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SLIGHTLY $\delta - \beta$-CONTINUOUS FUNCTIONS

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Abstract: The aim of this paper is to introduce and characterize a new class of functions called slightly $\delta - \beta$-continuous functions in topological spaces by using $\delta - \beta$-open sets.

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1. Introduction

Continuity is an important concept in mathematics and many forms of continuous functions have been introduced over the years. Recently Hatir and Noiri [1] developed the concept of $\delta - \beta$-continuous functions with the aid of $\delta - \beta$-open sets [1]. After their work, many weakend forms of $\delta - \beta$-continuity have also been introduced. Rajesh et al. [2] introduced the notion of quasi-$\delta - \beta$-continuous functions. Jafari and Rajesh introduced the concept of almost $\delta - \beta$-continuous functions [3] and faintly $\delta - \beta$-continuous functions [4]. The present paper aims to develop a new concept of slightly $\delta - \beta$-continuous functions which is a weaker form of quasi-$\delta - \beta$ and faintly $\delta - \beta$-continuous functions. A set of conditions have been generated to characterize the function and relationships of the function with its stronger forms which already exist in literature have been investigated.

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2. Preliminaries

Throughout the present paper, X and Y are always topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of X. The interior of A, the closure of A, the δ-interior [5] of A and the δ-closure [5] of A are denoted by $int(A), cl(A), int_\delta(A), cl_\delta(A)$ respectively. A subset A of X is said to be regular open (resp. regular closed) [6] if $A = int(cl(A))$(resp. $A = cl(int(A))$). The δ-interior of a subset A of X is the union of all regular open sets of X contained in A. A subset A is called δ-open [5] if $A = int_\delta(A)$, i.e., a set is δ-open if it is the union of regular open sets. The complement of a δ-open set is called δ-closed [5], alternatively, a subset A of X is called δ-closed if $A = cl_\delta(A)$, where $cl_\delta(A) = \{x \in X : A \cap int(cl(U)) \neq \emptyset, U$ is open in X and $x \in U\}$. A set A is called θ-open [5] in X if for each $x \in A$ there exists an open set U containing x such that $cl(U) \subset A$. The complement of a θ-open set is called a θ-closed set, alternatively, a subset A of X is called θ-closed [5] if $A = cl_\theta(A)$, where $cl_\theta(A) = \{x \in X : A \cap cl(U) \neq \emptyset, U$ is open in X and $x \in U\}$. A subset A of X is said to be δ^*-open [7] if for each $x \in A$, there exists a clopen set U containing x such that $U \in A$. The complement of a δ^*-open set is called a δ^*-closed set [7]. The intersection of all δ^*-closed set containing A is called δ^*-closure [7] of A and is denoted by $cl_{\delta^*}(A)$. The union of all δ^*-open sets contained in A is called δ^*-interior [7] of A and is denoted by $int_{\delta^*}(A)$. A subset A of a topological space X is said to be $\delta - \beta$-open [1] if $A \subset cl(int(cl_\delta(A)))$. The complement of a $\delta - \beta$-open set is called $\delta - \beta$-closed [1]. The intersection of all $\delta - \beta$-closed sets containing A is called $\delta - \beta$-closure [1] of A and is denoted by $\beta cl_\delta(A)$. The $\delta - \beta$-interior [1] of A is defined by the union of all $\delta - \beta$-open sets contained in A and is denoted by $\beta int_\delta(A)$. The set of all $\delta - \beta$-open (closed) sets is denoted by $\delta\beta O(X)$ (resp. $\delta\beta C(X)$). containing x is denoted by $\delta\beta O(X, x)$. A subset N of a space X is said to be a $\delta - \beta$-neighbourhood [8] of $x \in X$ if there exists $U \in \delta\beta O(X, x)$ such that $U \subset N$.

Definition 2.1. A function $f : X \to Y$ is said to be

(a) $\delta - \beta$-continuous [1] at a point $x \in X$ if for each open set V in Y containing $f(x)$, there exists $U \in \delta\beta O(X, x)$ such that $f(U) \subset V$;

(b) $\delta - \beta$-continuous [1] if it is $\delta - \beta$-continuous at each point $x \in X$.

Definition 2.2. A function $f : X \to Y$ is said to be

(a) almost $\delta - \beta$-continuous [3] at a point $x \in X$ if for each open set V in Y containing $f(x)$, there exists $U \in \delta\beta O(X, x)$ such that $f(U) \subset int(cl(V))$;

(b) almost $\delta - \beta$-continuous [3] if it is almost $\delta - \beta$-continuous at each point $x \in X$.

Definition 2.3. A function $f : X \to Y$ is said to be

(a) quasi-$\delta - \beta$-continuous [2] at a point $x \in X$ if for each open set V in Y containing $f(x)$, there exists $U \in \delta\beta O(X, x)$ such that $f(U) \subset cl(V)$;

(b) quasi-$\delta - \beta$-continuous [2] if it is quasi-$\delta - \beta$-continuous at each point $x \in X$.

Definition 2.4. A function $f : X \to Y$ is said to be

(a) faintly $\delta - \beta$-continuous [4] at a point $x \in X$ if for each θ-open set V in Y containing $f(x)$, there exists $U \in \delta\beta O(X, x)$ such that $f(U) \subset V$;

(b) faintly $\delta - \beta$-continuous [4] if it is faintly $\delta - \beta$-continuous at each point $x \in X$.

Definition 2.5. A function $f : X \to Y$ is said to be

(a) slightly continuous [7] at a point $x \in X$ if for each clopen set V in Y containing $f(x)$, there exists an open set U in X containing x such that $f(U) \subset V$;

(b) slightly continuous [7] if it has the property at each $x \in X$.

3. Slightly $\delta - \beta$-continuous functions

Definition 3.1. A function $f : X \to Y$ is said to be

(a) slightly $\delta - \beta$-continuous at $x \in X$ if for each clopen set V of Y containing $f(x)$ there exists $U \in \delta\beta O(X, x)$ such that $f(U) \subset V$.

(b) $\delta - \beta$-continuous if it has the property at each $x \in X$.

Theorem 3.1. *If a function $f : X \to Y$ is quasi $\delta - \beta$-continuous, then it is slightly $\delta - \beta$-continuous function.*

Proof. Let $x \in X$ and let V be a clopen set in Y containing $f(x)$. Since, f is quasi $\delta - \beta$-continuous, therefore, there exists $U \in \delta\beta O(X, x)$ such that $f(U) \subset cl(V) = V$. Hence, f is slightly $\delta - \beta$-continuous. □

Remark 1. The converse of the above theorem is not true in general as can be seen from the following example.

Example 1. Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{X, \emptyset, \{c\}\}$. Then the identity function $i : (X, \tau) \to (X, \sigma)$ is slightly $\delta - \beta$-continuous but not quasi $\delta - \beta$-continuous.

Corollary 3.1.1. *Every $\delta - \beta$-continuous function is slightly $\delta - \beta$-continuous.*

Definition 3.2. A topological space X is said to be 0-dimensional if each point of X has a neibourhood basis consisting of clopen sets.

Theorem 3.2. *If $f : X \to Y$ is slightly $\delta - \beta$-continuous and Y is a 0-dimensional space, then f is $\delta - \beta$-continuous.*

Proof. Let $x \in X$ and let V be an open set in Y containing $f(x)$. Since, Y is 0-dimensional, therefore, there exists a clopen set W in Y such that $f(x) \in W \subset V$. Now, slightly $\delta - \beta$-continuity of f implies that there exists $U \in \delta\beta O(X, x)$ such that $f(U) \subset W \subset V$. Hence, f is $\delta - \beta$-continuous. □

Theorem 3.3. *If $f : X \to Y$ is faintly $\delta - \beta$-continuous, then f is slightly $\delta - \beta$-continuous.*

Proof. The result is obvious from the fact that every clopen set is θ-open. □

Remark 2. The converse of the theorem is not, however, true in general as given by the following example.

Example 2. The function $f : (\mathbb{R}, \tau) \to (\mathbb{R}, \sigma)$ defined by $f(x) = x$ for all $x \in \mathbb{R}$, where $\tau = \{G \subset \mathbb{R} : 0 \in \mathbb{R}\} \cup \{\emptyset\}$ and σ is the usual topology on $\mathbb{R}$, is slightly $\delta - \beta$-continuous but not faintly $\delta - \beta$-continuous.

Definition 3.3. A topological space X is said to be extremally disconnected if closure of every open set is open in X.

Theorem 3.4. *If $f : X \to Y$ is slightly $\delta - \beta$-continuous and Y is extemally disconnected, then f is faintly $\delta - \beta$-continuous.*

Proof. Let $x \in X$ and let V be a θ-open set in Y containing $f(x)$. Therefore, there exists an open set W containing $f(x)$ such that $cl(W) \subset V$. Since, Y is extremally disconnected, therefore, $cl(W)$ is clopen in Y containing $f(x)$. Since, f is slightly $\delta - \beta$-continuous, there exists $U \in \delta\beta O(X, x)$ such that $f(U) \subset cl(W) \subset V$. Hence, f is faintly $\delta - \beta$-continuous. □

Theorem 3.5. *Let (X, τ) and (Y, σ) be topological spaces. The following statements are equivalent for a function $f : X \to Y$.*

(a) f is slightly $\delta - \beta$-continuous,

(b) $f^{-1}(V) \in \delta\beta O(X)$ for each clopen set V in Y,

(c) $f^{-1}(V) \in \delta\beta C(X)$ for each clopen set V in Y,

(d) $f^{-1}(V)$ is $\delta - \beta$-clopen in X for each clopen set V in Y,

(e) $f^{-1}(V) \in \delta\beta O(X)$ for each δ^*-open set V in Y,

(f) $f^{-1}(V) \in \delta\beta C(X)$ for each δ^*-closed set V in Y

(g) $f(\beta cl_\delta(A)) \subset cl_{\delta^*}(f(A))$ for each $A \subset X$,

(h) $\beta cl_\delta(f^{-1}(B)) \subset f^{-1}(cl_{\delta^*}(B))$ for each $B \subset Y$.

Proof. (a)$\Rightarrow$ (b): Let V be a clopen set in Y such that $x \in f^{-1}(V)$. Thus $f(x) \in V$. Therefore, there exists $U \in \delta\beta O(X, x)$ such that $f(U) \subset V$. This implies $U \subset f^{-1}(V)$. Thus $f^{-1}(V) \in \delta\beta O(X)$.

(b)$\Rightarrow$ (c): Let V be a clopen set in Y. then $Y \setminus V$ is also a clopen set in Y. This implies $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V) \in \delta\beta O(X)$. Hence, $f^{-1}(V) \in \delta\beta C(X)$.

(c) $\Rightarrow$ (d): Obvious.

(d)$\Rightarrow$ (a): Obvious.

(a)$\Rightarrow$ (e): Let V be a δ^*-open set in Y and let $x \in f^{-1}(V)$. Thus $f(x) \in V$. Therefore, there exists a clopen set W in Y such that $f(x) \in W \subset V$. Slightly $\delta - \beta$-continuity of f implies that there exists $U \in \delta\beta O(X, x)$ such that $f(U) \subset W \subset V$. Thus $U \subset f^{-1}(V)$. Hence, $f^{-1}(V) \in \delta\beta O(X)$.

(e)$\Rightarrow$ (f): Let V be a δ^*-closed set in Y. Thus $Y \setminus V$ is δ^*-closed in Y. This implies that $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V) \in \delta\beta O(X)$. Hence, $f^{-1}(V) \in \delta\beta C(X)$.

(f)$\Rightarrow$ (g): $cl_{\delta^*}(f(A))$ is a δ^*-closed set in Y containing $f(A)$. Thus $f^{-1}(cl_{\delta^*}(f(A)))$ is a $\delta - \beta$-closed set in X containing A. Thus $\beta cl(A) \subset f^{-1}(cl_{\delta^*}(f(A)))$. Hence, $f(\beta cl_\delta(A)) \subset cl_{\delta^*}(f(A))$.

(g)$\Rightarrow$(h): Let $B \subset Y$. Thus $f(\beta cl_\delta(f^{-1}(B))) \subset cl_{\delta^*}(f(f^{-1}(B))) \subset cl_{\delta^*}(B)$. Hence, $\beta cl_\delta(f^{-1}(B)) \subset f^{-1}(cl_{\delta^*}(B))$.

(h)$\Rightarrow$(c): Let V be a clopen set in Y. Thus V is δ^*-closed in Y. Therefore $\beta cl_\delta(f^{-1}(V)) \subset f^{-1}(cl_{\delta^*}(V)) = f^{-1}(V)$. Hence, $f^{-1}(V) \in \delta\beta C(X)$. □

Lemma 3.1. *If $A \in \delta\beta O(X)$ and $U \in \delta O(X)$, then $A \cap U \in \delta\beta O(U)$ [8].*

Theorem 3.6. *If $f : X \to Y$ is slightly $\delta - \beta$-continuous and U is δ-open, then $f|_U : U \to Y$ is slightly $\delta - \beta$-continuous.*

Proof. Let $V \in \delta\beta O(Y)$. Since, f is slightly $\delta - \beta$-continuous, therefore, $f^{-1}(V) \in \delta\beta O(X)$. Now, $(f|_U)^{-1}(V) = f^{-1}(V) \cap U \in \delta\beta O(U)$. Therefore, $f|_U$ is slightly $\delta - \beta$-continuous. □

Lemma 3.2. *Let $A \subset U \subset X, A \in \delta\beta O(U)$ and U is open in X, then $A \in \delta\beta O(X)$.*

Proof. We have

$$A \subset cl_U(int_U(cl_{\delta U}(A))) \subset cl(int_U(cl_{\delta U}(A))) = cl(int_U(cl_{\delta U}(A)) \cap U)$$
$$= cl(int_U(cl_{\delta U}(A)) \cap int(U)) = cl(int(cl_{\delta U}(A))) \subset cl(int(cl_\delta(A))).$$

Hence, $A \in \delta\beta O(X)$. □

Theorem 3.7. *Let $f : X \to Y$ be a function and let $\{U_\alpha : \alpha \in \Lambda\}$ be an open cover of X. If $f|_{U_\alpha}$ is slightly $\delta - \beta$-continuous for each $\alpha \in \Lambda$, then f is a slightly $\delta - \beta$-continuous function.*

Proof. Suppose that V is any clopen set in Y. Since $f|_\alpha$ is slightly $\delta - \beta$-continuous for each $\alpha \in \Lambda$, it follows that $(f|_{U_\alpha})^{-1}(V) \in \delta\beta O(U_\alpha)$. We have, $f^{-1}(V) = \bigcup_{\alpha\in\Lambda}(f^{-1}(V) \cap U_\alpha) = \bigcup_{\alpha\in\Lambda}(f|_{U_\alpha}^{-1}(V))$. Then, by Lemma 3.2 we obtain, $f^{-1}(V) \in \delta\beta O(X)$. Hence, f is slightly $\delta - \beta$-continuous. □

Theorem 3.8. *Let $f : X \to Y$ be a function and let $x \in X$. If there exists an open set U in X such that $x \in U$ and $f|_U$ is a slightly $\delta - \beta$-continuous function at x, then f is slightly $\delta - \beta$-continuous at x.*

Proof. Suppose that V be a clopen set in Y containing $f(x)$. Since $f|_U$ is slightly $\delta - \beta$-continuous at x, there exists $W \in \delta\beta O(U, x)$ such that $f(W) = (f|_U(W)) \subset V$. Since, U is open in X containing x, it follows from Lemma 3.2 that $W \in \delta\beta O(X, x)$. This shows that f is slightly $\delta - \beta$-continuous at x. □

Theorem 3.9. *Let $f : X \to Y$ be a function and let $g : X \to X \times Y$ be the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. Then g is slightly slightly $\delta - \beta$-continuous if and only if f is slightly $\delta - \beta$-continuous.*

Proof. Let g be slightly $\delta-\beta$-continuous and let V be a clopen set in Y. Then $X\times V$ is clopen in $X\times Y$. Since g is slightly $\delta-\beta$-continuous, thus $f^{-1}(V)=g^{-1}(X\times V)\in\delta\beta O(X)$. Thus, f is slightly $\delta-\beta$-continuous.

Conversely, let $x\in X$ and let W be a closed subset of $X\times Y$ containing $g(x)$. Then $W\cap(\{x\}\times Y)$ is clopen in $\{x\}\times Y$ containing $g(x)$. Also $\{x\}\times Y$ homeomorphic to Y. Hence $\{y\in Y:(x,y)\in W\}$ is a clopen subset of Y. Since f is slightly $\delta-\beta$-continuous, $\bigcup\{f^{-1}(y):(x,y)\in W\}$ is a $\delta-\beta$-open subset of X. Further $x\in\bigcup\{f^{-1}(y):(x,y)\in W\}\subset g^{-1}(W)$. Then $g^{-1}(W)$ is $\delta-\beta$-open. Hence g is slightly $\delta-\beta$-continuous.

□

Definition 3.4. A function $f:X\to Y$ is called $\delta-\beta$-irresolute if for every $\delta-\beta$-open set G in Y, $f^{-1}(G)$ is a $\delta-\beta$-open set in X.

Definition 3.5. A function $f:X\to Y$ is called $\delta-\beta$-open if for every $\delta-\beta$-open set H in X, $f(H)$ is a $\delta-\beta$-open set in Y.

Remark 3. Composition of two slightly $\delta-\beta$-continuous functions may not be slightly $\delta-\beta$-continuous as given by the following example.

Example 3. Let $X=Y=Z=\{a,b,c\}$ and let

$$\tau=\{X,\emptyset,X,\{a\},\{b\},\{a,b\}\},$$

$$\sigma=\{Y,\emptyset,\{a\},\{c\},\{a,c\},\{a,b\}\},$$

$$\gamma=\{Z,\emptyset,\{b\},\{a,c\}\}.$$

Then the functions $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\gamma)$ defined by $f(a)=b,f(b)=c,f(c)=a$ and $g(a)=c,g(b)=a,g(c)=b$ respectively are both $\delta-\beta$-continuous but $g_of:(X,\tau)\to(Z,\gamma)$ is not $\delta-\beta$-continuous at c.

Theorem 3.10. *Let $f:X\to Y$ and $g:Y\to Z$ be functions. Then, the following properties hold:*

(a) *If f is $\delta-\beta$-irresolute and g is slightly $\delta-\beta$-continuous, then g_of is slightly $\delta-\beta$-continuous.*

(b) *If f is $\delta-\beta$-irresolute and g is $\delta-\beta$-continuous, then g_of is slightly $\delta-\beta$-continuous.*

(c) *If f is $\delta-\beta$-irresolute and g is slightly-continuous, then g_of is slightly $\delta-\beta$-continuous.*

Proof. (a) Let V be any clopen set in Z. Since g is slightly $\delta-\beta$-continuous, $g^{-1}(V)$ is $\delta-\beta$-open. Since, f is $\delta-\beta$-irresolute, $f^{-1}(g^{-1}(V)) = (g_o f)^{-1}(V)$ is $\delta-\beta$-open. Therefore, $g_o f$ is slightly $\delta-\beta$-continuous.

(b) It follows from the fact that every clopen set is open.

(c) It follows from the fact that every open set is $\delta-\beta$-open. □

Theorem 3.11. *If $f : X \to Y$ is $\delta-\beta$-open and surjective and $g_o f : X \to Z$ is slightly $\delta-\beta$-continuous, then $g : Y \to Z$ is slightly $\delta-\beta$-continuous.*

Proof. Let V be any clopen set in Z. Since $g_0 f$ is slightly γ-continuous, therefore, $(g_0 f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\delta-\beta$-open in X. Since, f is $\delta-\beta$-open and surjective, therefore, $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is $\delta-\beta$-open. Hence, g is slightly $\delta-\beta$-continuous. □

Remark 4. The condition **surjectiveness** can not be dropped from f as shown by the following example:

Example 4. Let $X = Y = Z = \{a, b, c\}$ and let $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$, $\sigma = \{Y, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$, $\gamma = \{Z, \emptyset, \{b\}, \{a, c\}\}$. Let the function $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \gamma)$ be defined by $f(a) = f(c) = a, f(b) = c$ and $g(x) = x$ for all $x \in Y$ respectively. Then f is $\delta-\beta$-open, $g_0 f$ is slightly $\delta-\beta$-continuous but g is not slightly $\delta-\beta$-continuous at $b \in Y$.

Theorem 3.12. *Let $f : X \to Y$ be surjective, $\delta-\beta$-irresolute and $\delta-\beta$-open and $g : Y \to Z$ be a function. Then $g_o f$ is slightly $\delta-\beta$-continuous if and only if g is slightly $\delta-\beta$-continuous.*

Definition 3.6. A net $\{x_\lambda : \lambda \in \Lambda\}$ is said to be $\delta-\beta$-convergent [8] to a point $x \in X$ if for any $U \in \delta\beta O(X, x)$, there exists $\lambda_0 \in \Lambda$ such that $x_\lambda \in U$ for $\lambda \geq \lambda_0$.

Definition 3.7. A net $\{x_\lambda : \lambda \in \Lambda\}$ is said to be co-convergent [9] to a point $x \in X$ if for any $U \in \delta\beta O(X, x)$, there exists $\lambda_0 \in \Lambda$ such that $x_\lambda \in U$ for $\lambda \geq \lambda_0$.

Theorem 3.13. *If a function $f : X \to Y$ is slightly $\delta-\beta$-continuous, then for each point $x \in X$ and each net $\{x_\lambda : \lambda \in \Lambda\}$ in X $\delta-\beta$-converging to x, the net $\{f(x_\lambda) : \lambda \in \Lambda\}$ co-converges to $f(x)$.*

Proof. Let $x \in X$ and let $\{x_\lambda : \lambda \in \Lambda\}$ be any net in X $\delta-\beta$-converges to x. Let V be a clopen subset of Y containing $f(x)$. Since, f is slightly $\delta-\beta$-continuous at x, there exists $U \in \delta\beta O(X, x)$ such that $f(U) \subset V$. Since, the net $\{x_\lambda : \lambda \in \Lambda\}$ $\delta-\beta$-converges to x, there exists $\lambda_0 \in \Lambda$ such that $x_\lambda \in U$ for all

$\lambda \geq \lambda_0$. Thus $f(x_\lambda) \in f(U) \subset V$ for all $\lambda \geq \lambda_0$. Hence, the net $\{f(x_\lambda) : \lambda \in \Lambda\}$ co-converges to $f(x)$. □

Definition 3.8. A topological space X is called $\delta-\beta$-connected [8] if X can not be expressed as the union of two disjoint non-empty $\delta-\beta$-open sets.

Theorem 3.14. *If $f : X \to Y$ is slightly $\delta-\beta$-continuous surjection and X is $\delta-\beta$-connected space, then Y is connected space.*

Proof. Suppose that Y is not a connected space. Then there exists non-empty disjoint open sets U and V such that $Y = U \cup V$. Therefore, U and V are clopen sets in Y. Since f is slightly $\delta-\beta$-continuous, then $f^{-1}(U), f^{-1}(V)$ are $\delta-\beta$-open in X. Moreover, $f^{-1}(U)$ and $f^{-1}(V)$ are non-empty disjoint and $X = f^{-1}(U) \cup f^{-1}(V)$. This shows that X is not $\delta-\beta$-connected. This is a contradiction. Hence, Y is connected. □

Definition 3.9. A topological space X is said to be

(a) $\delta-\beta-T_1$ [8] if for each pair of distinct points x and y of X, there exist $\delta-\beta$-open sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.

(b) $\delta-\beta-T_2$ ($\delta-\beta$-Hausdorff) [8] if for each pair of distinct points x and y of X, there exist disjoint $\delta-\beta$-open sets U and V such that $x \in U$ and $y \in V$.

(c) clopen T_1 [9] if for each pair of distinct points x and y of X, there exist clopen sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.

(d) clopen T_2 (clopen Hausdorff or ultra-Hausdorff) [9] if for each pair of distinct points x and y of X, there exist disjoint clopen sets U and V such that $x \in U$ and $y \in V$.

Theorem 3.15. *If $f : X \to Y$ is a slightly $\delta-\beta$-continuous injection and Y is clopen T_1, then X is $\delta-\beta-T_1$.*

Proof. Let Y be clopen T_1 and let x and y be two distinct points of X. Since, f is an injection, therefore, $f(x) \neq f(y)$ in Y. Also, clopen T_1-ness of Y implies that there exist clopen sets V and W in Y such that $f(x) \in V, f(y) \notin V, f(x) \notin W$ and $f(y) \in W$. Since, f is slightly $\delta-\beta$-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are $\delta-\beta$-open subsets of X such that $x \in f^{-1}(V), y \notin f^{-1}(V), x \notin f^{-1}(W)$ and $y \notin f^{-1}(W)$. This shows that X is $\delta-\beta-T_1$. □

Theorem 3.16. *If $f : X \to Y$ is a slightly $\delta - \beta$-continuous injection and Y is clopen T_2, then X is $\delta - \beta - T_2$.*

Proof. Let Y be a clopen T_2 space and let x, y be two distinct points of X. Since, f is an injection, therefore, $f(x) \neq f(y)$ in Y. Therefore, there exist disjoint clopen sets V and W in Y such that $f(x) \in V$ and $f(y) \in W$. Since, f is slightly $\delta - \beta$-continuous, therefore, $f^{-1}(V) \in \delta\beta O(X, x)$ and $f^{-1}(W) \in \delta\beta O(X, y)$ and $f^{-1}(V) \cap f^{-1}(W) = \emptyset$. Hence, X is $\delta - \beta - T_2$. □

Definition 3.10. A topological space X is said to be clopen regular [9] ($\delta - \beta$-regular) if for each clopen (resp. $\delta - \beta$-closed) set F and each point $x \notin F$, there exist disjoint open sets U and V such that $x \in U$ and $F \subset V$.

Definition 3.11. A topological space X is said to be clopen normal [9] ($\delta - \beta$-normal) if for every pair of disjoint clopen (resp. $\delta - \beta$-closed) sets F_1, F_2, there exist disjoint open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Theorem 3.17. *If f is slightly $\delta - \beta$-continuous injective open function from a $\delta - \beta$-regular space X onto a space Y, then Y is clopen regular.*

Proof. Let F be a clopen set in Y and let $y \notin F$. Since, f is onto, there exists $x \in X$ such that $f(x) = y$. Since f is slightly $\delta - \beta$-continuous, $f^{-1}(F)$ is a $\delta - \beta$-closed set. We have, $x \notin f^{-1}(F)$. Since, X is $\delta - \beta$-regular, there exist disjoint open sets U and V such that $f^{-1}(F) \subset U$ and $x \in V$. We obtain that $F = f(f^{-1}(F)) \subset f(U)$ and $y = f(x) \in f(V)$ such that $f(U)$ and $f(V)$ are disjoint open sets. This shows that Y is clopen regular. □

Theorem 3.18. *If f is slightly $\delta - \beta$-continuous injective open function from a $\delta - \beta$-normal space X onto a space Y, then Y is clopen normal.*

Proof. Let F_1 and F_2 be two disjoint clopen subsets of Y. Since f is slightly $\delta - \beta$-continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are disjoint $\delta - \beta$-closed sets. Since, X is $\delta - \beta$-normal, there exist disjoint open sets U and V such that $f^{-1}(F_1) \subset U$ and $f^{-1}(F_2) \subset V$. Thus, we obtain that $F_1 = f(f^{-1}(F_1)) \subset f(U)$ and $F_2 = f(f^{-1}(F_2)) \subset f(V)$ such that $f(U)$ and $f(V)$ are disjoint open sets. Thus, Y is clopen normal. □

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