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AN BRIEF INTRODUCTION TO ROBUST OPTIMIZATION APPROACH

Hong-Xia Yu[1], Li Jin[2 §]

[1]School of Mathematics and Pyhsics
University of Shanghai Electric Power
No. 28, Xue Hai Road, 201300, Shanghai, P.R. CHINA

[2]School of Mathematics, Physics and Information Science
Zhejiang Ocean University
Zhoushan, 316004, P.R. CHINA

Abstract: In this paper, we firstly give a brief introduction to robust optimization, a more recent approach to optimization under uncertainty. Then we compare it to the other two classical methods for treating uncertain problem, sensitivity analysis and stochastic optimization.

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1. Introduction

When a real-world application is formulated as a mathematical optimization problem, more often than not the problem data are not precisely known, but are subject to uncertainty due to their random nature, measurement errors, or many other reasons. As the Chinese proverb says "To be uncertain is to be uncomfortable, but to be certain is to be ridiculous". In application, one cannot

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§Correspondence author

ignore the uncertainty of data, Ben-Tal and Nemirovski [3] show that a small perturbation on data can make the nominal optimal solution to the problem completely meaningless from a practical viewpoint. Consequently, there exists a real need of a methodology capable of generating a robust solution, one that is immunized against the effect of data uncertainty.

2. Robust Optimization

Given an objective $f_0(x)$ to optimize, subject to constraints $f_i(s, u_i) \leq 0$ with uncertain parameters, $\{u_i\}$, the general Robust Optimization formulation is:

$$\begin{array}{ll} \min & f_0(x) \\ \text{s.t.} & f_i(x, u_i) \leq 0, \quad \forall u_i \in U_i,\ i = 1, \cdots, m. \end{array} \tag{1}$$

Here $x \in R^n$ is a vector of decision variables, the uncertainty parameters $u_i \in R^k$ are assumed to take arbitrary values in the uncertainty sets $U_i \in R^k$, which are closed convex sets. The goal of (1) is to compute minimum cost solutions x^* among all those solutions which are feasible for all realizations of the disturbances u_i within U_i.

It is worthwhile to notice the following facts about the problem formulation of (1):

- The fact that the objective function is unaffected by parameter uncertainty is without loss of generality; we can introduce an auxiliary variable t, and minimize t subject to the additional constraint $\max\limits_{u_0 \in U_0} f_0(x, u_0) \leq t$.
- Constraints without uncertainty are also captured in this framework by assuming the corresponding U_i to be singletons.

The roots of Robust Optimization can be found in the field of robust control and in the work of Soyster [9] as well as later works by Ben-Tal and Nemirovski [1,2] and independently by El-Ghaoui and Lebret [6] and El-Ghaoui et al [7]. Idea behind robust optimization is to consider the worst case scenario without a specific distribution assumption. When modeling the optimization problem with data uncertainty, robust optimization technique could provide a solution that is guaranteed to be good for all or most possible realizations of the uncertainty in the data. We refer the reader to the excellent reference [4].

3. Robust vs. Sensitivity Analysis

Sensitivity analysis is a classical method of addressing parameter uncertainty , which is typically applied as a post-optimization tool for quantifying the change in cost of the associated optimal solution with small perturbations in the underlying problem data. That is, sensitivity analysis is only a tool for analyzing the goodness of a solution. It is not particularly helpful for generating solutions that are robust to data changes. Furthermore, it is impractical to perform joint sensitivity analysis in models with large number of uncertain parameters. The interested reader is referred to the Boyd and Vandenberghe [5], Renegar [8] for more on sensitivity analysis.

Distinctly different than Sensitivity Analysis, Robust Optimization is to compute fixed solutions that ensure feasibility independent of the data. In other words, the obtained solutions have a priori ensured feasibility when the problem parameters vary within the prescribed uncertainty set, which may be large.

4. Robust vs. Stochastic Optimization

In stochastic optimization, the uncertain numerical data are assumed to be random. we express the feasibility of a solution using chance constraints. Assuming that we are given the distributions of the input parameters, the corresponding stochastic optimization problem is:

$$\begin{aligned} \min \quad & f_0(x) \\ \text{s.t.} \quad & Prob(f_i(x, u_i) \leq 0) \geq p_i, \quad i = 1, \cdots, m. \end{aligned} \tag{2}$$

Although the model (2) is expressively rich, there are some fundamental difficulties. We can rarely obtain the actual distributions of the uncertain parameters. Moreover, even if we know the distributions, it is still computationally challenging to evaluate the chance constraints, let alone to optimize the model. Furthermore, the chance constraint can destroy the convexity properties and elevate significantly the complexity of the original problem.

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