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# ON FEKETE-SZEGÖ PROBLEMS FOR CERTAIN SUBCLASS OF ANALYTIC FUNCTIONS

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**Abstract:** In this present investigation, the authors obtain Fekete-Szegö inequality for certain normalized analytic function $f(z)$ defined on the open unit disk for which $\frac{z(D^{n,m}_{\lambda_1,\lambda_2}f(z))'}{D^{n,m}_{\lambda_1,\lambda_2}f(z)}$,$(n, m \in N_0, \lambda_2 \geq \lambda_1 \geq 0)$ lies in a region starlike with respect to 1 and is symmetric with respect to the real axis. Also certain applications of the main result for a class of functions defined by Hadamard product (convolutionn)are given. As a special case of the result, Fekete-Szegö inequality for a class of functions defined through fractional derivatives is obtained. the motivation of this paper is to give a generalization of the Fekete-Szegö inequalities obtained by Srivasatava and Mishra by making use of $D^{n,m}_{\lambda_1,\lambda_2}f(z)$ the generalized Ruscheweyh derivatives operator introduced by authors [3].

**AMS Subject Classification:** 30C45
**Key Words:** analytic function, starlike functions, subordination, Fekete-Szegö inequality, derivative operator

## 1. Introduction

Let $A$ denote the class of all analytic function $f$ of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \quad (z \in U := \{z \in C : |z| < 1\}). \tag{1}$$

and $S$ be the subclass of functions $f \in A$ which are univalent. Let $\phi(z)$ be an

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analytic function with positive real part on $A$ with $\phi(0) = 1, \phi'(0) > 0$ which maps the unit disk $U$ onto a region starlike with respect to 1 which is symmetric with respect to the real axis. Let $S^*(\phi)$ be the class of functions in $f \in S$ for which

$$\frac{zf'(z)}{f(z)} \prec \phi(z) \qquad (z \in U),$$

and $C(\phi)$ be the class of functions in $f \in S$ for which

$$1 + \frac{zf''(z)}{f'(z)} \prec \phi(z) \qquad (z \in U),$$

where $\prec$ denotes the subordination between analytic functions. These classes were introduced and studied by Ma and Minda [16]. They have obtained the Fekete-Szegö inequality for the functions in the class $C(\phi)$.Since $f \in C(\phi)$if and only if $zf'(z) \in S^*(\phi)$, we get the Fekete-Szegö inequality for functions in the class $S^*(\phi)$. For a brief history of the Fekete-Szegö problem for class of starlike convex, and close-to convex functions, see the recent paper by Srivastava [5].

In the present paper, we obtain the Fekete-Szegö inequality for functions in a more general class $M^{n,m}_{\lambda_1,\lambda_2}$ of functions which we define below. Also we give applications of our results to certain functions defined by Hadamard product (or convolution) and in particular we consider a class $M^{n,m}_{\lambda_1,\lambda_2}$ of functions defined by fractional derivatives. The motivation of this paper is to give a generalization of the Fekete-Szegö inequalities of Srivastava and Mishra [6].

**Definition 1.1.** Let $\phi(z)$ be a univalent starlike function with respect to 1 which maps the unit disk $U$ onto a region in the right half plane which is symmetric with respect to the real axis, $\phi(0) = 1$ and $\phi'(0) > 0$. A function $f \in A$ is in the class $M^{n,m}_{\lambda_1,\lambda_2}(\phi)$ if

$$\frac{z(D^{n,m}_{\lambda_1,\lambda_2} f(z))'}{D^{n,m}_{\lambda_1,\lambda_2} f(z)} \prec \phi(z) \tag{2}$$

where $m, n \in N_0$ and $D^{n,m}_{\lambda_1,\lambda_2}$ denote the derivative operator

**Definition 1.2.** (see [3]) Let the function $f$ be in the class $A$. For $m$,$n \in N_0 = N \cup \{0\}, \lambda_2 \geq \lambda_1 \geq 0$ we define the following differential operator

$$D^{n,m}_{\lambda_1,\lambda_2} f(z) = z + \sum_{k=n+1}^{\infty} \left[\frac{1 + (\lambda_1 + \lambda_2)(k-1)}{1 + \lambda_2(k-1)}\right]^m C(n,k) a_k z^k \tag{3}$$

.

To prove our main result, we need the following.

**Lemma 1.3.** (see [8]) *If $p_1(z) = 1 + c_1 z + c_2 z^2 + ....$ is an analytic function with positive real part in $U$, then*

$$|c_2 - \upsilon c_1^2| \leq \begin{cases} -4\upsilon + 2, & \text{if} \quad \upsilon \leq 0, \\ 2, & \text{if} \quad 0 \leq \upsilon \leq 1, \\ 4\upsilon + 2, & \text{if} \quad \upsilon \geq 1. \end{cases}$$

*When $\upsilon < 0$ or $\upsilon > 1$, the equality holds if and only if $p_1(z)$ is $\frac{1+z}{1-z}$ or one of its rotations. If $0 < \upsilon < 1$, then equality holds if and only if $p_1(z)$ is $\frac{1+z^2}{1-z^2}$ or one of its rotations. If $\upsilon = 0$, the equality holds if and only if*

$$p_1(z) = \left(\frac{1}{2} + \frac{1}{2}\gamma\right)\frac{1+z}{1-z} + \left(\frac{1}{2} - \frac{1}{2}\gamma\right)\frac{1-z}{1+z} \qquad (0 \leq \gamma \leq 1),$$

*or one of its rotations. If $\upsilon = 1$, the equality holds if and only if $p_1(z)$ is the reciprocal of one of the functions such that the equality holds in the case of $\upsilon = 0$. Also the above upper bound is sharp, it can be improved as follows when $0 < \upsilon < 1$:*

$$|c_2 - \upsilon c_1^2| + \upsilon |c_1|^2 \leq 2 \qquad (0 < \upsilon \leq \frac{1}{2}),$$

$$|c_2 - \upsilon c_1^2| + (1-\upsilon)|c_1|^2 \leq 2 \qquad (\frac{1}{2} < \upsilon \leq 1).$$

## 2. Fekete-Szegö Problem

Our first result is the following :

**Theorem 2.1.** *Let $\phi(z) = 1 + B_1 z + B_2 + .....$ If $f(z)$ is given by (1) belongs to $M^{n,m}_{\lambda_1,\lambda_2}\phi(z)$, then*

$$|a_3 - \mu a_2^2| \leq$$

$$\begin{cases} \dfrac{(1+\lambda_2)^m B_2}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m} - \dfrac{(1+\lambda_2)^m \mu B_1^2}{(n+1)^2(1+\lambda_1+\lambda_2)^{2m}} \\ + \dfrac{(1+\lambda_2)^m B_1^2}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m}, & \text{if} \quad \mu \leq \delta_1, \\ \dfrac{(1+\lambda_2)^m B_1}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m}, & \text{if } \delta_1 \leq \mu \leq \delta_2, \\ -\dfrac{(1+\lambda_2)^m B_2}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m} + \dfrac{(1+\lambda_2)^m \mu B_1^2}{(n+1)^2(1+\lambda_1+\lambda_2)^{2m}} \\ -\dfrac{(1+\lambda_2)^m B_1^2}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m}, & \text{if} \quad \mu \geq \delta_2, \end{cases}$$

*where*

$$\delta_1 := \frac{(n+1)^2(1+\lambda_1+\lambda_2)^{2m}\{(B_2-B_1)+B_1^2\}}{(n+2)(n+1)(1+\lambda_2)^m(1+2(\lambda_1+\lambda_2))^m},$$

$$\delta_2 := \frac{(n+1)^2(1+\lambda_1+\lambda_2)^{2m}\{(B_2+B_1)+B_1^2\}}{(n+2)(n+1)(1+\lambda_2)^m(1+2(\lambda_1+\lambda_2))^m B_1^2}$$

*the result is sharp.*

*Proof.* For $f \in M^{n,m}_{\lambda_1,\lambda_2}\phi(z)$, let

$$p(z) = \frac{z(D^{n,m}_{\lambda_1\lambda_2}f(z))'}{D^{n,m}_{\lambda_1\lambda_2}f(z)} = 1 + b_1 z + b_2 z^2 + .....$$

where

$$\left[\frac{1+\lambda_1+\lambda_2}{1+\lambda_2}\right]^m (n+1)a_2 = b_1$$

and

$$\left[\frac{1+(2(\lambda_1+\lambda_2))}{1+\lambda_2}\right]^m (n+2)(n+1)a_3 = \left[\frac{1+\lambda_1+\lambda_2}{1+\lambda_2}\right]^{2m} (n+1)^2 a_2^2 + b_2$$

Since $\phi(z)$ is univalent and $p \prec \phi$ the function

$$p_1(z) = \frac{1+\phi^{-1}(p(z))}{1-\phi^{-1}(p(z))} = 1 + c_1 z + c_2 z^2 + .....$$

is analytic and has a positive real part in $U$. Also we have

$$p(z) = \phi\left(\frac{p_1(z)-1}{p_1(z)-1}\right),$$

and also

$$\begin{aligned} 1 + b_1 z + b_2 z^2 + ..... &= \phi\left(\frac{c_1 z + c_2 z^2 + ...}{2 + c_1 z + c_2 z^2 + ...}\right) \\ &= \phi[\frac{1}{2}c_1 z + \frac{1}{2}(c_2 - \frac{1}{2}c_1^2)z^2 + ...] \\ &= 1 + B_1\frac{1}{2}C_1 Z + B_1\frac{1}{2}(c_2 - \frac{1}{2}c_1^2)z^2 + B_2\frac{1}{4}c_1^2 z^2 ... \end{aligned}$$

we obtain

$$b_1 = \frac{1}{2}B_1 c_1 \quad \textit{and} \quad b_2 = \frac{1}{2}B_1(c_2 - \frac{1}{2}c_1^2) + \frac{1}{4}B_2 c_1^2.$$

From the last equations we get

$$a_2 = \frac{B_1 c_1 (1+\lambda_2)^m}{2(1+\lambda_2+\lambda_1)^m (n+1)}$$

and

$$\begin{aligned} a_3 =& \frac{(\frac{1}{4}B_1^2 c_1^2 + \frac{1}{2}B_1(c_2 - \frac{1}{2}c_1^2) + \frac{1}{4}B_2 c_1^2)(1+\lambda_2)^m}{[1+2(\lambda_1+\lambda_2)]^m (n+2)(n+1)} \\ a_3 - \mu a_2^2 =& \frac{B_1(1+\lambda_2^m)}{2(1+2(\lambda_1+\lambda_2)^m (n+2)(n+1))}\{c_1 - c_1^2[\frac{1}{2} \\ & \left(1 - \frac{B_2}{B_1} + \right. \\ & \left.\left.\left.\frac{(n+2)(n+1)(1+\lambda_2^n)(1+2(\lambda_1+\lambda_2))^m \mu - (1+\lambda_2+\lambda_1)^{2m}(n+1)^2}{(1+\lambda_2+\lambda_1)^{2m}(n+1)^2} B_1\right)\right]\right\} \\ =& \frac{B_1(1+\lambda_2)^m}{2(1+2(\lambda_1+\lambda_2))^m (n+2)(n+1)}[c_2 - v c_1^2] \end{aligned}$$

where

$$v = \frac{1}{2}\left(1 - \frac{B_2}{B_1} + \frac{(n+2)(n+1)(1+\lambda_2^n)(1+2(\lambda_1+\lambda_2))^m \mu - (1+\lambda_2+\lambda_1)^{2m}(n+1)^2}{(1+\lambda_2+\lambda_1)^{2m}(n+1)^2} B_1\right).$$

If $\mu \leq \sigma_1$, then by applying Lemma 1.3 we get

$$|a_3 - \mu a_2^2| \leq \frac{(1+\lambda_2)^m B_2}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m}$$

$$-\frac{\mu B_1^2 (1+\lambda_2)^m}{(1+\lambda_1+\lambda_2)^{2m}(n+1)^2} + \frac{(1+\lambda_2)^m B_1^2}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m}.$$

Similarly, if $\mu \leq \sigma_2$ we get

$$|a_3 - \mu a_2^2| \leq -\frac{(1+\lambda_2)^m B_2}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m}$$

$$+\frac{\mu B_1^2 (1+\lambda_2)^m}{(1+\lambda_1+\lambda_2)^{2m}(n+1)^2} - \frac{(1+\lambda_2)^m B_1^2}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m}.$$

Finally, we see that

$$|a_3 - \mu a_2^2| = \frac{(1+\lambda_2)^m B_1}{2(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m}|c_2 - c_1^2[\frac{1}{2}(1 - \frac{B_2}{B_1} +$$

$$\frac{(n+2)(n+1)(1+\lambda_2^n)(1+2(\lambda_1+\lambda_2))^m\mu-(1+\lambda_2+\lambda_1)^{2m}(n+1)^2}{(1+\lambda_2+\lambda_1)^{2m}(n+1)^2}B_1)]|$$

and

$$\max|\frac{1}{2}(1-\frac{B_2}{B_1}$$
$$+\frac{(n+2)(n+1)(1+\lambda_2^n)(1+2(\lambda_1+\lambda_2))^m\mu-(1+\lambda_2+\lambda_1)^{2m}(n+1)^2}{(1+\lambda_2+\lambda_1)^{2m}(n+1)^2}B_1)]|$$
$$(\delta_1\leq\mu\leq\delta_2)$$

therefore using Lemma 1.3 we get

$$|a_3-\mu a_2^2|=\frac{(1+\lambda_2)^m B_1|c_1|}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m}$$
$$\leq\frac{(1+\lambda_2)^m B_1}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m}\quad(\delta_1\leq\mu\leq\delta_2).$$

## 3. Applications to Functions Defined by Fractional Derivatives

For two analytic functions $f(z)=z+\sum_{k=2}^{\infty}a_kz^k$ and $g(z)=z+\sum_{k=2}^{\infty}b_kz^k$, their convolution (or Hadamard product) is defined to be the function $(f*g)(z)$ given by $(f*g)(z)=f(z)*(z)=z+\sum_{k=2}^{\infty}a_kb_kz^k$. For fixed $g\in A$, let $M_{\lambda_1,\lambda_2}^{n,m,g}(\phi)$ be the class of functions $f\in A$ for which $(f*g)\in M_{\lambda_1,\lambda_2}^{n,m}(\phi)$.

**Definition 3.1.** (see [14]) Let $f$ be analytic in a simply connected region of the $z-plane$ containing the origin. the functional derivative of $f$ of order $\gamma$ is defined by

$$D_z^{\gamma}f(z)=\frac{1}{\Gamma(1-\gamma)}\frac{d}{dz}\int_0^z\frac{f(\zeta)}{(z-\zeta)^{\gamma}}d\zeta\qquad(0\leq\gamma<1).$$

Where the multiplicity of $(z-\zeta)^{\gamma}$ is removed by requiring that $log(z-\zeta)$ is real for $z-\zeta>0$. Using the above definition 3.1 and its known extensions involving fractional derivatives and fractional integrals, Owa and Srivastava [14] introduced the $\Omega^{\gamma}:A\rightarrow A$ defined by

$$\Omega^{\gamma}f(z)=\Gamma(2-\gamma)z^{\gamma}D_z^{\gamma}f(z).\qquad(\gamma\neq 2,3,4,5,...)$$

The class $M_\delta^{m,n,\gamma}(\phi)$ consists of functions $f \in A$ for which $\Omega^\gamma f \in M_\delta^{m,n}(\phi)$. Note that $M_\delta^{m,n,\gamma}(\phi)$ is the special case of the class $M_{\lambda_1,\lambda_2}^{m,n,g}(\phi)$ when

$$g(z) = z + \sum_{k=2}^{\infty} \frac{\Gamma(k+1)\Gamma(2-\gamma)}{\Gamma(k+1-\gamma)} z^k.$$

Let

$$g(z) = z + \sum_{k=2}^{\infty} g_k z^k \qquad (g_k > 0).$$

Since $D_{\lambda_1,\lambda_2}^{n,m} f(z) \in M_{\lambda_1,\lambda_2}^{n,m,g}(\phi)$ if and only if $D_{\lambda_1,\lambda_2}^{n,m} f(z) * g(z) \in M_{\lambda_1,\lambda_2}^{n,m}(\phi)$, we obtain the coefficient estimate for functions in the class $M_{\lambda_1,\lambda_2}^{n,m,g}(\phi)$, from the corresponding estimate for functions in the class $M_{\lambda_1,\lambda_2}^{n,m}(\phi)$. Applying Theorem 2.1 for the function $D_{\lambda_1,\lambda_2}^{n,m} f(z) * g(z) = z + [\frac{1+\lambda_1+\lambda_2}{1+\lambda_2}]^m (n+1) a_2 g_2 z^2 + ....$ we get the following Theorem 3.2 after an obvious change of the parameter $\mu$.

**Theorem 3.2.** *Let $g(z) = z + \sum_{n=2}^{\infty} g_n z^n, (g_n > 0)$ and let the function $\phi(z)$ be given by $\phi(z) = 1 + \sum_{k=1}^{\infty} B_k z^k$. If $D_{\lambda_1,\lambda_2}^{n,m} f(z)$ given by (3) belongs to $M_{\lambda_1,\lambda_2}^{n,m,g}$, then*

$$|a_3 - \mu a_2^2| \leq$$
$$\begin{cases} \frac{1}{g_3}[\frac{(1+\lambda_2)^m B_2}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m} \\ -\frac{(1+\lambda_2)^m \mu g_3 B_1^2}{(n+1)^2(1+\lambda_1+\lambda_2)^{2m} g_2^2} + \frac{(1+\lambda_2)^m B_1^2}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m}], & \text{if} \quad \mu \leq \delta_1, \\ \frac{1}{g_3}[\frac{(1+\lambda_2)^m B_1}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m}], & \text{if} \quad \delta_1 \leq \mu \leq \delta_2, \\ \frac{1}{g_3}[-\frac{(1+\lambda_2)^m B_2}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m} \\ +\frac{(1+\lambda_2)^m \mu g_3 B_1^2}{(n+1)^2(1+\lambda_1+\lambda_2)^{2m} g_2^2} - \frac{(1+\lambda_2)^m B_1^2}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m}] & \text{if} \quad \mu \geq \delta_2. \end{cases}$$

$$\delta_1 := \frac{g_2^2(n+1)^2(1+\lambda_1+\lambda_2)^{2m}\{(B_2-B_1)+B_1^2\}}{g_3(n+2)(n+1)(1+\lambda_2)^m(1+2(\lambda_1+\lambda_2))^m},$$

$$\delta_2 := \frac{g_2^2(n+1)^2(1+\lambda_1+\lambda_2)^{2m}\{(B_2+B_1)+B_1^2\}}{g_3(n+2)(n+1)(1+\lambda_2)^m(1+2(\lambda_1+\lambda_2))^m B_1^2}.$$

The result is sharp. Since

$$(\Omega^\gamma D_{\lambda_1,\lambda_2}^{n,m} f)(z) = z + \sum_{k=2}^{\infty} \frac{\Gamma(k+1)\Gamma(2-\gamma)}{\Gamma(k+1-\gamma)} \left[\frac{1+(\lambda_1+\lambda_2)(k-1)}{1+\lambda_2(k-1)}\right]^m C(n,k) z^k$$

we have

$$g_2 := \frac{\Gamma(3)\Gamma(2-\gamma)}{\Gamma(3-\gamma)} = \frac{2}{(2-\gamma)}$$

and

$$g_3 := \frac{\Gamma(4)\Gamma(3-\gamma)}{\Gamma(4-\gamma)} = \frac{6}{(2-\gamma)(3-\gamma)}$$

For $g_2$ and $g_3$ given by above inequalities, Theorem 3.2 reduces to the following:

**Corollary 3.3.** *Let $g(z) = z + \sum_{n=2}^{\infty} g_n z^n$, ($g_n > 0$) and let the function $\phi(z)$ be given by $\phi(z) = 1 + \sum_{k=1}^{\infty} B_k z^k$. If $D^{n,m}_{\lambda_1,\lambda_2} f(z)$ given by (3) belong to $D^{n,m,\gamma}_{\lambda_1,\lambda_2} f(\phi)$, then*

$$|a_3 - \mu a_2^2| \leq$$

$$\begin{cases} \frac{(2-\gamma)(3-\gamma)}{6}\Big[\frac{(1+\lambda_2)^m B_2}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m} \\ -\frac{3(2-\gamma)(1+\lambda_2)^m \mu B_1^2}{2(3-\gamma)(n+1)^2(1+\lambda_1+\lambda_2)^{2m}} \\ +\frac{(1+\lambda_2)^m B_1^2}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m}\Big], & \text{if} \quad \mu \leq \delta_1, \\ \frac{(2-\gamma)(3-\gamma)}{6}\Big[\frac{(1+\lambda_2)^m B_1}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m}\Big], & \text{if} \quad \delta_1 \leq \mu \leq \delta_2, \\ \frac{(2-\gamma)(3-\gamma)}{6}\Big[-\frac{(1+\lambda_2)^m B_2}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m} \\ +\frac{3(2-\gamma)(1+\lambda_2)^m \mu B_1^2}{2(3-\gamma)(n+1)^2(1+\lambda_1+\lambda_2)^{2m}} \\ -\frac{(1+\lambda_2)^m B_1^2}{(n+2)(n+1)(1+2(\lambda_1+\lambda_2))^m}\Big], & \text{if} \quad \mu \geq \delta_2, \end{cases}$$

$$\delta_1 := \frac{2(3-\gamma)(n+1)^2(1+\lambda_1+\lambda_2)^{2m}\{(B_2 - B_1) + B_1^2\}}{3(2-\gamma)(n+2)(n+1)(1+\lambda_2)^m(1+2(\lambda_1+\lambda_2))^m},$$

$$\delta_2 := \frac{2(3-\gamma)(n+1)^2(1+\lambda_1+\lambda_2)^{2m}\{(B_2 + B_1) + B_1^2\}}{3(2-\gamma)(n+2)(n+1)(1+\lambda_2)^m(1+2(\lambda_1+\lambda_2))^m B_1^2}$$

*the result is sharp.*

**Remark 3.4** When $\lambda_1 = \lambda_2 = m = n = 0, B_1 = \frac{8}{\pi^2}, B_2 = \frac{16}{3\pi^2}$ the above Corollary 3.3 reduces to a recent result of Srivastava and Mishra [5] for a class of functions for which $\Omega^\gamma f(z)$ is a parabolic starlike functions (see [1], [4]). For other work related to the Fekete-Szegó problem can be read in the following articles (see for examples [2], [7], [9], [10], [11], [12], [13] and [15]).

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**References**

[1] A.W. Goodman, Uniformly convex functions, *Ann. Polon. Math.*, **56** (1991), 87-92.

[2] A. Mohammed, M. Darus, On Fekete-Szego problem for certain subclass of analytic functions, *Int. J. Open Problems Compt. Math.* **3**, No. 4 (2010), 510-520.

[3] E.A. Eljamal, M. Darus, Subordination results defined by a new differential operator, *Acta Universitatis Apulensis*, No. 27/2011, inprint.

[4] F. Ronning, Uniformly convex functions and a corresponding class of starlike functions, *Proc, Amer. math. Soc.*, **118** (1993), 189-196.

[5] H.M. Srivastava, A.K. Mishra, M.K. Das, The Fekete-Szegö problem for a subclass of close-to-convex functions, *Complex variables, Theory Appl.*, **44** (2001),145-163.

[6] H.M. Srivastava, A.K. Mishra, Applications of fractional calculus to parabolic starlike and unifromly convex functions, *Computer Math. Appl.*, **39** (2000), 57-69.

[7] I. Faisal, M. Darus, On Fekete-Szego theorem for certain analytic functions, *Journal of Ouality Measurement and Analysis,*, **5**, No. 1 (2009), 85-91.

[8] K. Al-Shaqsi, M. Darus, On Fekete-Szegö Problems for certain Subclass of Analytic Functions, *Applied Mathematical Science*, **2** (2008), 9, 431-441.

[9] M. Darus, The Fekete-Szegö problems for close-to-convex functions of the class $K_{sh}(alpha, beta)$, *Acta Mathematica Academiae Paedagogicae Nyiregyhaziensis*, **18**, No. 1 (2002), 13-18.

[10] M. Darus, T.N. Shanmugam, S. Sivasubramanian, The Fekete-Szegö inequality for a certain class of analytic functions, *MATHEMATICA(Cluj)*, Tome 49 (72), No. 1 (2007), 29-34.

[11] M.H. Al-Abbadi, M. Darus, The Fekete-Szegö theorem for a certain class of analytic functions, *Sains Malaysiana,*, **40**, No. 4 (2011), 385-389.

[12] N. Tuneski, M. Darus, Fekete-Szegö Functional for non-Bazilevich functions, *Acta Mathematica Academiae Paedagogicae Nyiregyhaziensis*, **18**, No. 2 (2002), 63-65.

[13] S.F. Ramadan, M. Darus, On the Fekete-Szegö inequality for a class of analytic functions defined by using generalized differential operator, *Acta Universitatis Apulensis*, **26** (2011), 167-178.

[14] S. Owa, H.M. Srivastava, Univalent and starlike generalized hypergeometric functions, *Canad. J. Math.*, **39**, No. 5 (1987), 1057-1077.

[15] V. Ravichandran, A. Gangadharan, M. Darus, On Fekete-Szegö inequality for certain class of Bazilevic functions, *Far East Journal of Mathematical Sciences (FJMS)*, **15**, No. 2 (2004), 171-180.

[16] W. Ma, D. Minda, A unified treatment of some special classes of univalent functions, In: *Proceedings of the Conference on Complex Analysis* (Ed-s: Z. Li, F. Ren, L. Yang, S. Zhang), Int. Press (1994), 157-169.