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ANALYSIS OF AN ECO-EPIDEMICAL MODEL WITH IVLEV FUNCTIONAL RESPONSE AND IMPULSIVE PERTURBATION

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Abstract: In this paper,an impulsive predator-prey model with disease in the predator is investigated for the purpose of integrated pest management. We consider an Ivlev-type functional response and assume that the periods of releasing natural enemies and spraying pesticide are different. We get the sufficient conditions for the local asymptotic stability of pest-eradication periodic solution by applying the Floquet theorem. We also get the sufficient condition for the permanence of the system, which means if the releasing amount of natural enemies and the spraying amount of pesticide satisfy the condition, then the prey and the predator will coexist.

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1. Introduction

Pest outbreak often cause serious ecological and economic problem, and the warfare between human and pests(such as locust, Aphis and cotton bollworm)has sustained for thousands of years. With the development of society and the progress of science and technology, man has adopted some advanced and modern weapons such as chemical pesticides, biological pesticides, remote sensing and measuring, and so on. Some brilliant achievements have been obtained. However, the warfare is not over, and will continue. A great deal of and a large

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variety of pesticides were used to control pests, because they can quickly kill a significant portion of a pest population and sometimes provide the only feasible method for preventing economic loss, which is called chemical control. Biological control, including microbial control with pathogens, can be important natural controls of some pests. Insects, like humans and other animals, can be infected by disease-causing organisms such as bacteria, viruses and fungi. Under appropriate conditions, such as high humidity or high pest abundance, these naturally occurring organisms may multiply to cause disease outbreaks or epizootics that can decimate an insect population. Biological control and chemical control are often used for pest control in the past. But each of them has advantages and disadvantages, see [1]-[4]. Therefore, many scholars put forward integrated pest management (IPM), see [5], [6]. IPM is an effective and environmentally sensitive approach to pest management that relies on a combination of common-sense practices, see [7]. It is proved in practice that IPM is more effective than using any method separately, see [8]. Paper [9] studied a model with disease in the predator, it consider Holling II functional response and assume that the periods of releasing natural enemies and spraying pesticide are different. It proved that there existed a stable pest-eradication periodic solution and the condition for the permanence of the system was also given. Paper [10] discussed the dynamics of a Beddington-type system with impulsive control strategy. Conditions for the system to be extinct were given and it proved that the system was permanent via the method of comparison involving multiple Liapunov functions. Paper [11] studied the complexities of the related food chain system based on paper [10]. It discussed the boundedness of solution the stability of mid-level predator eradication periodic solutions. And the numerical simulation is given. Paper [12] considered a special functional response and assumed that the periods of releasing natural enemies and spraying pesticide are different.

2. Model Formulation

Motivated by the above references, in this paper, we study the following model for integrated pest management.

$$\begin{cases} \left.\begin{array}{l} \dot{x}(t) = rx(t)(1 - \frac{x(t)}{K}) - (1 - e^{-ax})s(t) \\ \dot{s}(t) = p(1 - e^{-ax})s(t) - ds(t) - \beta s(t)i(t) \\ \dot{i}(t) = \beta s(t)i(t) - gi(t) \end{array}\right\}, & \begin{array}{l} t \neq nT, \\ t \neq (n+k-1)T, n \in N, \end{array} \\ \left.\begin{array}{l} x(t^+) = (1-\delta_1)x(t), s(t^+) = (1-\delta_2)s(t) \\ i(t^+) = (1-\delta_3)i(t) \end{array}\right\}, & t = (n+k-1)T, n \in N, \\ x(t^+) = x(t), \quad s(t^+) = s(t) + q, \quad i(t^+) = i(t) \ , & t = nT, n \in N. \end{cases} \tag{2.1}$$

Here $x = x(t)$ denote the total population density of the prey(pest), the total predator population is composed of two classes: susceptible predator $s(t)$, and infected predator $i(t)$, we assume that the infected predator can't catch prey. Here r is the intrinsic growth rate for the prey, $K(0 < K < 1)$ is the environmental carrying capacity, p is the converting coefficient, d, g are the death rates of susceptible and infected predator, respectively. The incidence is assumed to be the simple mass action incidence $\beta si, (1 - e^{-ax(t)})$ represents Ivlev-type functional response, T is the impulsive period, $\delta_i (0 \leq \delta_i < 1, i = 1, 2, 3)$ represent the fraction of prey, susceptible predator and infected predator which die due to the pesticide at $t = (n + k - 1)T$ respectively, q is the release amount of predator at $t = nT$. All the coefficient are positive constants.

3. Preliminary

Let $R_+ = [0, \infty), R^3_+ = \{x \in R^3 : x \geq 0\}, \Omega = \mathrm{int} R^3_+$. The map $f = (f_1, f_2, f_3)^T$ is defined by the right hand of the first three equations of system (2.1).

It is easy to prove the following lemmas:

Lemma 3.1. *Let $X(t) = (x(t), s(t), i(t))$ be a solution of system (2.1) with $X(0^+) \geq 0$, then $X(t) \geq 0$ for all $t \geq 0$.*

Lemma 3.2. *There exists $M > 0$ such that $x(t) \leq M, s(t) \leq M, i(t) \leq M$ for each solution $(x(t), s(t), i(t))$ of (2.1) with t large enough.*

Proof. Define $V(t) = px(t) + s(t) + i(t)$, when $t \neq (n + k - 1)T, t \neq nT$, choose $0 < l \le d$, ($d \le g$ obvious), we have

$$\begin{aligned} D^+V(t) + lV(t) &= p(r+l)x + (l-d)s - \frac{prx^2}{K} + (l-g)i \\ &\le p(r+l)x - \frac{prx^2}{K} \\ &\le \frac{pK(r+l)^2}{4r} \triangleq M_1. \end{aligned}$$

When $t = (n+k-1)T$, $V((n+k-1)T^+) \le V((n+k-1)T)$.
When $t = nT$, $V(nT^+) = V(nT) + q$.
By Lemma 2.2 of Bainov and Simeonov [13], for $t \ge 0$, we have

$$\begin{aligned} V(t) &\le V(0)e^{-lt} + \frac{M_1}{l}(1 - e^{-lt}) + q\frac{e^{-l(t-T)}}{1 - e^{lT}} + q\frac{e^{lT}}{e^{lT-1}} \\ &\to \frac{M_1}{l} + q\frac{e^{lT}}{e^{lT-1}}, \quad \text{as } t \to \infty. \end{aligned}$$

So $V(t)$ is uniformly bounded. By the definition of $V(t)$, there exists a constant $M > 0$ such that $x(t) \le M, s(t) \le M, i(t) \le M$ for t large enough. The proof is complete.

Next, we give some basic properties about the following subsystem of (2.1)

$$\begin{cases} \dot{s}(t) = -ds, & t \ne (n+k-1)T,\ t \ne nT, \\ \Delta s = -\delta_2 s, & t = (n+k-1)T, \\ \Delta s = q, & t = nT. \end{cases} \tag{3.1}$$

System (3.1) is a periodically forced linear system. It is easy to obtain that

$$\tilde{s}(t) = \begin{cases} \dfrac{q\exp(-d(t-(n-1)T))}{1-(1-\delta_2)\exp(-dT)}, & (n-1)T < t \le (n+k-1)T, \\ \dfrac{q(1-\delta_2)\exp(-d(t-(n-1)T))}{1-(1-\delta_2)\exp(-dT)}, & (n+k-1)T < t \le nT. \end{cases} \tag{3.2}$$

and $\tilde{s}(0^+) = \tilde{s}(nT^+) = \frac{q}{1-(1-\delta_2)\exp(-dT)}$, $\tilde{s}(kT^+) = \frac{q(1-\delta_2)\exp(-dkT)}{1-(1-\delta_2)\exp(-dT)}$ is a positive periodic solution of system (3.1). Since

$$s(t) = \begin{cases} (1-\delta_2)^{n-1}[s(0^+) - \dfrac{q}{1-(1-\delta_2)\exp(-dT)}]\exp(-dt) + \tilde{s}(t), \\ \qquad\qquad (n-1)T < t \le (n+k-1)T, \\ (1-\delta_2)^{n}[s(0^+) - \dfrac{q}{1-(1-\delta_2)\exp(-dT)}]\exp(-dt) + \tilde{s}(t), \\ \qquad\qquad (n+k-1)T < t \le nT. \end{cases}$$

is the solution of (3.1)with initial value $s(0^+) > 0$, We have

Lemma 3.3. *System (3.1) has a positive periodic solution $\tilde{s}(t)$ and for every solution $s(t)$ of (3.1) with positive initial condition, we have $s(t) \to \tilde{s}(t)$, as $t \to \infty$.*

So system (2.1) has a pest, infected predator eradication periodic solution $(0, \tilde{s}(t), 0)$.

4. Main Results

Theorem 4.1. *Let $(x(t), s(t), i(t))$ be any solution of (3.1), then $(0, \tilde{s}(t), 0)$ is locally asymptotically stable if*

$$T < \frac{aq(1-(1-\delta_2)\exp(-dT) - \delta_2 \exp(-dkT))}{rd(1-(1-\delta_2)\exp(-dT))} + \frac{1}{r} \ln \frac{1}{1-\delta_1},$$

and

$$T > \frac{\beta q(1-(1-\delta_2)\exp(-dT) - \delta_2 \exp(-dkT))}{gd(1-(1-\delta_2)\exp(-dT))} - \frac{1}{g} \ln \frac{1}{1-\delta_3}.$$

Proof. Let $x(t) = u(t), s(t) = v(t) + \tilde{s}(t), i(t) = w(t)$, then

$$\begin{pmatrix} u(t) \\ v(t) \\ w(t) \end{pmatrix} = \Phi(t) \begin{pmatrix} u(0) \\ v(0) \\ w(0) \end{pmatrix}, 0 \le t < T,$$

where $\Phi(t)$ satisfies

$$\frac{d\Phi}{dt} = \begin{pmatrix} r - a\tilde{s} & 0 & 0 \\ ap\tilde{s} & -d & -\beta\tilde{s} \\ 0 & 0 & \beta\tilde{s} - g \end{pmatrix} \Phi(t),$$

and $\Phi(0) = I$, the identity matrix.

The impulsive perturbations of (2.1) become

$$\begin{pmatrix} u((n+k-1)T^+) \\ v((n+k-1)T^+) \\ w((n+k-1)T^+) \end{pmatrix} = \begin{pmatrix} 1-\delta_1 & 0 & 0 \\ 0 & 1-\delta_2 & 0 \\ 0 & 0 & 1-\delta_2 \end{pmatrix} \begin{pmatrix} u((n+k-1)T) \\ v((n+k-1)T \\ w((n+k-1)T \end{pmatrix}$$

and

$$\begin{pmatrix} u(nT^+) \\ v(nT)^+ \\ w(nT)^+ \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u(nT) \\ v(nT) \\ w(nT) \end{pmatrix}.$$

The stability of the periodic solution $(0, \tilde{s}(t), 0)$is determined by the eigenvalues of

$$M = \begin{pmatrix} 1-\delta_1 & 0 & 0 \\ 0 & 1-\delta_2 & 0 \\ 0 & 0 & 1-\delta_2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Phi(T),$$

where $\mu_1 = (1-\delta_1)\exp(\int_0^T (r - a\tilde{s})dt)$, $\mu_2 = (1-\delta_2)\exp(-dT) < 1$, $\mu_3 = (1-\delta_3)\exp(\int_0^T (\beta\tilde{s} - g)dt)$. From the conditions of Theorem 4.1, we know that $|\mu_1| < 1, |\mu_3| < 1$, According to Floquet theory, $(0, \tilde{s}(t), 0)$ is locally stable The proof is complete.

Theorem 4.2. *System (2.1) is permanent if*

$$T > \frac{aq(1-(1-\delta_2)\exp(-dT) - \delta_2\exp(-dkT))}{rd(1-(1-\delta_2)\exp(-dT))} + \frac{1}{r}\ln\frac{1}{1-\delta_1},$$

and

$$T < \frac{\beta q(1-(1-\delta_2)\exp(-dT) - \delta_2\exp(-dkT))}{gd(1-(1-\delta_2)\exp(-dT))} - \frac{1}{g}\ln\frac{1}{1-\delta_3}.$$

Proof. Suppose $X(t) = (x(t), s(t), i(t))$is a solution of (2.1)with $X(0) > 0$.

Firstly, By Lemma 3.2, we may assume that there exists $M > 0$, such that $x(t) \le M, s(t) \le M, i(t) \le M$ for t large enough and $M > \frac{r}{a}$. From (2.1), we get

$$\frac{ds}{dt} \ge -(d + \beta M)s.$$

Consider the impulsive differential equation

$$\begin{cases} \dot{y}(t) = -(d+\beta M)y, & t \ne (n+k-1)T, t \ne nT, \\ \Delta y = -\delta_2 y, & t = (n+k-1)T, \\ \Delta y = q, & t = nT, \\ y(0^+) = s(0^+), \end{cases} \tag{4.1}$$

$$\tilde{y}(t) = \begin{cases} \dfrac{q\exp\{-(d+\beta M)[t-(n-1)T]\}}{1-(1-\delta_2)\exp[-(d+\beta M)T]}, \\ \qquad (n-1)T < t \le (n+k-1)T, \\ \dfrac{q(1-\delta_2)\exp\{-(d+\beta M)[t-(n-1)T]\}}{1-(1-\delta_2)\exp[-(d+\beta M)T]}, \\ \qquad (n+k-1)T < t \le nT, \end{cases} \tag{4.2}$$

and

$$\tilde{y}(0^+) = \tilde{y}(nT^+) = \frac{q}{1-(1-\delta_2)\exp[-(d+\beta M)T]},$$

$$\tilde{y}(kT^+) = \frac{q(1-\delta_2)\exp[-(d+\beta M)kT]}{1-(1-\delta_2)\exp[-(d+\beta M)T]}$$

is a positive periodic solution of (4.1). By the same method of lemma 3.3, we have $y(t) \to \tilde{y}(t)$, as $t \to \infty$. By the comparison theorem of impulsive equation [13], we can conclude that for $\varepsilon > 0, \exists t_0 > 0$ such that $s(t) \geq y(t) > \tilde{y}(t) - \varepsilon$ hold for $t > t_0$. thus $s(t) \geq \frac{q(1-\delta_2)\exp[-(d+\beta M)T]}{1-(1-\delta_2)\exp[-(d+\beta M)T]} - \varepsilon \triangleq m_2$ for $t > t_0$.

Secondly, we will prove that there exists $m_1 > 0$ such that $x(t) \geq m_1$ for t large enough. we will do it in the following two steps

Step 1. we can choose $0 < m_3 < \frac{d}{pa}, \varepsilon_0 > 0$ be small enough such that

$$\begin{aligned}\eta \triangleq\ & (1-\delta_1)\exp\{rT - \frac{r}{k}m_3T - a\varepsilon_1 T \\ & -\frac{aq\{1-(1-\delta_2)\exp((-d+pam_3)T) - \delta_2\exp((-d+pam_3)kT)\}}{(d-pam_3)(1-(1-\delta_2))\exp((-d+pam_3)T))}\} > 1.\end{aligned}$$

We will prove $x(t) < m_3$ can't hold for all $t \geq 0$. Otherwise, we have

$$\begin{cases} \dot{s}(t) \leq (-d+pam_3)s, & t \neq (n+k-1)T, t \neq nT, \\ \Delta s = -\delta_2 s, & t = (n+k-1)T, \\ \Delta s = q, & t = nT. \end{cases} \tag{4.3}$$

Then $s(t) \leq z(t)$ and $z(t) \to \tilde{z}(t), ast \to \infty$, where $z(t)$is the solution of

$$\begin{cases} \dot{z}(t) = (-d+pam_3)z, & t \neq (n+k-1)T, t \neq nT, \\ \Delta z = -\delta_2 z, & t = (n+k-1)T, \\ \Delta z = q, & t = nT, \\ z(0^+) = s(0^+), \end{cases} \tag{4.4}$$

$$\tilde{z}(t) = \begin{cases} \dfrac{q\exp((-d+pam_3)(t-(n-1)T))}{1-(1-\delta_2)\exp((-d+pam_3)T)}, \\ \qquad\qquad (n-1)T < t \leq (n+k-1)T, \\ \dfrac{q(1-\delta_2)\exp\{(-d+pam_3)(t-(n-1)T)\}}{1-(1-\delta_2)\exp((-d+pam_3)T)}, \\ \qquad\qquad (n+k-1)T < t \leq nT. \end{cases}$$

So there exists a constant $T_0 > 0$ such that $s(t) \leq z(t) < \tilde{z}(t) + \varepsilon_0$. when $t > T_0$

$$\begin{cases} \dot{x}(t) \geq (r - \dfrac{rm_3}{k} - a(\tilde{z}(t)+\varepsilon_0))x(t), & t \neq (n+k-1)T, \\ \Delta x = -\delta_1 x(t), & t = (n+k-1)T. \end{cases} \tag{4.5}$$

Let $N_0 \in N$, and $(N_0+k-1)T \geq T_0$,Integrating (4.5) on $((n+k-1)T,(n+k)T], n \geq N_0$, then

$$\begin{aligned}
&x((n+k)T) \\
&\quad \geq x((n+k-1)T)(1-\delta_1)\exp\Big(\int_{(n+k-1)T}^{(n+k)T}(r-\frac{rm_3}{k}-a(\tilde{z}(t)+\varepsilon_0))dt\Big) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = x((n+k-1)T)\eta.
\end{aligned}$$

Therefore, $x((N_0+k-1)T) \geq x((N_0+k)T)\eta^n \to \infty$ as $n \to \infty$, which is a contradiction to $x(t) \leq M$. Hence, there exists $t_1 > 0$ such that $x(t_1) \geq m_3$.

Step 2. If $x(t) \geq m_3$ for all $t \geq t_1$, then our aim is achieved. Otherwise, let $t^* = \inf\limits_{t>t_1}\{x(t) < m_3\}$, there are two possible cases for t^*:

Case 1) $t^* = (n'+k-1)T, n' \in N$, Then $x(t) \geq m_3$ and

$$(1-\delta_1)m_3 \leq x(t^{*+}) = (1-\delta_1)x(t^*) < m_3, \quad \text{for} \quad t \in [t_1, t^*].$$

Choose n_2, n_3, such that $(n_2-1)T > \frac{\ln(\frac{\varepsilon_0}{M+q})}{pam_3-d}$ and

$$(1-\delta_1)^{n_2}\eta^{n_3}\exp(n_2\eta_1 T) > (1-\delta_1)^{n_2}\eta^{n_3}\exp[(n_2+1)\eta_1 T] > 1,$$

where $\eta_1 = r - \frac{rm_3}{k} - aM < 0$.

Let $\bar{T} = n_2T + n_3T$, we claim that there must be a $t_2 \in (t^*, t^*+\bar{T}]$ such that $x(t_2) > m_3$.

Otherwise, consider (4.4) with $z(t^{*+}) = s(t^{*+})$, we have

$$z(t) = \begin{cases} (1-\delta_2)^{n-(n_1+1)}\Big(z(n_1T^+) - \dfrac{q}{1-(1-\delta_2)\exp((-d+pam_3)T)}\Big) \\ \times\exp((-d+pam_3)(t-n_1T)) + \tilde{z}(t), \\ \qquad\qquad\qquad\qquad\qquad\qquad (n-1)T < t \leq (n+k-1)T, \\ (1-\delta_2)^{n-n_1}\Big(z(n_1T^+) - \dfrac{q}{1-(1-\delta_2)\exp((-d+pam_3)T)} \\ \times\exp((-d+pam_3)(t-n_1T)) + \tilde{z}(t), \\ \qquad\qquad\qquad\qquad\qquad\qquad (n+k-1)T < t \leq nT, \end{cases}$$

and $(n_1+1) \leq n \leq n_1+n_2+n_3+1$.

Then if $n_1T + (n_2-1)T \leq t \leq t^*+\bar{T}$, we have

$$|z(t) - \tilde{z}(t)| < (M+q)\exp[(-d+pam_3)(t-n_1T)] < \varepsilon_0,$$

and

$$s(t) \le z(t) < \tilde{z}(t) + \varepsilon_0,$$

which implies that (4.5) holds for $t^* + n_2T \le t \le t^* + \bar{T}$.

As the first step, we have

$$x(t^* + \bar{T}) \ge x(t^* + n_2T)\eta^{n_3}. \tag{4.6}$$

For system (2.1), when $t \in [t^*, t^* + n_2T]$,

$$\begin{cases} \dot{x}(t) \ge (r - \frac{rm_3}{k} - aM)x(t), & t \ne (n+k-1)T, \\ \Delta x = -\delta_1 x(t), & t = (n+k-1)T. \end{cases} \tag{4.7}$$

Integrating (4.7) on $[t^*, t^* + n_2T]$, we have

$$x(t^* + n_2T) \ge m_3(1-\delta_1)^{n_2} \exp(n_2\eta_1 T). \tag{4.8}$$

Therefore, $x(t^* + \bar{T}) \ge m_3(1-\delta_1)^{n_2} \exp(n_2\eta_1 T)\eta^{n_3} > m_3$, which is a contradiction.

Let $\tilde{t} = \inf\limits_{t>t^*}\{x(t) > m_3\}$, and then $x(t) \le m_3$ and $x(\tilde{t}) = m_3$ for $t \in (t^*, \tilde{t})$. For $t \in (t^*, \tilde{t})$, we have

$$x(t) \ge m_3(1-\delta_1)^{n_2+n_3} \exp[(n_2+n_3)\eta_1 T] \triangleq m_1^{'}.$$

So $x(t) \ge m_1^{'}$.

For $t > \tilde{t}$, the same arguments can be continued since $x(\tilde{t}) \ge m_3$.

Case 2. $t^* \ne (n+k-1)T, n \in N$, Then $x(t) \ge m_3$ and $x(t^*) = m_3$, for $t \in [t_1, t^*)$.

Let $t^* \in ((n_1^{'} + k - 1)T, (n_1^{'} + k)T), n_1^{'} \in N$, there are two possible cases for $t \in (t^*, (n_1^{'} + k)T)$:

Case 2a. If $x(t) \le m_3$ for all $t \in (t^*, (n_1^{'} + k)T)$, similar to Case 1), we can prove there must be a $t_2^{'} \in [(n_1^{'} + k)T, (n_1^{'} + k)T + \bar{T}]$ such that $x(t_2^{'}) > m_3$. Here we omit it.

Let $\bar{t} = \inf\limits_{t>t^*}\{x(t) > m_3\}$, then $x(t) \le m_3$ and $x(\bar{t}) = m_3$ for $t \in (t^*, \bar{t})$. For $t \in (t^*, \bar{t})$, we have

$$x(t) \ge m_3(1-\delta_1)^{n_2+n_3} \exp[(n_2+n_3+1)\eta_1 T] \triangleq m_1.$$

So $x(t) \ge m_1$ for $t \in (t^*, \bar{t})$. For $t > \bar{t}$, the same arguments can be continued since $x(\bar{t}) \ge m_3$.

Case 2b. There exists a $t \in (t^*, (n_1^{'} + k)T)$ such that $x(t) > m_3$.

Let $\bar{t} = \inf_{t>t^*} \{x(t) > m_3\}$, then $x(t) \le m_3$ and $x(\bar{t}) = m_3$, for $t \in (t^*, \bar{t})$. for $t \in (t^*, \bar{t})$, (4.7)holds true. Thus $x(t) \ge x(t^*)\exp(\eta_1(t - t^*)) \ge m_3 \exp(\eta_1 T) > m_1$. for $t > \bar{t}$, the same arguments can be continued since $x(\bar{t}) \ge m_3$.

So $x(t) \ge m_1$ for all $t \ge t_1$.

Finally, we will prove there exists $m_5 > 0$ such that $i(t) \ge m_5$ for t large enough. we will do it in the following two steps:

1) From the condition of the theorem, choose $m_4, \varepsilon_1 > 0$ be small enough such that

$$\zeta \triangleq (1-\delta_3)\times \exp\{\frac{\beta q(1-(1-\delta_2)\exp((-d-\beta m_4)T) - \delta_2 \exp((-d-\beta m_4)kT))}{(d+\beta m_4)(1-(1-\delta_2)\exp((-d-\beta m_4)T))} - \beta\varepsilon_1 T - gT\} > 1.$$

We will prove $i(t) < m_4$ can't hold for all $t \ge 0$. Otherwise

$$\begin{cases} \dot{s}(t) \ge -(d+\beta m_4)s, & t \ne (n+k-1)T, t \ne nT, \\ \Delta s = -\delta_2 s, & t = (n+k-1)T, \\ \Delta s = q, & t = nT. \end{cases}$$

So we have $s(t) \ge \phi(t)$ and for $t \to \infty$, we have $\phi(t) \to \tilde{\phi}(t)$, where $\phi(t)$ is the solution of

$$\begin{cases} \dot{\phi}(t) = -(d+\beta m_4)\phi, & t \ne (n+k-1)T, t \ne nT, \\ \Delta \phi = -\delta_2 \phi, & t = (n+k-1)T, \\ \Delta \phi = q, & t = nT, \\ \phi(0^+) = s(0^+). \end{cases}$$

$$\tilde{\phi}(t) = \begin{cases} \dfrac{q\exp((-d-\beta m_4)(t-(n-1)T))}{1-(1-\delta_2)\exp((-d-\beta m_4)T)}, & (n-1)T < t \le (n+k-1)T, \\ \dfrac{q(1-\delta_2)\exp((-d-\beta m_4)(t-(n-1)T))}{1-(1-\delta_2)\exp((-d-\beta m_4)T)}, & (n+k-1)T < t \le nT. \end{cases}$$

Therefore, there exists a $T_1 > 0$ such that $s(t) \ge \phi(t) > \tilde{\phi}(t) - \varepsilon_1$ and for $t > T_1$, we have

$$\begin{cases} \dot{i}(t) \ge (-g + \beta(\tilde{\phi}(t) - \varepsilon_1))i, & t \ne (n+k-1)T, \\ \Delta i = -\delta_3 i, & t = (n+k-1)T. \end{cases}$$

2) Similar to the method of Step 2, we can obtain that exist $t_3 > 0$ such that $i(t_3) \geq m_4$, and $i(t) \geq m_5$ for all $t \geq t_3$.

Thus, when t is large enough, we have $m \leq x(t), s(t), i(t) \leq M$, where $m = \min\{m_1, m_2, m_5\}$, so the system is permanent. The proof is complete.

5. Concluding Remarks

In this paper, an impulsive predator-prey model with disease in the prey is investigated for the purpose of integrated pest control. By lemma 3.2, we get the boundedness of the system (2.1) with t large enough, and by lemma 3.3, we get a pest-eradication periodic solution.In Theorem 4.1, we get the sufficient conditions of local asymptotic stability of pest-eradication periodic solution by applying the Floquet theorem and comparison theorem of impulsive differential equation when $T < T_1$ and $T > T_2$, where

$$T_1 = \frac{aq(1-(1-\delta_2)\exp(-dT)-\delta_2\exp(-dkT))}{rd(1-(1-\delta_2)\exp(-dT))} + \frac{1}{r}\ln\frac{1}{1-\delta_1},$$

$$T_2 = \frac{\beta q(1-(1-\delta_2)\exp(-dT)-\delta_2\exp(-dkT))}{gd(1-(1-\delta_2)\exp(-dT))} - \frac{1}{g}\ln\frac{1}{1-\delta_3}.$$

In Theorem 4.2, we also get the sufficient condition for the permanence of the system when $T > T_1$ and $T < T_2$. Therefore, our mathematical results present a more prior strategy for pest management.

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