International Journal of Pure and Applied Mathematics
Volume 72 No. 4 2011, 555-564

TRANSLATES OF INTUITIONISTIC FUZZY SUBGROUPS

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Abstract: As an abstraction of the geometric notion of translation, we introduce two operators $T_{\alpha+}$ and $T_{\alpha-}$ called the intuitionistic fuzzy translation operator on the intuitionistic fuzzy set A. We investigate their properties and also studied their action on Intuitionistic fuzzy subgroups of a group.

AMS Subject Classification: 03F55, 20M12, 08A72
Key Words: intuitionistic fuzzy set (IFS), intuitionistic fuzzy subgroup (IFSG), intuitionistic fuzzy normal subgroup (IFNSG), translation, operator, coset

1. Introduction

The concept of Translates in fuzzy subgroups has been discussed by Souriar Sebastian [4]. Here we extend this study to the Intuitionistic fuzzy subgroups. First we define the operator $T_{\alpha+}$ and $T_{\alpha-}$ on Intuitionistic fuzzy sets and derive some of their properties. Then we investigate their action on Intuitionistic fuzzy subgroups of a group. We study the interaction of these operators with intuitionistic fuzzy coset formation..

2. Preliminaries

Atanassov introduced in [1] the concept of intuitionistic fuzzy sets defined on a non-empty set X as objects having the form

$$A = \{<x, \mu_A(x), \nu_A(x)>; x \in X\},$$

where the functions μ_A: X $\to$ [0, 1] and ν_A : X $\to$ [0, 1] denote the degree of

Received: August 14, 2011

membership and the degree of non-membership of each element $x \in X$ to the set A, respectively. Here $0 \leq \mu_A(x), \nu_A(x) \leq 1$, for all $x \in X$. Such defined objects are studied by many authors and have many interesting applications in mathematics.

Let A and B be two intuitionistic fuzzy subsets of a set X. Then the following expressions are defined in [1], [2].

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$,

(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,

(iii) $A^C = \{ <x, \nu_A(x), \mu_A(x)>; x \in X \}$,

(iv) $A \cap B = \{ <x, \min\{ \mu_A(x), \mu_B(x)\}, \max\{ \nu_A(x), \nu_B(x)\}>: x \in X\}$,

(v) $A \cup B = \{ <x, \max\{ \mu_A(x), \mu_B(x)\}, \min\{ \nu_A(x), \nu_B(x)\}>: x \in X\}$.

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the intuitionistic fuzzy subset $A = \{ <x, \mu_A(x), \nu_A(x)>; x \in X \}$

Definition 2.1. (see [3], [5]) An IFS $A = (\mu_A, \nu_A)$ of a group G is said to be *intuitionistic fuzzy subgroup* of G (in short IFSG) of G if

1. $\mu_A(xy) \geq \text{Min} \{ \mu_A(x), \mu_A(y)\}$
2. $\mu_A(x^{-1}) = \mu_A(x)$
3. $\nu_A(xy) \leq \text{Max} \{ \nu_A(x), \nu_A(y)\}$
4. $\nu_A(x^{-1}) = \nu_A(x)$, for all $x, y \in G$

In other words. An IFS A of X is IFSG of G iff $\mu_A(xy^{-1}) \geq \text{Min} \{ \mu_A(x), \mu_A(y)\}$and $\nu_A(xy^{-1}) \leq \text{Max} \{ \nu_A(x), \nu_A(y)\}$holds for all $x, y \in G$.

Definition 2.2. (see [3], [5]) An IFSG $A = (\mu_A, \nu_A)$ of a group G said to be *intuitionistic fuzzy normal subgroup* of G (in short IFNSG) of G if

1. $\mu_A(xy) = \mu_A(yx)$
2. $\nu_A(xy) = \nu_A(yx)$, for all $x, y \in G$

Definition 2.3. (see [3], [5]) Let G be a group and A be IFSG of group G. Let $x \in G$ be a fixed element. Then for every element $g \in G$, we define

1. $(xA)(g) = (\mu_{xA}(g), \nu_{xA}(g))$, where $\mu_{xA}(g) = \mu_A(x^{-1}g)$ and $\nu_{xA}(g) = \nu_A(x^{-1}g)$ is called intuitionistic fuzzy left coset of G determined by A and x

2. $Ax(g) = (\mu_{Ax}(g), \nu_{Ax}(g))$, where $\mu_{Ax}(g) = \mu_A(gx^{-1})$ and $\nu_{Ax}(g) = \nu_A(gx^{-1})$ is called the intuitionistic fuzzy right coset of G determined by A and x.

3. The Operators $T_{\alpha+}$ and $T_{\alpha-}$

Definition 3.1. Let $A = (\mu_A, \nu_A)$ be an IFS of X and $\alpha \in [0, 1]$. We define

$$T_{\alpha+}(A)(x) = (\mu_{T\alpha+}(x), \nu_{T\alpha+}(x))$$

and

$$T_{\alpha-}(A)(x) = (\mu_{T\alpha-}(x), \nu_{T\alpha-}(x)),$$

where

$\mu_{T\alpha+}(x) = \text{Min}\{ \mu_A(x) + \alpha, 1\}$, $\nu_{T\alpha+}(x) = \text{Max}\{ \nu_A(x) - \alpha, 0 \}$

and

$\mu_{T\alpha-}(x) = \text{Max}\{ \mu_A(x) - \alpha, 0\}$, $\nu_{T\alpha-}(x) = \text{Min}\{ \nu_A(x) + \alpha, 1 \}$

$T_{\alpha+}(A)$ and $T_{\alpha-}(A)$ are respectively called the α- up and α- down intuitionistic fuzzy operators of A. We shall call $T_{\alpha+}$ and $T_{\alpha-}$ as the intuitionistic fuzzy operator.

Results 3.2. The following results can be easily verified from our definitions:

$$(i) T_{0+}(A) = T_{0-}(A) = A, \quad (ii) T_{1+}(A) = 1^{\sim}, \quad (iii) T_{1-}(A) = 0^{\sim}.$$

Remark 3.3. It can be easily checked that if A is IFS of X, then both $T_{\alpha+}(A)$ and $T_{\alpha-}(A)$ are IFS of X. In other words $0 \leq \mu_{T\alpha+}(x) + \nu_{T\alpha+}(x) \leq 1$ and $0 \leq \mu_{T\alpha-}(x) + \nu_{T\alpha-}(x) \leq 1$, for all $x \in X$.

Example 3.4. Let $X = \{ 1, \omega, \omega^2 \}$. Let $A = \{ <1, 0.3, 0.4>, <\omega, 0.1, 0.25>, <\omega^2, 0.5, 0.3>\}$be an IFS of X Take $\alpha = 0.2$, then:

$$T_{\alpha+}(A) = \{\{<1, 0.5, 0.2>, <\omega, 0.3, 0.05>, <\omega^2, 0.7, 0.1>\}\}$$

and

$$T_{\alpha-}(A) = \{\{<1, 0.1, 0.6>, <\omega, 0, 0.45>, <\omega^2, 0.3, 0.5>\}\}.$$

Proposition 3.5. *For any IFS A of X and $\alpha \in [0,1]$, we have*

$$T_{\alpha+}(A^c) = (T_{\alpha-}(A))^c, (ii) T_{\alpha-}(A^c) = (T_{\alpha+}(A))^c$$

Proof. Let A = (μ_A, ν_A) be an IFS of X and let x $\in$X, $\alpha \in$ [0,1]. Then $A^c = (\mu_A^c, \nu_A^c) = (\nu_A, \mu_A)$ be the complement of A.

1. Let $T_{\alpha+}(A^c)(x) = (\mu'_{T\alpha+}(x), \nu\prime_{T\alpha+}(x))$,

where $\mu'_{T\alpha+}(x)$ = Min$\{ \mu_A^c(x) + \alpha, 1 \}$= Min $\{ \nu_A(x) + \alpha, 1 \} = \nu_{T\alpha-}(x)$ and $\nu\prime_{T\alpha+}(x)$ = Max $\{ \nu_A^c(x) - \alpha, 0\}$= Max $\{ \mu_A(x) - \alpha , 0 \} = \mu_{T\alpha+}(x)$.

Thus $T_{\alpha+}(A^c)(x) = (\nu_{T\alpha-}(x), \mu_{T\alpha+}(x)) = (T_{\alpha-}(A))^c(x)$.

Hence $T_{\alpha+}(A^c) = (T_{\alpha-}(A))^c$.

(ii) Let $T_{\alpha-}(A^c)(x) = (\mu''_{T\alpha-}(x), \nu\prime\prime_{T\alpha-}(x))$, where $\mu''_{T\alpha-}(x)$ = Max$\{ \mu_A^c(x) - \alpha, 0 \}$= Max $\{ \nu_A(x) - \alpha , 0 \} = \nu_{T\alpha+}(x)$ and $\nu\prime\prime_{T\alpha-}(x)$ = Min $\{ \nu_A^c(x) + \alpha, 1\}$= Min $\{ \mu_A(x) + \alpha, 1 \} = \mu_{T\alpha+}(x)$.

Thus $T_{\alpha-}(A^c)(x) = (\nu_{T\alpha+}(x), \mu_{T\alpha+}(x)) = (T_{\alpha+}(A))^c(x)$.

Hence $T_{\alpha-}(A^c) = (T_{\alpha+}(A))^c$.

Next, we show that; in general, for any given $\alpha \in$ [0,1], the operators $T_{\alpha+}$ and $T_{\alpha-}$ donot commute each other. In other words $T_{\alpha-}(T_{\alpha+}(A))$ and $T_{\alpha+}(T_{\alpha-}(A))$ may be different from A.

Example 3.6. Let X = $\{ 1, \omega, \omega^2 \}$.Let A = $\{ <1, 0.3, 0.4 >, <\omega, 0.1, 0.25 >, <\omega^2, 0.5, 0.3 >\}$be an IFS of X Take $\alpha = 0.3$, then

$$T_{\alpha+}(A) = \{\{<1, 0.6, 0.1>, <\omega, 0.4, 0>, <\omega^2, 0.8, 0>\}$$

and $T_{\alpha-}(A) = \{ \{ <1, 0, 0.7 >, <\omega, 0, 0.55 >, <\omega^2, 0.2, 0.6 >\}$.

Now $T_{\alpha-}(T_{\alpha+}(A)) = \{ <x, \mu^*_{T\alpha-}(x), \nu*_{T\alpha-}(x) >; x \in X \}$, where $\mu^*_{T\alpha-}(x)$ = Max $\{ \mu_{T\alpha+}(x) - \alpha, 0 \}$and $\nu*_{T\alpha-}(x)$ = Min $\{ \nu_{T\alpha+}(x) + \alpha, 1 \}$.

Therefore $T_{\alpha-}(T_{\alpha+}(A)) = \{ <1, 0.3, 0.4 >, <\omega, 0, 0.3 >, <\omega^2, 0.5, 0.3 >\} \neq$ A and $T_{\alpha+}(T_{\alpha-}(A)) = \{ <x, \mu^{**}_{T\alpha+}(x), \nu**_{T\alpha+}(x) >; x \in X \}$, where

$\mu^{**}_{T\alpha+}(x)$ = Min $\{ \mu_{T\alpha-}(x) + \alpha, 1 \}$and $\nu**_{T\alpha+}(x)$ = Max $\{ \nu_{T\alpha-}(x) - \alpha, 0 \}$

Therefore $T_{\alpha+}(T_{\alpha-}(A)) = \{ <1, 0.3, 0.4 >, <\omega, 0.3, 0.25 >, <\omega^2, 0.5, 0.3 >\} \neq$ A

It may be checked that if we take $\alpha = 0.2$, then $T_{\alpha-}(T_{\alpha+}(A))$ = A

Theorem 3.7. *For any IFS A of X and $\alpha \in$ [0,1]*

1. *$T_{\alpha-}(T_{\alpha+}(A))$ = A $\Leftrightarrow \alpha \leq$ Min $\{ 1 - p, q \}$*

where p = Max $\{ \mu_A(x) : x \in X \}$and q = Min $\{ \nu_A(x) : x \in X \}$

1. *$T_{\alpha+}(T_{\alpha-}(A))$ = A $\Leftrightarrow \alpha \leq$ Min $\{ p\prime, 1 - q\prime \}$*

Where $p\prime$ = *Min* { $\mu_A(x) : x \in X$ }*and* $q\prime$ = *Max* { $\nu_A(x) : x \in X$ }

Proof. (i) An easy calculation shows that, for any $x \in X$,
$T_{\alpha-}(T_{\alpha+}(A))(x) = ($ Max{ Min{ $\mu_A(x) + \alpha$,1}-α,0}, Min{ Max{ $\nu_A(x)$-α, 0 }+ α, 1})
Now $T_{\alpha-}(T_{\alpha+}(A)) = A \Leftrightarrow$ Max{ Min{ $\mu_A(x) + \alpha$, 1}, 0}= $\mu_A(x)$ and
Min{ Max{ $\nu_A(x)$- α, 0 }+α, 1}= $\nu_A(x)$ for all $x \in X$
$\Leftrightarrow$ Max{ Min{ $\mu_A(x)$,1- α), 0}= $\mu_A(x)$ and Min{ Max{ $\nu_A(x)$, α), 1}= $\nu_A(x)$
$\Leftrightarrow \mu_A(x) \leq$ 1- α and $\nu_A(x) \geq \alpha$ for all $x \in X$
$\Leftrightarrow$ Max { $\mu_A(x) : x \in X$ }$\leq$ 1- α and Min { $\nu_A(x) : x \in X$ }$\geq \alpha$
$\Leftrightarrow$ p $\leq$ 1- α and q $\geq \alpha \Leftrightarrow \alpha \leq$ 1- p and $\alpha \leq$ q
$\Leftrightarrow \alpha \leq$ Min { 1 - p, q }
(ii) An easy calculation shows that, for any $x \in X$,
$T_{\alpha+}(T_{\alpha-}(A))(x) =$ (Min{ Max{ $\mu_A(x)$ - α,0}+α,1}, Max{ Min{ $\nu_A(x) + \alpha$, 1 } - α, 0})
$T_{\alpha+}(T_{\alpha-}(A)) = A \Leftrightarrow$ Min{ Max{ $\mu_A(x)$- α,0}+α ,1}= $\mu_A(x)$ and
Max{ Min{ $\nu_A(x) + \alpha$,1 }- α, 0}= $\nu_A(x)$ for all $x \in X$
$\Leftrightarrow$ Min{ Max{ $\mu_A(x)$, α }, 1}= $\mu_A(x)$ and Max{ Min{ $\nu_A(x)$,1-α}, 0}= $\nu_A(x)$
$\Leftrightarrow$ Max{ $\mu_A(x)$, α }= $\mu_A(x)$ and Min{ $\nu_A(x)$,1-α}for all $x \in X$
$\Leftrightarrow \mu_A(x) \geq \alpha$ and $\nu_A(x) \leq$ 1-α for all $x \in X$
$\Leftrightarrow$ Min { $\mu_A(x) : x \in X$ }$\geq \alpha$ and Max { $\nu_A(x) : x \in X$ }$\leq$ 1-α
$\Leftrightarrow$ p$\prime \geq \alpha$ and q$\prime \leq$ 1-$\alpha \Leftrightarrow \alpha \leq$ p$\prime$ and $\alpha \leq$ 1 - q$\prime$
$\Leftrightarrow \alpha \leq$ Min { p$\prime$, 1 - q$\prime$ }.

Proposition 3.8. *For any IFS A of X and* $\alpha, \beta \in$ *[0,1]*

1. $$T_{\alpha+}(T_{\beta+}(A)) = T_{\beta+}(T_{\alpha+}(A)) = \begin{cases} T_{(\alpha+\beta)+}(A) & if \alpha+\beta¡1 \\ 1^{\sim} & if \alpha+\beta = 1 \end{cases}$$
2. $$T_{\alpha-}(T_{\beta-}(A)) = T_{\beta-}(T_{\alpha-}(A)) = \begin{cases} T_{(\alpha+\beta)-}(A) & if \alpha+\beta¡1 \\ 0^{\sim} & if \alpha+\beta = 1 \end{cases}$$

Proof. The proof is obvious.

Remark 3.9. For any IFS's A and B of X with $A \subseteq B$, we have
$T_{\alpha+}(A) \subseteq T_{\alpha+}(B)$ and $T_{\alpha-}(A) \subseteq T_{\alpha-}(B)$, for all $\alpha \in [0,1]$
Thus the Intuitionistic fuzzy operators are isotones.

4. Translation of Intuitionistic Fuzzy Groups

Here we study the action of $T_{\alpha+}$ and $T_{\alpha-}$ on IFSG of a group G. We prove that these operators takes on IFSG to an IFSG and preserve some properties of Intuitionistic fuzzy group.

Theorem 4.1. *If A is IFSG of a group G, then $T_{\alpha+}(A)$ and $T_{\alpha-}(A)$ are IFSG of G, for all $\alpha \in [0,1]$.*

Proof. Let A be IFSG of a group G and $\alpha \in [0,1]$.

Let x, y $\in$ G be any elements, we have $T_{\alpha+}(A)(xy^{-1}) = (\mu_{T\alpha+}(xy^{-1}), \nu_{T\alpha+}(xy^{-1}))$.

Here

$\mu_{T\alpha+}(xy^{-1}) = \text{Min}\{\mu_A(xy^{-1}) + \alpha, 1\}$
$\geq \text{Min}\{\text{Min}\{\mu_A(x), \mu_A(y)\}+\alpha, 1\}$
$= \text{Min}\{\text{Min}\{\mu_A(x) + \alpha, \mu_A(y) + \alpha\}, 1\}$
$= \text{Min}\{\text{Min}\{\mu_A(x) + \alpha, 1\}, \text{Min}\{\mu_A(y) + \alpha, 1\}\}$
$= \text{Min}\{\mu_{T\alpha+}(x), \mu_{T\alpha+}(y)\}$

i.e. $\mu_{T\alpha+}(xy^{-1}) \geq \text{Min}\{\mu_{T\alpha+}(x), \mu_{T\alpha+}(y)\}$. Similarly:

$\nu_{T\alpha+}(xy^{-1}) = \text{Max}\{\nu_A(xy^{-1}) - \alpha, 0\}$
$\leq \text{Max}\{\text{Max}\{\nu_A(x), \nu_A(y)\} - \alpha, 0\}$
$= \text{Max}\{\text{Max}\{\nu_A(x) - \alpha, \mu_A(y) - \alpha\}, 0\}$
$= \text{Max}\{\text{Max}\{\mu_A(x) - \alpha, 0\}, \text{Max}\{\mu_A(y) - \alpha, 0\}\}$
$= \text{Max}\{\nu_{T\alpha+}(x), \nu_{T\alpha+}(y)\}$

i.e. $\nu_{T\alpha+}(xy^{-1}) \leq \text{Max}\{\nu_{T\alpha+}(x), \nu_{T\alpha+}(y)\}$.

Thus $T_{\alpha+}(A)$ is IFSG of group G (follows from Proposition 2.1).

Similarly, we can show that $T_{\alpha-}(A)$ is IFSG of group G.

Remark 4.2. If $T_{\alpha+}(A)$ or $T_{\alpha-}(A)$ is IFSG of group G, for a particular value of $\alpha \in [0,1]$, then it cannot be deduced that A is IFSG of group G.

Example 4.3. Let G be the Klein 4-group { e, a, b, ab }, where $a^2 = b^2 = e$ and ab = ba. Define A = { <e, 0.9, 0.1 >, <a, 0.65, 0.3 >, <b, 0.61, 0.29 >, <ab, 0.6, 0.31 >}be IFS in G. Take $\alpha = 0.4$, then

$$T_{\alpha+}(A) = \{<e,1,0>, <a,1,0>, <b,1,0>, <ab,1,0>\} = 1^{\sim}.$$

Clearly, $T_{\alpha+}(A)$ is an IFSG of G, however A is not IFSG of G.

Proposition 4.4. (see [3]) *Let G be a group with identity element e and A be any IFS of G. Then set $G_A = \{x \in G ; \mu_A(x) = \mu_A(e)$ and $\nu_A(x) = \nu_A(e)\}$ is a subgroup of G.*

Proposition 4.5. *Let A be IFS of a group G such that $T_{\alpha+}(A)$ be IFSG of G, for some $\alpha \in [0,1]$ with $\alpha <$Min { 1- p″, q″ }, then A is IFSG of G, where*

$$p'' = Max\{\mu_A(x) : x \in G - G_A\}$$

and

$$q'' = Min\{\nu_A(x) : x \in G - G_A\}.$$

Proof. Let $T_{\alpha+}(A)$ be IFSG of G, for some $\alpha \in [0,1]$ with $\alpha <$Min { 1- p″, q″ }

Therefore, for any x, y ∈ G, we have

$T_{\alpha+}(A)(xy^{-1}) = (\mu_{T\alpha+}(xy^{-1}), \nu_{T\alpha+}(xy^{-1}))$, Where $\mu_{T\alpha+}(xy^{-1}) \geq$ Min$\{\mu_{T\alpha+}(x), \mu_{T\alpha+}(y)\}$and $\nu_{T\alpha+}(xy^{-1}) \leq$ Max$\{\nu_{T\alpha+}(x), \nu_{T\alpha+}(y)\}$(*)

Case (i) When $\mu_{T\alpha+}(x) = 1$ and $\mu_{T\alpha+}(y) = 1$

As $0 \leq \mu_{T\alpha+}(x) + \nu_{T\alpha+}(x) \leq 1$ and $0 \leq \mu_{T\alpha+}(y) + \nu_{T\alpha+}(y) \leq 1$, for all x, y ∈G

Therefore $\nu_{T\alpha+}(x) = 0$ and $\nu_{T\alpha+}(y) = 0$

Min $\{\mu_A(x) + \alpha, 1\} = 1$, Min $\{\mu_A(y) + \alpha, 1\} = 1$ and

Max $\{\nu_A(x) - \alpha, 0\} = 0$, Max $\{\nu_A(y) - \alpha, 0\} = 0$

$\Rightarrow \mu_A(x) + \alpha \geq 1$, $\mu_A(y) + \alpha \geq 1$ and

$\nu_A(x) - \alpha \leq 0$, $\nu_A(y) - \alpha \leq 0$

$\Rightarrow \mu_A(x) \geq 1 - \alpha$, $\mu_A(y) \geq 1 - \alpha$ and $\nu_A(x) \leq \alpha$, $\nu_A(y) \leq \alpha$ -(1)

Since $\alpha <$Min { 1- p″, q″ }$\Rightarrow \alpha <$1- p″ and $\alpha <$q″

$\Rightarrow$ p″<1 - α and q″ >α

$\Rightarrow$ Max $\{\mu_A(x) : x \in G - G_A\} < 1 - \alpha$ and Min $\{\nu_A(x) : x \in G - G_A\} > \alpha$ and Max $\{\mu_A(y) : y \in G - G_A\} < 1 - \alpha$ and Min $\{\nu_A(y) : y \in G - G_A\} > \alpha$

Therefore from (1), we get $x \in G_A$ and $y \in G_A$, but G_A is a subgroup of G

$\Rightarrow xy^{-1} \in G_A$ and so

$\mu_A(xy^{-1}) = \mu_A(e) =$ Min $\{\mu_A(e), \mu_A(e)\} =$ Min $\{\mu_A(x), \mu_A(y)\}$

i.e. $\mu_A(xy^{-1}) \geq$ Min $\{\mu_A(y), \mu_A(y)\}$

also $\nu_A(xy^{-1}) = \nu_A(e) =$ Max $\{\nu_A(e), \nu_A(e)\} =$ Max $\{\nu_A(x), \nu_A(y)\}$

i.e. $\nu_A(xy^{-1}) \leq$ Max $\{\nu_A(x), \nu_A(y)\}$.

Case (ii) When $\mu_{T\alpha+}(x) = 1$ and $\mu_{T\alpha+}(y) < 1$

As in case (i), we get $x \in G_A$ and so $\mu_A(x) = \mu_A(e)$ and $\nu_A(x) = \nu_A(e)$

From (*), we get

$\mu_{T\alpha+}(xy^{-1}) \geq$ Min$\{\mu_{T\alpha+}(x), \mu_{T\alpha+}(y)\}=$ Min$\{1, \mu_{T\alpha+}(y)\}= \mu_{T\alpha+}(y)\}$

$\Rightarrow$ Min$\{\mu_A(xy^{-1}) + \alpha, 1\} \geq$ Min$\{\mu_A(y) + \alpha, 1\}$

$\Rightarrow \mu_A(xy^{-1}) + \alpha \geq \mu_A(y) + \alpha$
$\Rightarrow \mu_A(xy^{-1}) \geq \mu_A(y) = \text{Min}\{\mu_A(e), \mu_A(y)\} = \text{Min}\{\mu_A(x), \mu_A(y)\}$
Thus $\mu_A(xy^{-1}) \geq \text{Min}\{\mu_A(x), \mu_A(y)\}$
Also $\nu_{T\alpha+}(xy^{-1}) \leq \text{Max}\{\nu_{T\alpha+}(x), \nu_{T\alpha+}(y)\} = \text{Max}\{0, \nu_{T\alpha+}(y)\} = \nu_{T\alpha+}(y)$
$\Rightarrow \text{Max}\{\nu_A(xy^{-1}) - \alpha, 0\} \leq \text{Max}\{\nu_A(y) - \alpha, 0\}$
$\Rightarrow \nu_A(xy^{-1}) - \alpha \leq \nu_A(y) - \alpha$
$\Rightarrow \nu_A(xy^{-1}) \leq \nu_A(y) = \text{Max}\{\nu_A(e), \nu_A(y)\} = \text{Max}\{\nu_A(x), \nu_A(y)\}$
Thus $\nu_A(xy^{-1}) \leq \text{Max}\{\nu_A(x), \nu_A(y)\}$.

Case (iii) When $\mu_{T\alpha+}(x) < 1$ and $\mu_{T\alpha+}(y) < 1$
$\text{Min}\{\mu_A(x) + \alpha, 1\} < 1$ and $\text{Min}\{\mu_A(y) + \alpha, 1\} < 1$
$\Rightarrow \mu_A(x) + \alpha < 1$ and $\mu_A(y) + \alpha < 1$. From (*), we get
$\mu_{T\alpha+}(xy^{-1}) \geq \text{Min}\{\mu_{T\alpha+}(x), \mu_{T\alpha+}(y)\}$ and $\nu_{T\alpha+}(xy^{-1}) \leq \text{Max}\{\nu_{T\alpha+}(x), \nu_{T\alpha+}(y)\}$
$\Rightarrow \text{Min}\{\mu_A(xy^{-1}) + \alpha, 1\} \geq \text{Min}\{\text{Min}\{\mu_A(x) + \alpha, 1\}, \text{Min}\{\mu_A(y) + \alpha, 1\}\}$
$= \text{Min}\{\mu_A(x) + \alpha, \mu_A(y) + \alpha\}$
$= \text{Min}\{\mu_A(x), \mu_A(y)\} + \alpha$
Thus $\mu_A(xy^{-1}) + \alpha \geq \text{Min}\{\mu_A(x), \mu_A(y)\} + \alpha$
i.e. $\mu_A(xy^{-1}) \geq \text{Min}\{\mu_A(x), \mu_A(y)\}$
also $\text{Max}\{\nu_A(xy^{-1}) - \alpha, 0\} \leq \text{Max}\{\text{Max}\{\nu_A(x) - \alpha, 0\}, \text{Max}\{\nu_A(y) - \alpha, 0\}\}$
$= \text{Max}\{\nu_A(x) - \alpha, \nu_A(y) - \alpha\}$
$= \text{Max}\{\nu_A(x), \nu_A(y)\} - \alpha$
Thus $\nu_A(xy^{-1}) - \alpha \leq \text{Max}\{\nu_A(x), \nu_A(y)\} - \alpha$
i.e. $\nu_A(xy^{-1}) \leq \text{Max}\{\nu_A(x), \nu_A(y)\}$
Hence A is IFSG of G.

Proposition 4.6. *Let A be IFS of a group G such that $T_{\alpha-}(A)$ be IFSG of G, for some $\alpha \in [0,1]$ with $\alpha < \text{Min}\{1 - p^*, q^*\}$, then A is IFSG of G, where $p^* = \text{Max}\{\nu_A(x) : x \in G - G_A\}$ and $q^* = \text{Min}\{\mu_A(x) : x \in G - G_A\}$*

Proof. Similar to the proof of Proposition (4.5)

Theorem 4.7. *If A is IFNSG of a group G, if and only if $T_{\alpha+}(A)$ and $T_{\alpha-}(A)$ are IFNSG of G, for all $\alpha \in [0,1]$.*

Proof. Firstly let A be IFNSG of group G and $\alpha \in [0,1]$ be any real number.
A is IFSG of G and $\mu_A(xy) = \mu_A(yx)$, $\nu_A(xy) = \nu_A(yx)$ holds for all x, y $\in$G.
By Theorem (4.1), $T_{\alpha+}(A)$ and $T_{\alpha-}(A)$ are IFSG of G. Now for x, y $\in$G., we have

$T_{\alpha+}(A)(xy) = (\mu_{T\alpha+}(xy), \nu_{T\alpha+}(xy)) = ($ Min$\{ \mu_A(xy) + \alpha, 1\}$, Max$\{ \nu_A(xy)$-$\alpha, 0\})$

$= ($ Min$\{ \mu_A(yx) + \alpha, 1\}$, Max$\{ \nu_A(yx)$-$\alpha, 0\})$

$= (\mu_{T\alpha+}(yx), \nu_{T\alpha+}(yx))$

$= T_{\alpha+}(A)(yx)$

Hence $T_{\alpha+}(A)$ is IFNSG of G. Similarly we can show $T_{\alpha-}(A)$ is also IFNSG of G.

Conversely, let $T_{\alpha+}(A)$ and $T_{\alpha-}(A)$ are IFNSG of group G for all $\alpha \in [0,1]$

Take $\alpha = 0$, we get $T_{0+}(A) = A$ and $T_{0-}(A) = A$

Hence A is IFNSG of group G.

Proposition 4.8. *Let A be IFS of a group G such that $T_{\alpha+}(A)$ be IFSNG of G, for some $\alpha \in [0,1]$ with $\alpha <$Min $\{ 1- p'', q'' \}$, then A is IFNSG of G, where*

$p'' =$ Max $\{ \mu_A(x) : x \in G - G_A \}$ and $q'' =$ Min $\{ \nu_A(x) : x \in G - G_A \}$

Proof. Follows from Propositions 4.5 and 4.7.

Proposition 4.9. *Let A be IFS of a group G such that $T_{\alpha-}(A)$ be IFNSG of G, for some $\alpha \in [0,1]$ with $\alpha <$Min $\{ 1- p^*, q^* \}$, then A is IFNSG of G, where*

$p^* =$ Max $\{ \nu_A(x) : x \in G - G_A \}$ and $q^* =$ Min $\{ \mu_A(x) : x \in G - G_A \}$

Proof. Follows from Propositions (4.6) and (4.7)

Proposition 4.10. *For any IFSG of a group G and $x \in G$, $\alpha \in [0,1]$, we have*

1. *$(T_{\alpha+}(A))x = T_{\alpha+}(Ax)$ (ii) $x(T_{\alpha+}(A)) = T_{\alpha+}(xA)$*

Proof. For any $g \in G$, we have

(i) $((T_{\alpha+}(A))x)(g) = (T_{\alpha+}(A))(gx^{-1})$

$= (\mu_{T\alpha+}(gx^{-1}), \nu_{T\alpha+}(gx^{-1}))$

$= ($ Min$\{ \mu_A(gx^{-1}) + \alpha, 1\}$, Max$\{ \nu_A(gx^{-1})$-$\alpha, 0\})$

$= ($ Min$\{ \mu_{Ax}(g) + \alpha, 1\}$, Max$\{ \nu_{Ax}(g)$-$\alpha, 0\})$

$= T_{\alpha+}(Ax)(g)$

Thus $(T_{\alpha+}(A))x = T_{\alpha+}(Ax)$

(ii) $(x(T_{\alpha+}(A)))(g) = (T_{\alpha+}(A))(x^{-1}g)$

$= (\mu_{T\alpha+}(x^{-1}g), \nu_{T\alpha+}(x^{-1}g))$

$= ($ Min$\{ \mu_A(x^{-1}g) + \alpha, 1\}$, Max$\{ \nu_A(x^{-1}g)$-$\alpha, 0\})$

$= ($ Min$\{ \mu_{xA}(g) + \alpha, 1\}$, Max$\{ \nu_{xA}(g)$-$\alpha, 0\})$

$= (T_{\alpha+}(xA))(g)$

Thus $x(T_{\alpha+}(A)) = T_{\alpha+}(xA)$

Proposition 4.11. *For any IFSG of a group G and* $x \in G$*,* $\alpha \in [0,1]$*, we have*

1. $(T_{\alpha-}(A))x = T_{\alpha-}(Ax)$ *(ii)* $x(T_{\alpha-}(A)) = T_{\alpha-}(xA)$

Proof. The proof is similar to Proposition 4.10.

From Propositions 4.10 and 4.11 we see that intuitionistic fuzzy translation operators commute with intuitionistic fuzzy coset formation.

5. Conclusion

We observed that some intuitionistic fuzzy group theoretical concepts are well-behaved with respect to the intuitionistic fuzzy translation operators, in the sense that, they either remain invariant under the action of the operators or commute with them. But it is yet to see those properties which donot behave like these.

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