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PARAMETER ESTIMATION OF THE POISSON SHOCK MODEL USING MASKED DATA

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Abstract: If masking happens, we can only observe the system life time and the set of components that containing the real culprit for the failure. Estimation of the parameters included in the lifetime distribution of the individual components in a Poisson shock model is introduced in this paper by using masked system life data. We deduce the maximum likelihood and Bayes estimation of these parameters, as well as the survival function of each component. And a numerical simulation study is introduced to compare the influence of the masking level on the accuracy of the estimation.

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Key Words: Poisson shock model, maximum likelihood estimation, Bayes estimation, masked data

1. Introduction

In reliability analysis, it is interesting to estimate the unknown parameters included in the lifetime distribution of the individual components in a multi-component system. Ideally, the system's lifetime data not only includes the specific time of the system, but also includes which leads to the failure, however, in real world, due to lack of diagnostic tools as well as the devastating failure , the exact cause of system failure might be unknown. Referring to such situations as being masked. Then the exact cause of the failure may be isolated to a subset of the system components.

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Under an assumption that the components are independent, Lin, Usher and Guess derived the MLE of the parameters in a three-component system using some extra conditions [2], and they also considered the Bayes point-estimates in a 2-component series system of exponential components using discrete-step priors [3]. Usher considered that the cause of system failure can be isolated to some subset of components and the likelihood, while presented for complete data, can be extended to censoring with various component life distributions [4], he later derived maximum likelihood estimations of the parameters in the case of a two-component series system under the assumption that the components have Weibull lifetime distribution, and illustrated the approach with a numerical example [5]. Sarhan had done a lot of work on masked data, he derived the maximum likelihood estimates of the parameters included in the case of a two-component and a three-component series system under the assumption that the lifetime distributions of the components are both Weibull [6]. He presented a new conjugate prior family called a conjugate convex tent family in Bayes procedure for estimating the unknown parameter indexed to some of one parameter exponential family distributions, and introduced a numerical simulation study by using the Monte Carlo method to verify the accuracy of the new method [7]. By using the new conjugate prior family, he considered the maximum likelihood and Bayes estimations of the reliability of a two-component series system when the components had linear failure rate [8], and a two-component parallel system as well [9]. He also introduced the maximum likelihood and Bayes estimation of component's reliability when the components have constant failure rates, and the Bayes estimation, which based on the respective percentage errors, was assumed that the component's reliability were independent random variables having piecewise linear prior distributions [10].

All the discussion is based on independent-component systems, here a Poisson shock model [1] proposed by Marshall and Olkin where the components are dependent is considered. And a numerical example is introduced to compare the maximum likelihood and Bayes estimation.

This paper is derived into four sections. Section 2 considers the maximum likelihood approach, Bayes estimation is discussed in Section 3, and we give two numerical examples in Section 4, Section 5 is conclusion.

2. Maximum Likelihood Estimations

We first introduce the famous multivariate exponential distribution [1] proposed by Marshall and Olkin.

The system consists of two nonidentical components subjected to a sequence of independent shocks occurring randomly in time as events in three Poisson processes with rates $\lambda_1, \lambda_2, and \lambda_{12}$, respectively. The first shock leads component 1 to fail with probability p_1, the second shock leads component 2 to fail with probability p_2, and the third shock leads component 1 to fail with probability p_{10}, probability p_{01}leads component 2 to fail, probability p_{11} leads both component 1 and 2 to fail at the same time, and probability p_{00} stops component 1 and 2 from failing. Obiviously, $p_{10} + p_{01} + p_{11} + p_{00} = 1$, and $p_1, p_2, p_{10}, p_{01}, p_{11}, p_{00}$are known, then the joint survival function of the system is

$$\begin{aligned} P(X_1 > x_1, X_2 > x_2) &= \exp\{-(\lambda_1 p_1 + \lambda_{12} p_{10})x_1 - (\lambda_2 p_2 + \lambda_2 p_{01})x_2 \\ &\quad -\lambda_3 p_{11} \max(x_1, x_2)\}. \end{aligned} \tag{1}$$

Assuming there are n such systems participating in the life test. The test will be terminated when all the systems are failed. For each system i, the observed data t_i is the lifetime of the system, $i = 1, 2 \cdots, n$.

For a given system on the test, the observed failure reasons may be (1)component 1 fails while component 2 doesn't, (2) component 2 fails while component 1 doesn't, (3)both component 1 and 2 fails, and(4)we can't find the exact culprit for the failure, for example, the system burst, then we say the failure reason is masked, and we have masked data, we can only attribute the failure reason to a subset of the system components$1, 2$. Now, there are n systems participating in the life test, and the observed failure reason may be one of the four above.

We can see that either the first shock or the third shock destroys component 1 at time t when the observed failure reason is (1). Then, we have the following lemma:

Lemma 3.1. *Assume the system fails at time t, and we do observe that it is the first shock destroy component 1, then the probability of the case happening is*

$$P_{11} = p_1 \lambda_1 e^{-(\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 (1 - p_{00}))t}. \tag{2}$$

Proof. When we do observe that it is the first shock destroying component 1, that is to say, beyond time t, the system survives the $i-1$ first shocks , j second shocks and k third shocks , and the system fails when the first shock destroys component 1 at time t, then the probability of the case happens is

$$
\begin{aligned}
P_{11} &= \sum_{i=1}^{\infty} \frac{(\lambda_1 t)^{i-1}}{(i-1)!} e^{-\lambda_1 t} (1-p_1)^{i-1} p_1 \lambda_1 \cdot \sum_{j=0}^{\infty} \frac{(\lambda_2 t)^j}{(j)!} e^{-\lambda_2 t} (1-p_2)^j \\
&\quad \cdot \sum_{k=0}^{\infty} \frac{(\lambda_{12} t)^k}{(k)!} e^{-\lambda_3 t} p_{00}^k \\
&= p_1 \lambda_1 e^{-(\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 (1-p_{00}))t}. \qquad (3)
\end{aligned}
$$

□

The same as lemma 3. 1, if we do observe that it is the third shock that destroys component 1 at time t, then the probability of the case happens is

$$P_{31} = p_{10} \lambda_3 e^{-(\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 (1-p_{00}))t}. \qquad (4)$$

We can see that either the second shock or the third shock destroys component 2 at time t when the observed failure reason is (2). Similarly, if we do observe that it is the second shock destroys component 2 at time t, then the probability of the case happens is

$$P_{22} = p_2 \lambda_2 e^{-(\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 (1-p_{00}))t}. \qquad (5)$$

If we do observe that it is the third shock destroys component 2 at time t, then the probability of the case happens is

$$P_{32} = p_{01} \lambda_{12} e^{-(\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 (1-p_{00}))t}. \qquad (6)$$

If the observed failure reason is (3), there is only one case that the third shock destroys both component 1 and 2 at time t, then the probability is

$$P_3 = p_{11} \lambda_3 e^{-(\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 (1-p_{00}))t}. \qquad (7)$$

If the failure reason is masked, for example, an explosion takes place, then we can't find the exact culprit, the failure reason may be one of the reasons analyzed above. In this case, the probability is

$$
\begin{aligned}
P_m &= P_{11} + P_{31} + P_{22} + P_{32} + P_3 \\
&= p_1 \lambda_1 e^{-(\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 (1-p_{00}))t} + p_{10} \lambda_3 e^{-(\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 (1-p_{00}))t} \\
&\quad + p_2 \lambda_2 e^{-(\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 (1-p_{00}))t} + p_{01} \lambda_3 e^{-(\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 (1-p_{00}))t} \\
&\quad + p_{11} \lambda_3 e^{-(\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 (1-p_{00}))t} \\
&= (p_1 \lambda_1 + p_{10} \lambda_{12} + p_2 \lambda_2 + p_{01} \lambda_3 + p_{11} \lambda_3) e^{-(\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 (1-p_{00}))t}. \qquad (8)
\end{aligned}
$$

Under the analysis above, we divide the n systems into five groups, and the failure data are given in Table 1.

It can be easily seen that $n_{11} + n_{31} + n_{22} + n_{32} + n_3 + n_m = n$.

Failed component	responsible shock	quantity of failed systems	observed lifetime
1	1	n_{11}	$X_{11}, X_{12}, \cdots, X_{1n_{11}}$
1	3	n_{31}	$X_{31}, X_{32}, \cdots, X_{3n_{31}}$
2	2	n_{22}	$Y_{21}, Y_{22}, \cdots, Y_{2n_{22}}$
2	3	n_{32}	$Y_{31}, Y_{32}, \cdots, Y_{3n_{32}}$
1, 2	3	n_3	$Z_1, Z_2, \cdots, Z_{n_3}$
masked	masked	n_m	$Z_{m1}, Z_{m2}, \cdots, Z_{mn_m}$

Table 1: Failure data of the five groups

Theorem 1. *The maximum likelihood estimation (MLE) of* $\lambda_1, \lambda_2, \lambda_{12}$ *is*

$$\hat{\lambda}_1 = \frac{n_{11}n}{p_1(n-n_m)T}, \hat{\lambda}_2 = \frac{n_{22}n}{p_2(n-n_m)T}, \hat{\lambda}_3 = \frac{(n_{31}+n_{32}+n_3)n}{(1-p_{00})(n-n_m)T}. \tag{9}$$

Proof. Under the analysis above, we can get the maximum likelihood function

$$\begin{aligned} L(data, \lambda_1, \lambda_2, \lambda_3) &= \prod_{i=1}^{n_{11}} p_1\lambda_1 e^{-(\lambda_1 p_1+\lambda_2 p_2+\lambda_3(1-p_{00}))x_{1i}} \\ &\cdot \prod_{i=1}^{n_{31}} p_{10}\lambda_3 e^{-(\lambda_1 p_1+\lambda_2 p_2+\lambda_3(1-p_{00}))x_{3i}} \\ &\cdot \prod_{i=1}^{n_{22}} p_2\lambda_2 e^{-(\lambda_1 p_1+\lambda_2 p_2+\lambda_3(1-p_{00}))y_{2i}} \cdot \prod_{i=1}^{n_{32}} p_{01}\lambda_3 e^{-(\lambda_1 p_1+\lambda_2 p_2+\lambda_3(1-p_{00}))y_{3i}} \\ &\cdot \prod_{i=1}^{n_3} p_{11}\lambda_3 e^{-(\lambda_1 p_1+\lambda_2 p_2+\lambda_3(1-p_{00}))z_i} \\ &\cdot \prod_{i=1}^{n_m} (p_1\lambda_1 + p_{10}\lambda_3 + p_2\lambda_2 + p_{01}\lambda_3 + p_{00}\lambda_3) e^{-(\lambda_1 p_1+\lambda_2 p_2+\lambda_3(1-p_{00}))z_{mi}} \\ &= p_1^{n_{11}} p_2^{n_{22}} p_{10}^{n_{31}} p_{01}^{n_{32}} p_{11}^{n_3} \lambda_1^{n_{11}} \lambda_2^{n_{22}} \lambda_3^{n_{31}+n_{32}+n_3} \\ &\cdot (p_1\lambda_1 + p_{10}\lambda_3 + p_2\lambda_2 + p_{01}\lambda_3 + p_{00}\lambda_3)^{n_3} e^{-(p_1\lambda_1+p_2\lambda_2+(1-p_{00})\lambda_3)T}, \end{aligned} \tag{10}$$

and $T = \sum_{i=1}^{n} t_i$.

So, the log-likelihood function is

$$lnL(data, \lambda_1, \lambda_2, \lambda_3) = n_{11}lnp_1 + n_{22}lnp_2 + n_{31}lnp_{10} + n_{32}lnp_{01} + n_3lnp_{11}$$

$$+n_{11}ln\lambda_1 + n_{22}ln\lambda_2 + (n_{31} + n_{32} + n_3)ln\lambda_3$$
$$+n_3ln(p_1\lambda_1 + p_{10}\lambda_3 + p_2\lambda_2 + p_{01}\lambda_3 + p_{00}\lambda_3)$$
$$-(p_1\lambda_1 + p_2\lambda_2 + (1 - p_{00})\lambda_3)T, \tag{11}$$

Let $\frac{\partial lnL(data,\lambda_1,\lambda_2,\lambda_3)}{\partial\lambda_j} = 0, j = 1, 2, 3$, we can get the result. □

When $p_1 = p_2 = p_{11} = 1$, the joint distribution function of the two components is the bivariate exponential distribution, and the the survival functions of the two components are

$$\bar{F}_j = e^{(-\lambda_j+\lambda_3)t}, \quad j = 1, 2. \tag{12}$$

Using theorem 3. 1, we obtain the MLE of the the survival functions at any given time t_0.

$$\hat{\bar{F}}_1 = e^{(-\hat{\lambda}_1+\hat{\lambda}_3)t_0}, \hat{\bar{F}}_2 = e^{(-\hat{\lambda}_2+\hat{\lambda}_3)t_0}. \tag{13}$$

3. Bayes Approach

In this section we consider the Bayes estimation of $\lambda_j, j = 1, 2, 3$, Bayes posterior hazard, and the Bayes estimation of $\bar{F}_j(t), j = 1, 2$ at t_0 when $p_1 = p_2 = p_{11} = 1$, meanwhile $n_{31} = n_{32} = 0$. Firstly, we give two assumptions:

Assumption 1. The parameters $\lambda_1, \lambda_2, \lambda_{12}$ are independent random variables with prior Gamma distributions. And the prior probability density function of λ_j is

$$g_j(\lambda_j) = \frac{\beta_j^{\alpha_j}}{\Gamma(\alpha_j)}\lambda_j^{\alpha_j-1}e^{-\beta_j\lambda_j}, \alpha_j > 0, \beta_j > 0, \quad j = 1, 2, 3. \tag{14}$$

Assumption 2. $\tilde{\lambda}_j$ is the Bayes estimation of λ_j, and the loss function is given as:

$$L(\tilde{\lambda}_j, \lambda_j) = k(\tilde{\lambda}_j - \lambda_j)^2, \quad j = 1, 2, 3. \tag{15}$$

Let

$$h(p, j) =(T + \beta_1)^{n_{11}+\alpha_1}(T + \beta_2)^{n_{22}+\alpha_2}(T + \beta_3)^{n_3+n_m+\alpha_3}$$
$$\cdot [\sum_{k=0}^{n_m}\sum_{i=0}^{k} C_{n_m}^k C_k^i \frac{\Gamma(i + n_{11} + \alpha_1 + p\delta_{1j})}{(T + \beta_1)^{i+p\delta_{1j}}}$$

$$\cdot \frac{\Gamma(k-i+n_{22}+\alpha_2+p\delta_{2j})}{(T+\beta_2)^{k-i+p\delta_{2j}}} \cdot \frac{\Gamma(n_m-k+n_3+\alpha_3+p\delta_{3j})}{(T+\beta_3)^{-k+p\delta_{3j}}}], \quad (16)$$

where $\delta_{ij} = \{^{1.i=j}_{0,else}, j = 1, 2, 3, p = 1, 2, 3, \ldots$

Theorem 2. *Under the assumption 2, the Bayes estimation of* λ_j *is*

$$\tilde{\lambda}_j = h(1, j)/h(0, j), j = 1, 2, 3 \quad (17)$$

Proof. Using assumption 1, the joint prior pdf of $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ is

$$g(\lambda) = \prod_{j=1}^{3} \frac{\beta_j^{\alpha_j}}{\Gamma(\alpha_j)} \lambda_j^{\alpha_j-1} e^{-\beta_j\lambda_j}, \quad (18)$$

According to Bayes theory and (10), we can obtain the joint posterior of λ

$$\begin{aligned} g(\lambda|data) =& \frac{L(data, \lambda)g(\lambda)}{\int\int\int L(data, \lambda)g(\lambda)d\lambda_1 d\lambda_2 d\lambda_3} \\ =& \frac{1}{h(0, j)} \sum_{k=0}^{n_m}\sum_{i=0}^{k} C_{n_m}^k C_k^i \lambda_1^{i+n_{11}+\alpha_1-1} e^{-(T+\beta_1)\lambda_1} \lambda_2^{k-i+n_{22}+\alpha_2-1} \\ & \cdot e^{-(T+\beta_2)\lambda_2} \lambda_3^{-k+n_m+n_3+\alpha_3-1} e^{-(T+\beta_3)\lambda_3}, \end{aligned} \quad (19)$$

under the Assumption 2, the Bayes estimation of λ_j is

$$\tilde{\lambda}_j = \int_0^\infty \int_0^\infty \int_0^\infty \lambda_j g(\lambda|data) d\lambda_1 d\lambda_2 d\lambda_3, \quad (20)$$

Bring (19) into (20), calculating $(\lambda_1 + \lambda_2 + \lambda_3)^{n_m}$, and using the properties of Gamma distribution, we can get (17). □

Corollary 3. *The Bayes posterior hazard* R_{λ_j} *which is just the variance of the estimation 0f* λ_j *is*

$$R_{\lambda_j} = \frac{h(2, j)}{h(0, j)} - (\frac{h(1, j)}{h(0, j)})^2 \quad (21)$$

It is easy to deduce the result from Theorem 2, and we omit the proof.

Let

$$\begin{aligned} & \varphi(p, j) \\ =& \sum_{k=0}^{n_m}\sum_{i=0}^{k} C_{n_m}^k C_k^i \frac{\Gamma(i+n_{11}+\alpha_1)}{(T+\beta_1+pt_0\delta_{1j})^{i+n_{11}+\alpha_1}} \cdot \frac{\Gamma(k-i+n_{22}+\alpha_2)}{(T+\beta_2+pt_0\delta_{2j})^{k-i+n_{22}+\alpha_2}} \end{aligned}$$

i	t_i	Failure reason	i	t_i	Failure reason	i	t_i	Failure reason
1	0.742213	(2)	11	1.39275	(2)	21	0.214151	(4)
2	0.726588	(3)	12	0.273116	(1)	22	0.675581	(2)
3	0.126385	(2)	13	1.84216	(4)	23	0.21651	(3)
4	0.248553	(2)	14	0.7488	(1)	24	0.107996	(1)
5	1.74967	(4)	15	0.0326426	(2)	25	0.107996	(1)
6	2.0036	(2)	16	1.26628	(2)	26	0.254499	(1)
7	1.02518	(2)	17	0.286518	(1)	27	0.284961	(2)
8	0.21969	(4)	18	0.127114	(2)	28	0.213653	(3)
9	0.623118	(2)	19	0.511684	(4)	29	0.0642288	(2)
10	0.339241	(2)	20	0.337575	(2)	30	0.815889	(4)

Table 2: Simulated system-lifetime data

$$\cdot \frac{\Gamma(n_m - k + n_3 + \alpha_3)}{(T + \beta_3 + pt_0)^{n_m - k + n_3 + \alpha_3}}, \quad (22)$$

where $\delta_{ij} = \{^{1.i=j}_{0,else}, j = 1, 2, p = 1, 2, 3, \ldots$. Then we have:

Theorem 4. *The Bayes estimation of the components' survival function at time t_0 is*

$$\tilde{\bar{F}}_j = \frac{\varphi(1, j)}{h(0, j)}. \quad (23)$$

Proof. As in Theorem 2,

$$\tilde{\bar{F}}_j(t) = \int_0^\infty \int_0^\infty \int_0^\infty \bar{F}_j(t) g(\lambda|data) d\lambda_1 d\lambda_2 d\lambda_3, \quad j = 1, 2, \quad (24)$$

using (12)and (19), we can obtain the result. □

4. Numerical Study

In this section, we give some numerical examples to illuminate the result.

Example 1. Assuming there are 30 systems participating in the life test. Let $\lambda_1 = \frac{1}{3}, \lambda_2 = 1, \lambda_3 = \frac{1}{5}, \alpha_1 = 1.5, \alpha_2 = 2, \alpha_3 = 0.5, \beta_1 = 4.5, \beta_2 = 2, \beta_3 = 2.5$. The Simulated test data is in Table 2.

Parameter	Method				
	MLE		Bayes		
	Estimation	PE(%)	Estimation	PE(%)	hazard
λ_1	0.345579	3.6737	0.343424	3.0272	0.017466
λ_2	1.10585	10.585	1.09481	9.481	0.0583711
λ_3	0.207347	3.6735	0.206635	3.33175	0.0118623
$\bar{F}_1(0.5)$	0.758462	0.974765	0.762219	0.484249	0.0039642
$\bar{F}_1(0.5)$	0.518462	5.5028	0.526012	4.15443	0.00447238

Table 3: The MLE and Bayes estimation

masking level(%)	λ_1		λ_2		λ_3	
s	MLE	Bayes	MLE	Bayes	MLE	Bayes
0	0. 0034	0. 0023	0. 0356	0. 0284	0. 0014	0. 0011
20	0. 0039	0. 0025	0. 0418	0. 0330	0. 0015	0. 0013
30	0. 0043	0. 0027	0. 0437	0. 0347	0. 0026	0. 0018
50	0. 0104	0. 0052	0. 0693	0. 0513	0. 0077	0. 0051
70	0. 0387	0. 0131	0. 1613	0. 0909	0. 0298	0. 0126

Table 4: The MSE of $\lambda_j, j = 1, 2, 3$ with the increasing of masking level

Obviously, we can see $T = 18.0856, n = 30, n_{11} = 5, n_{22} = 16, n_3 = 3, n_m = 6$ from the table. Using (9), (13), (17), (21), (23), we can derive the MLE and Bayes estimations of $\lambda_j, (j = 1, 2, 3)$ and $\bar{F}_j(0.5), (j = 1, 2)$.

Defining $PE = \frac{|\bar{\lambda}_j - \lambda_j|}{\lambda_j} \times 100\%$, $\bar{\lambda}_j$ is the estimation of $\lambda_j, j = 1, 2, 3$, then we have Table 3.

Example 2. In order to consider the effect of masking level $s = n_m/n$ on the accuracy of estimation, we simulate 5000 data as in Table 1, and define $MSE_{\lambda_j} = \frac{\sum_{i=1}^{5000} (\bar{\lambda}_j^{(i)} - \lambda_j)^2}{5000}$, $\bar{\lambda}_j^{(i)}$ is the estimation of $\lambda_j, j = 1, 2, 3$ in the ith simulation. Then we obtain:

The MSE of $\lambda_j, j = 1, 2, 3$ with the increasing of masking level in Table 4.

From Table 4 , we can see that:

(1) The MSE of Bayes estimation of $\lambda_j, j = 1, 2, 3$ is smaller than MLE's at all masking levels;

(2) The MSE of either Bayes estimation or MLE of $\lambda_j, j = 1, 2, 3$ is monotonic increasing with the masking level s;

(3) The amount of increasing in the value of MSE of Bayes estimation is smaller than the MLE's;

From (9), we can see that when $s = 100\%$, MLE fails to give the estimation of λ_j, $j = 1, 2, 3$.

5. Conclusion

Here we discuss the parameter estimation of Poisson shock model in which components are dependent by using masked data, and provided evidence of their goodness of use. It would be interesting to extend the work of this paper to other dependent-component system.

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