frac{1}{4}c\left[\left(I\left(\lambda_0 + i\right) - P^{(0)}\right)^{-1} + \left(I\left(\lambda_0 - i\right) - P^{(0)}\right)^{-1}\right]^{-1} \cdot ca^{(0,0)} + \left(\frac{c}{2} - I\lambda_2\right) a^{(0,0)}. \tag{3.24}$$

The matrices within the sequare bracket are complex conjugate so that only the real parts of either needs to be considered, then equation (3.24) can be written as:

$$\left(I\lambda_0 - P^{(0)}\right) a^{(2,0)} = \left[\left\{\frac{1}{2}cR_e\left(I\lambda_0 + i\right) - P^{(0)}\right\}^{-1} + \left(\frac{c}{2} - I\lambda_2\right)^{-1}\right] a^{(0,0)} \tag{3.25}$$

After some mathematical manipulations, we from (3.25) obtain the value of λ_2 given by

$$\lambda_2 = -\left[\frac{\left(Q^2 - 4R - 16\right)\lambda_0^2 + A_0F_0 + A_1F_1 + A_2F_2}{4\left(Q^2 - 4R - 16\right)\lambda_0^2 + 32R}\right]\lambda_0 = A\lambda_0, \tag{3.26}$$

where

$$A_0 = \left[(Q+4)^2(Q-4) - 4QR\right]\lambda_0^2 - R(Q+4) - 4R^2,$$

$$A_1 = -8\lambda_0 R\left[2\lambda_0^2 - (Q+4)\right],$$

$$A_2 = -\lambda_0^2 R\left[Q^2 - 4R - 16\right],$$

$$F_0 = \frac{1}{N}\left[\lambda_0^2(Q+1) + (Q+1) + 2R\right],$$

$$F_1 = -\frac{\lambda_0}{N}\left[2\lambda_0^2 + (Q+3)\right],$$

$$F_2 = -\frac{1}{N}\left[\lambda_0^2 - (Q+1)\right],$$

$$N = \lambda_0^2\left[4Q^2 + 8Q + 4 - 16R\right] - 4R + (Q+1)^2. \tag{3.27}$$

Using this value of

$$Q = -2 - 2A_1 + \mu + \mu A_1 - \frac{87}{8}\mu A_2 + \in\left(2A_1 - \frac{9}{2}\mu A_1\right),$$

We find the value of the parameter A, which is calculated in appendix. Hence

$$-A = \left[\frac{\left(Q^2 - 4R - 16\right)\lambda_0^2 + A_0F_0 + A_1F_1 + A_2F_2}{4\left(Q^2 - 4R - 16\right)\lambda_0^2 + 32R}\right], \tag{3.28}$$

thus

$$\lambda_2 = A\lambda_0, \tag{3.29}$$

where A is given by the equation (3.28). Hence the solution of this system becomes

$$\lambda = \lambda_0 + e^2\lambda_0, \tag{3.30}$$

where λ_2 is given by this equation (3.29).

4. Transition Curves Separating Stable and Unstable Regions

The transition curves separating stable and unstable regions,which describes the stability of the triangular equilibrium points in the elliptical restricted three body problem under the stable primaries, can be found by simply equating the expression for the characteristic roots or exponents to the value of periodic solutions. In the range $0 \leq \mu \leq \frac{1}{2}$, the periodic solution provides:

$$\lambda^{\star} = \pm\frac{i}{2}, \tag{4.1}$$

replacing λ by $\lambda^{\star}$in (3.30), we obtain:

$$\pm\frac{i}{2} = \lambda_0 + e^2 A\lambda_0,$$

$$\pm\frac{i}{2} = \left(1 + e^2 A\right)\lambda_0.$$

Squaring both the sides we obtain:

$$\left(1 + e^2 A\right)^2 \lambda_0^2 = -\frac{1}{4},$$

$$\left(1 + e^2 A\right) = \pm\left(-\frac{1}{4}\lambda_0^2\right)^{\frac{1}{2}},$$

$$e^2 = \left[\pm\left(-\frac{1}{4\lambda_0^2}\right)^{\frac{1}{2}} - 1\right]\left(\frac{1}{A}\right). \tag{4.2}$$

Now evaluating the values of A from the equation (3.28), which is mentioned in this appendix and those of λ_0^2 from (3.19), we can evaluate $'e'$ easily for different values of μ. The dependence of μ upon e given by equation (4.2) is shown in graph for various values of oblateness parameters A_1 and A_2. The triangular equilibrium points are unstable in the region between these curves. In the region outside these curves the equilibrium points are stable. The effects of oblateness of the primaries introduce a visible left shift in the bifurcation points from its value for the classical problem.

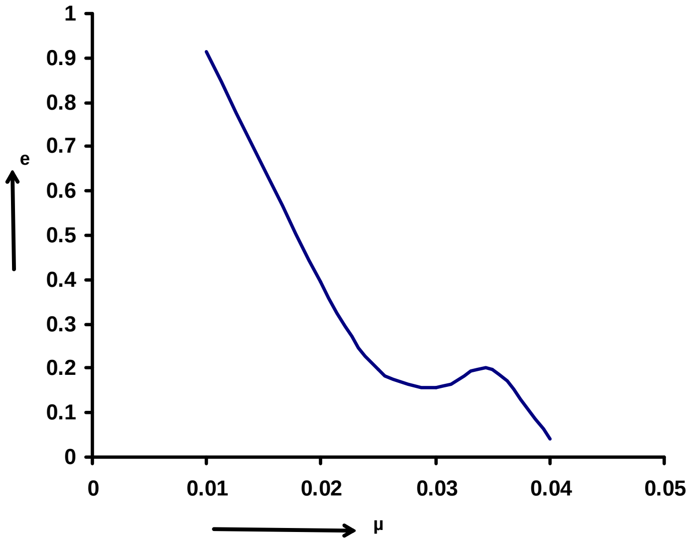

Figure 1: Transition curve $A_1 = A_2 = 0$, $\varepsilon = .0001$

5. Discussion and Conclusion

The stability of the triangular equilibrium points of the planar elliptical restricted three body problem under the influence of oblateness of the primaries has been investigated using an analytical method for determination of characteristic exponents in a system with periodic coefficients. The problem is studied under the assumption that the eccentricity of the orbit of the gravitating bodies is small. The oblateness of the more massive primary does not affect the motion of the smaller primary due to its larger mass, whereas it affects the motion of infinitesimal body. In this problem, we have determined the characteristic exponents upto the second order of approximation in 'e', we have drawn the transition curves separating stable and unstable region by equating to the values for periodic solutions. It is observed that the regions within the transition curves are unstable whereas, the regions outside these curves are stable.

We arrive at the conclusion that the determination of characteristic exponents up to the order of introduces a correction resulting in the decrease of range of stability.

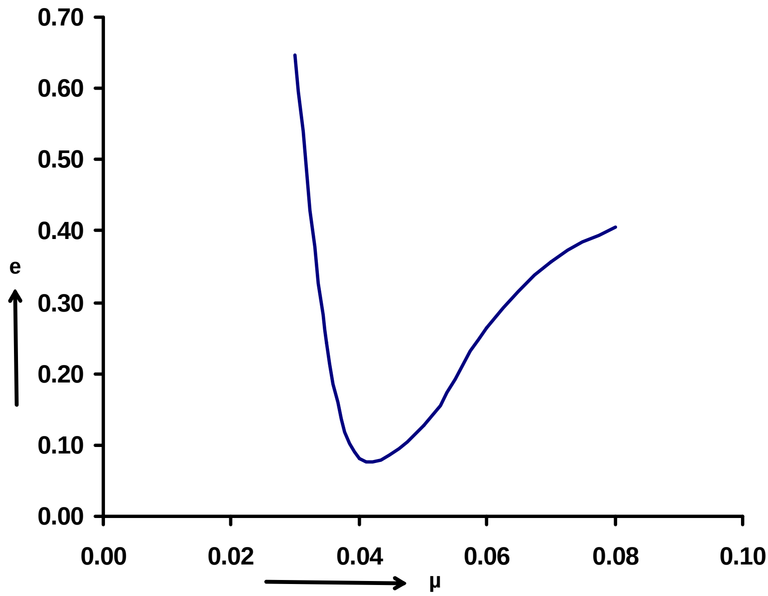

Figure 2: Transition curve $A_1 = A_2 = 0.001$, $\varepsilon = .0001$

References

[1] J.M.A. Danby, Stability of the triangular points in elliptic restricted problem of three bodies, *The Astronomical Journal*, **69** (1964), 165-172.

[2] W.H. Jefferys, Doubly syymmetric periodic orbits in the three dimensional restricted problem, *Astron. J.*, **70**, No. 6 (1965), 393-394.

[3] W.H. Jefferys, A new class of periodic solutions of the three dimesnional restricted problem, *Astron. J.*, **71**, No. 2 (1966), 99-102.

[4] J. Gyorgrey, On the non-linear stability of motion's around L_5 in the elliptic restricted problem of the three bodies, *Clestial Mech., Dyn. Astro.*, **36**, No. 3 (1985), 281-285.

[5] V. Kumar, R.K. Choudhary, Non linear stability of the triangular libration points for the photogravitational elliptic restricted problem of three bodies, *Celestial Mech. and Dyn. Astro.*, **48**, No. 4 (1990), 299-317.

[6] S.N. Khasan, Liberation Solutions to the photogravitational restricted three body problem, *Cosmic Research*, **34**, No. 2 (1966), 146-151.

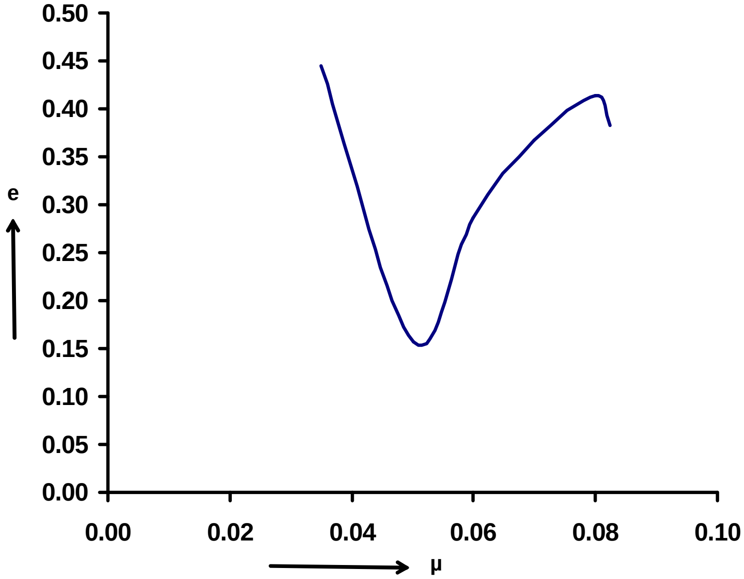

Figure 3: Transition curve $A_1 = A_2 = 0.001$, $\varepsilon = .0002$

[7] S.N. Khasan, Three dimensional periodic solutions to the photogravitational Hill problem, *Cosmic Research*, **34**, No. 5 (1990), 299-317.

[8] D. Selaru, C. Cucu-Dumitreacu, An analysis asymptotical solution in the three body problem, *Bom Astron. J.*, **4**, No. 1 (1994), 59-67.

[9] D. Selaru, C. Cucu-Dumitrescu, Infinitesimal orbit around Lagrange points in the elliptic restricted three body problem, *Celest. Mech. Dyn. Astron.*, **61**, No. 4 (1995), 333-346.

[10] G.E. Roberts, Linear stability of the elliptic Lagrangian triangle solution in the three body problem, *J. Differential Equation*, **182** (2002), 191-218.

[11] A.P. Markeev, One special case of parametric resonance in problem of celestial mechanics, *Astronomy Letter*, **31**, No. 5 (2005), 300-356.

[12] A.P. Markeev, *Libration points in celestial mechanics and cosmodynamics*, Nauk Moscow (1978).

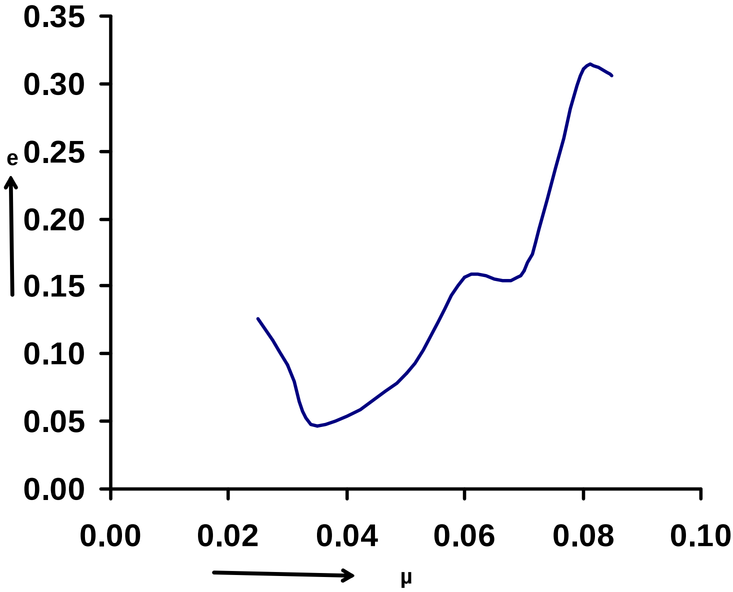

Figure 4: Transition curve $A_1 = A_2 = 0.002$, $\varepsilon = .0001$

[13] Sandoor Zsoft, Robert Balla, Ferenc Teger, Baliant Erdi, Short time Lyapunov, *Indicator in the Restricted Three Body Problem*, **79**, Celest. Mech and Dyn. Astr. (2001), 29-40.

[14] Conxita Pinyol, Ejection collision orbits with the more massive primary in the planar elliptic restricted three body problem, *Celest – Mech and Dyn Astro.*, **61** (1995), 315-331.

[15] P.P. Halan, N. Rana, The existence and stability of equilibrium points in the Robe's restricted three body problem, *Celest. Mech. and Dyn. Astro.*, **79** (2001), 145-155.

[16] Sandoor Zsoft, B. Erdi, Sympletic mapping for the Trojan-type motion in the elliptic restricted three body problem, *Celest. Mech. and Dyn. Astro.*, **86** (2003), 301-319.

[17] L. Floria, On an analytical solutions in the planar elliptic restricted three body problem, *Monografsem. Mat. Caracia de Galdeano*, **31** (2004), 135-144.

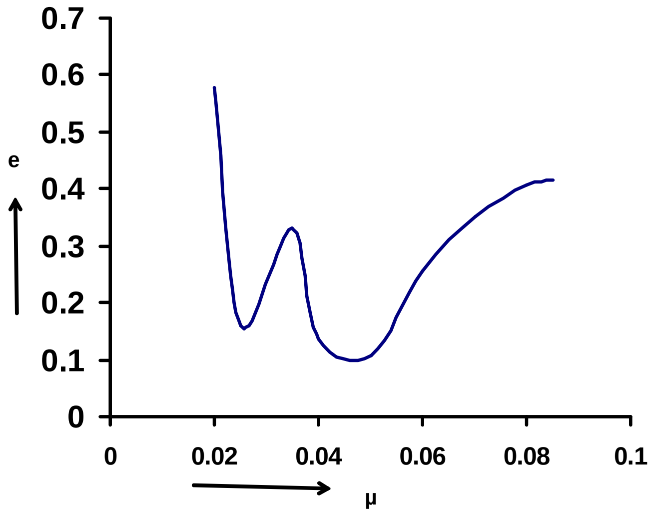

Figure 5: Transition curve $A_1 = 0.002$, $A_2 = 0.001$, $\varepsilon = .0003$

[18] J.F. Palacian, P. Yanguas, From the circular to the spatial elliptic restricted three body problem, *Celest. Mech. and Dyn. Astro.*, **95** (2006), 81-99.

[19] Alexander D. Bruno, Victor P. Varin, On families of periodic solution of the restricted three body problem, *Celest. Mech. and Dyn. Astro.*, **95** (2006), 27-54.

[20] M.K. Ammar, The effect of solar radiation pressure on the Lagrangian points in the elliptic restricted three body problems, *Astrophys Space Sci.*, **313** (2008), 393-408.

[21] V. Szebebely, *Theory of Orbits*, Academic, New York (1967).

[22] A. Bennett, Characteristic Exponent of the five equilibrium solutions in elliptically restricted problems, *Icarus*, **4** (1965), 177-187.

[23] A. Narayan, C. Ramesh, Stability of triangular points in the generalized restricted three body problem, *Journal of Modeling Ex-B*, France, **77** (2008).

[24] A. Narayan, C. Ramesh, Resonance stability of triangular equilibrium points of the planar elliptical restricted three body problem in which both

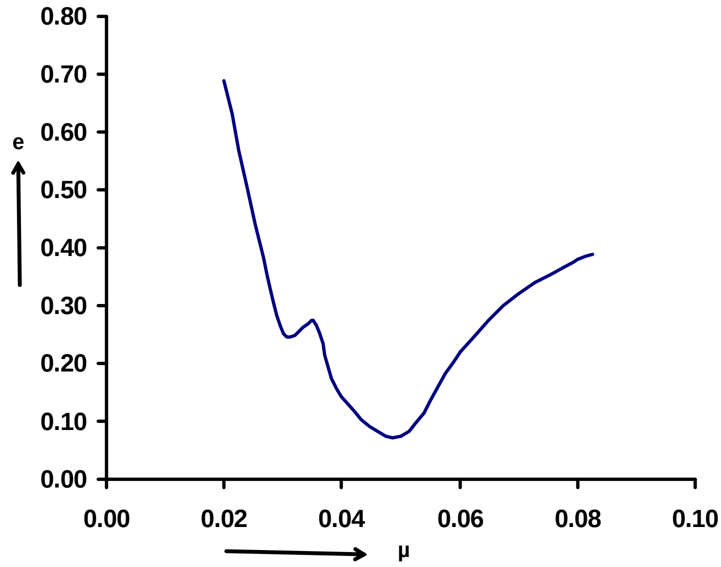

Figure 6: Transition curve $A_1 = 0.003$, $A_2 = 0.001$, $\varepsilon = .0002$

of the primaries are oblate spheroid, *Journal of Modeling Ex-B*, France, **78** (2008), 55.

[25] A. Narayan, C. Ramesh, Parametric resonance stability of triangular equilibrium points of the planar elliptical restricted three body problem around the primaries of oblate spheroid, *Journal of Modeling Ex-B*, France, **78** (2009).

Appendix

$$A_0F_0 = \frac{1}{N}\Bigg[-R\Big(22+6A_1-3\mu-18\mu^2+57\mu A_1+261\mu A_2\Big)+\in\left(-6A_1+\frac{219}{2}\mu A_1\right)\Bigg\}$$
$$+R^2\Bigg\{\left(8+36A_1-18\mu-26\mu A_1+\frac{87}{4}\mu A_2\right)+\in\left(-36A_1+\frac{185}{2}\mu A_1\right)$$
$$-\lambda_0^2\Bigg\{\left(24+56A_1-28\mu-14\mu^2+140\mu A_1+\frac{771}{2}\mu A_2\right)+\in(-72A_1+454\mu A_1)\Bigg\}$$

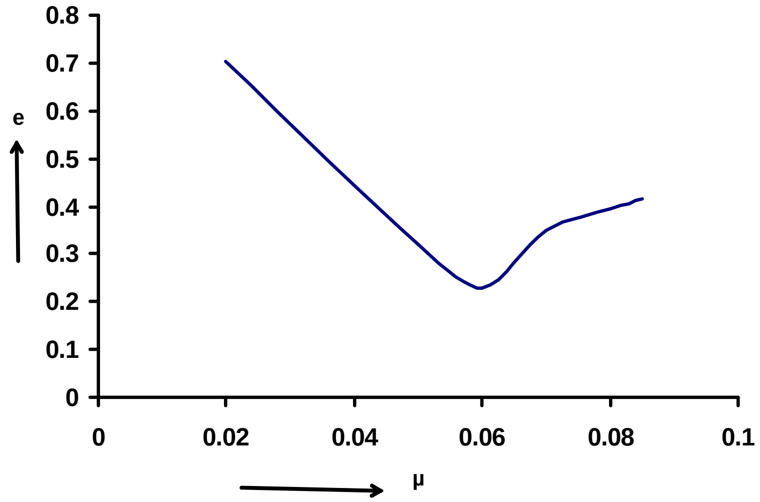

Figure 7: Transition curve $A_1 = 0.004,\ A_2 = 0.001,\ \varepsilon = .0002$

$$+ R\lambda_0^2\{-30 + 146A_1 - 73\mu - 53\mu A_1 - 16\mu^2 + 3567\mu A_2 + \in(-96A_1 - 104\mu A_1)\}$$
$$+ R^2\lambda_0^2\left\{\left(20 + 24A_1 - 12\mu - 12\mu A_1 - \frac{261}{2}\mu A_2\right) - \in(24A_1 - 54\mu A_1)\right\} - 8R^3\Big].$$

Again

$$A_1F_1 = \frac{8R\lambda_0^2}{N}\Big\{4\lambda_0^4 - 2\lambda_0^2 - 2 + 6A_1 - 3\mu + \mu A_1 - \mu^2 + \frac{261\mu A_2}{4} - \in\left(6A_1 - \frac{19}{2}\mu A_1\right)\Big\},$$

and

$$A_2F_2 = \frac{\lambda_0^2 R}{N}\Big[\left(-12 + 8A_1 - 4\mu + \mu^2 - 8\mu A_1 + \frac{87}{8}\mu A_2 - 4R\right)\lambda_0^2 + \Big(-12 - 16\mu + 5\mu^2$$
$$- 16A_1 - 12\mu A_1 - 87\mu A_2\Big) + \in(16A_1 - 16\mu A_1) - 4R\Big(1 - \mu + 2A_1 - \mu A_1$$
$$+ \frac{87}{8}\mu A_2 - \in\left(2A_1 - \frac{9}{2}\mu A_1\right)\Big)\Big].$$

Thus

$$A_1F_1 + A_2F_2 = \frac{\lambda_0^2 R}{N}\Bigg[-4R\left\{\left(9-\mu+2A_1-\mu A_1+\frac{87}{8}\mu A_2\right)-\in\left(2A_1-\frac{9}{2}\mu A_1\right)\right\} \lambda_0^2\left(-92-56A_1+28\mu+\mu^2+26\mu A_1-\frac{609}{2}\mu A_2-4R\right)+\Big(-28+32A_1-40\mu - 4\mu A_1-3\mu^2+435\mu A_2\Big)+\in(-32A_1+60\mu A_1)\Bigg].$$

Hence

$$Q^2-4R-16=\left(-12+\mu^2+8A_1-4\mu-8\mu A_1+\frac{87}{2}\mu A_2\right)-\in\left(2A_1-\frac{9}{2}\mu A_1\right),$$

$$A=-\left[\frac{\left(Q^2-4R-16\right)\lambda_0^2+A_0F_0+A_1F_1+A_2F_2}{4\left(Q^2-4Q-4R\right)\lambda_0^2+32R}\right]$$

N^r of

$$A=\left(Q^2-4R-16\right)\lambda_0^2+A_0F_0+A_1F_1+A_2F_2 =\left\{\left(-12+\mu^2+8A_1-4\mu-8\mu A_1+\frac{87}{2}\mu A_2\right)-\in\left(2A_1-\frac{9}{2}\mu A_1\right)\right\}\lambda_0^2 +A_0F_0+A_1F_1+A_2F_2,$$

D^r of

$$A=4\left(Q^2-4Q-4R\right)\lambda_0^2+32R =4\Big[\left(12+16A_1-8\mu-12\mu A_1+\mu^2+87\mu A_2\right)+4\in\mu A_1-4R\Big]\lambda_0^2+32R,$$

$$A=\frac{N^r\, of\, A}{D^r\, of\, A}$$

$$A=\frac{\left\{\left(-12+\mu^2+8A_1-4\mu-8\mu A_1+\frac{87}{2}\mu A_2\right)-\in\left(2A_1-\frac{9}{2}\mu A_1\right)\right\}}{4\Big[(12+16A_1-8\mu-12\mu A_1+\mu^2+87\mu A_2)+4\in\mu A_1-4R\Big]\lambda_0^2+32R} \times\left(\lambda_0^2+A_0F_0+A_1F_1+A_2F_2\right).$$