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A RUMINATION OF THE GOLDBACH CONJECTURE

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Abstract: The Goldbach conjecture is a challenging prime number riddle. This paper offers some thoughts on the conjecture, provides some numerical evidence and describes some approaches.

AMS Subject Classification: 11-XX
Key Words: regularity of pattern, recurrent, identical mathematical structures, building-blocks, "reflection" principle

1. Introduction

The problem of whether there is an infinitude of cases of even numbers which are each the sum of 2 primes is an inherently difficult one to solve, as infinity (normally symbolised by: ∞) is a difficult concept and is against common sense. It is impossible to count, calculate or live to infinity, perhaps with the exception of God. Infinity is a nebulous idea and appears to be only an abstraction devoid of any actual practical meaning. How do we quantify infinity? How big is infinity? The difficulty of the problem of infinity has been compounded by Georg Cantor who proved that there are actually different sizes to infinity, an idea so bizarre to many mathematicians that he was attacked for his ideas during much of his career. The attack was so serious that he suffered mental

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illness and severe depression. However, after his death his ideas became widely accepted as the only consistent, accurate and powerful definition of infinity. Hilbert had honoured him by saying, "No one shall drive us from the paradise Cantor has created for us." Nevertheless, in this paper offering solutions for infinity, in this case the infinity of the even numbers which are each the sum of 2 primes, incontrovertible evidence that the peculiar characteristics of the prime numbers themselves contribute to the infinite "generation" of such even numbers would be put forward.

2. Solution

Christian Goldbach, tutor to the teenage Czar Peter II, had examined dozens of even numbers and noticed that he could split all of them into the sum of 2 primes. Thus, his conjecture that every even number after the number 2 is the sum of 2 prime numbers, for example:

$4 = 2 + 2$
$6 = 3 + 3$
$8 = 3 + 5$
$10 = 3 + 7$ and $5 + 5$
$12 = 5 + 7$
$14 = 3 + 11$ and $7 + 7$
$16 = 3 + 13$ and $5 + 11$
$18 = 5 + 13$ and $7 + 11$
$20 = 3 + 17$ and $7 + 13$
$50 = 19 + 31$
$100 = 53 + 47$
$21{,}000 = 17 + 20{,}983$
$\vdots$

Cmputer searches completed in 2000 had verified that all even numbers up to 400 trillion (4 x 10^{14}), which is not a small list, are sums of 2 primes, while in 2008, a distributed computer search ran by Tomas Oliveira e Silva, a researcher at the University of Aveiro, Portugal, had further verified the Goldbach conjecture up to 12 x 10^{17}. But is the conjecture valid?

Theorem. *Every even number after 2 is the sum of 2 primes.*

Proof. By Euclid's proof, there is an infinitude of primes; that is, the list of primes: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37,..., continues to infinity.

Goldbach's conjecture states that every even number after the number 2 is the sum of 2 primes. How do we prove this?

First of all, we ask a "reversed" question here (as opposed to Goldbach's conjecture). We ask whether all the prime numbers in the infinite list of prime numbers would combine with each other to form a regular, continuous (*without breaks or gaps*) and infinite list of even numbers. This would lead to our proof.

Let us now take a subset of primes from the infinite set of prime numbers, say, all the primes found in the set of integers ranging from 1 to 50, that is, the following subset of prime numbers:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 and 47.

Then, we conduct a close examination of how this subset of prime numbers "behaves", that is, how the prime numbers combine (one-to-one) with each other to form even numbers, and observe whether there is any "regularity of pattern" in the way they do so. Here, we look at how the prime numbers from 2 To 47 combine with each other to form even numbers.

We could observe the primes from 2 to 47 "generating" a regular, continuous (*without breaks or gaps*) list of even numbers from 4 to 94. This is a regular, continuous list of even numbers, the even numbers becoming evidently progressively more repetitious. For example, there are 5 discernable combinations of primes/partitions for the even number 48, which is as follows:

a) $19 + 29 = 48$
b) $17 + 31 = 48$
c) $11 + 37 = 48$
d) $5 + 43 = 48$
e) $7 + 41 = 48$

and, there are 5 discernable combinations of primes/partitions for the even number 54, which is as follows:

1. $17 + 37 = 54$
2. $13 + 41 = 54$
3. $11 + 43 = 54$
4. $7 + 47 = 54$
5. $23 + 31 = 54$

And many others.

There appears to be a "regularity of pattern" in the way the even numbers "pop up".

From this, we could thus conclude the following characteristic or "pattern" of the prime numbers: They would combine (one-to-one) with each other to form a regular, continuous (*without breaks or gaps*) list of even numbers, with "overwhelming repetitions" all over the place.

Next, we select another subset of primes from the infinite set of prime numbers. We would here take the prime numbers from the set of integers 51 to 100, which is just "next to" the set of integers 1 to 50 from which we have taken our first subset of prime numbers, to be our second subset of primes. This second subset of prime numbers is as follows:

53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

As usual, we conduct a close examination of how this second subset of prime numbers "behaves," that is, how they combine (one-to-one) with each other to form even numbers, and observe whether there is "regularity of pattern" in the way they do so.

Here, as earlier, we could observe the primes from 53 to 97 "generating" a regular, continuous (*without breaks or gaps*) list of even numbers ranging from 56 to 194. This regular, continuous list of even numbers is also evidently progressively more repetitious. For example, there are 8 discernable combinations of primes/partitions for the even number 90, which is as follows:

1. $53 + 37 = 90$
2. $59 + 31 = 90$
3. $61 + 29 = 90$
4. $67 + 23 = 90$
5. $71 + 19 = 90$
6. $73 + 17 = 90$
7. $79 + 11 = 90$
8. $83 + 7 = 90$

and, there are 8 discernable combinations of primes/partitions for the even number 120, which is as follows:

1. $53 + 67 = 120$
2. $59 + 61 = 120$

3. $67 + 53 = 120$
4. $73 + 47 = 120$
5. $79 + 41 = 120$
6. $83 + 37 = 120$
7. $89 + 31 = 120$
8. $97 + 23 = 120$

And many others.

There appears to be a "regularity of pattern" in the way the even numbers "pop up" here - as a matter of fact, this "regularity of pattern" resembles that found in the earlier listing.

The list of even numbers "generated" by this second subset of prime numbers, that is, the regular, continuous list of even numbers ranging from 56 to 194, even overlaps (by a wide margin) the list of even numbers "generated" by the first subset of primes (2 to 47), that is, the regular, continuous list of even numbers ranging from 4 to 94.

From this listing also, from the "characteristics" found in all these listings, where the "regularity of pattern" of the appearance of the even numbers is evident, we could deduce the following characteristic of the prime numbers: The prime numbers would combine (one-to-one) with each other to form a regular, continuous (*without breaks or gaps*) list of even numbers, with "overwhelming repetitions" all over the place. This could be further confirmed by studying the even numbers "generated", e.g., by the 3 subsets of primes by combining with other prime numbers, including the prime numbers before them, for the 3 consecutive sets of integers, 101 To 150, 151 To 200, and, 201 To 250, with "overwhelming repetitions" all over the place (see Item (1) in the data below, where there are also much further examples).

Lemma. The well-established self-similarity concept, which was developed by Mitchell Feigenbaum in the 1970s and which brought him fame, upon which the method of renormalization in perturbation theory is based, postulates that there is a tendency of identical mathematical structures to recur on many levels. Within a given structure, there would be smaller copies of the same structure, their sizes being determined by the scaling factor. Feigenbaum found that at the utmost tips of the fig-tree, there is some mathematical structure which remains the same when its size is changed (enlarged) by a scaling factor of 4.669, which is found to be a constant like pi (3.142); this structure is the shape of the fig-tree itself; in other words, little whorls could be found within big whorls. Renormalization has been a well-established technique in chaos theory/fractal geometry and is a mathematical trick which functions rather like a microscope, zooming in on the self-similar structure, removing any approximations, and filtering out everything else. All this shows the universality of some features of chaos. That is, some kind of order or pattern could be found in or is inherent in disorder or chaos. In other words, the elements of an infinite subset of an infinite set contain all the recursive significant properties of that set unless the process which selects the elements of the subset directly excludes a property.

To make it simpler, we re-phrase this concept as follows: The characteristic of a mountain or infinite volume of sand is reflected in the characteristic of some grains of sand found there so that studying the characteristic of some grains of sand found there is sufficient for deducing the characteristic of the mountain or infinite volume of sand. Likewise, if x is a subset of y and if x is a list of prime numbers while y is another list of prime numbers, the characteristic presence of the even numbers “generated” by all the primes in x suggests (or reflects) the characteristic presence of even numbers “generated” by all the primes in y, so that if y is an infinite list of prime numbers, whence the prime numbers in it run to infinity, so do the even numbers “generated” by all the primes in it. What is described here is actually the “reflection” principle.

Therefore, by the above-mentioned principle, all the above-mentioned selected subsets of primes in the infinite set of primes would each reflect (present an image of, or, display something which has similarity with) the characteristic of this infinite set of primes; that is, all the infinite primes in the infinite set of primes, including its infinite subsets such as the selected subsets mentioned above, would combine (one-to-one) with each other to form a regular, continuous (*without breaks or gaps*) and infinite (implied by the infinitude of the primes (vide Euclid’s proof) and the even numbers themselves) list of even numbers. It is evident that the higher up the infinite list of primes we go, the more “overwhelming” or dense the (one-to-one) combinations of primes (in the formation of even numbers) would become, the number of permutations of the combinations of primes tending towards infinity (with the infinity of the prime numbers), as a study with further selected subsets of primes would reveal. A study of the even numbers “generated” by all these subsets of primes would show that the higher up the subsets of primes we go the more “overwhelmingly” the even numbers “generated” would repeat themselves and overlap. This is (very) significant. Though the infinitude of the prime numbers would ensure that there would always be new even numbers being “generated”, there is also the “fear” that there might be gaps, breaks or lack of continuity in the even numbers thus ”generated”. But, it is evident that these more and more profuse repetitions and overlaps of the even numbers thus “generated” by the primes the higher up the infinite list of prime numbers we go “ensure” that such gaps or breaks would not appear between the even numbers “generated” - they “ensure” that the even numbers thus “generated” by the primes in the infinite list of primes would be regular, continuous, *without breaks or gaps*, and, in consecutive running order. Also (very) significant is the great number of new even numbers that each of the primes in these subsets of primes helps to “generate”. This “profuse generation” of “regular batches” of even numbers by the prime numbers represents a characteristic or feature of the prime numbers, a universal “pattern” or feature of the “chaotic” infinite prime numbers, or, recurrent, identical mathematical structures, which is all in accordance with the above lemma. (This “pattern of behaviour” of the prime numbers, as described in the paper, is analogous to the “self-similar mathematical pattern or structure” (which is the shape of the fig-tree itself) of the various parts of the fig-tree, that is, its trunk to bough section, bough to branch section, branch to twig section and twig to twiglet section, in Feigenbaum’s famous fig-tree example, and, such self-similar mathematical pattern or structure, or, fractal characteristic, could also be found in other aspects of nature, for example, waves, turbulence or chaos, the structures of viruses and bacteria, polymers and ceramic materials, the universe and many others, even the movements of prices in financial markets, the growths of populations, the sound of music, the flow of blood through our circulatory system, the behaviour of people en masse, etc., which have all spawned a relatively new and important branch of mathematics with wide practical applications known as fractal geometry, which has been pioneered by Benoit Mandelbrot. As a matter of fact, self-similarity or fractal characteristic could be regarded as the fundamental mathematical aspect found in practically everything in nature including the numbers such as the prime numbers and the even numbers which are the subjects of our investigation here, and, this new branch of mathematics, fractal geometry, besides having a great practical impact on us also gives us a deeper vision of the universe in which we live and our place in it.) In other words, by the above lemma the infinity of the prime numbers implies the infinity of the “profuse generation” of “regular batches” of even numbers by the prime numbers, that is, the validity of the Goldbach conjecture.

Here, we take a close look at the following data:-

1. No. Of Old/Repeated (Also Appeared Earlier) Even Numbers/Overlaps “Generated” (By The Additions/Combinations Of 2 Primes), For Integers 1 To 1,250 (*See Appendix 1 For Example Of* Computation Method)

(a) Set Of Integers, 1 To 50, With 14 Primes Within It = Not Applicable
(aa) Percentage Increase In Repetition = Not Applicable

1. Set Of Integers, 51 To 100, With 10 Primes Within It = **20** Repeated Even Nos.

(bb) Percentage Increase In Repetition = Not Applicable

1. Set Of Integers, 101 To 150, With 10 Primes Within It = 46 Repeated Even Nos.
   (a) Percentage Increase In Repetition = (46 - 20) ÷20 x 100% = **130%**
2. Set Of Integers, 151 To 200, With 11 Primes Within It = 73 Repeated Even Nos.
   (a) Percentage Increase In Repetition = (73 - 46) ÷46 x 100% = 58.7%

(e) Set Of Integers, 201 To 250, With 7 Primes Within It = 93 Repeated Even Nos.

1. Percentage Increase In Repetition = (93 - 73) ÷73 x 100% = 27.4%
2. Set Of Integers, 251 To 300, With 9 Primes Within It = 115 Repeated Even Nos.
3. Percentage Increase In Repetition = (115 - 93) ÷93 x 100% = 23.66%
4. Set Of Integers, 301 To 350, With 8 Primes Within It = 139 Repeated Even Nos.
5. Percentage Increase In Repetition = (139 - 115) ÷115 x 100% = 20.87%
6. Set Of Integers, 351 To 400, With 8 Primes Within It = 172 Repeated Even Nos.
7. Percentage Increase In Repetition = (172 - 139) ÷139 x 100% = 23.74%
8. Set Of Integers, 401 To 450, With 9 Primes Within It = 196 Repeated Even Nos.
9. Percentage Increase In Repetition = (196 - 172) ÷172 x 100% = 13.95%
10. Set Of Integers, 451 To 500, With 8 Primes Within It = 220 Repeated Even Nos.

11. Percentage Increase In Repetition = (220 - 196) ÷196 x 100% = 12.24%
12. Set Of Integers, 501 To 550, With 6 Primes Within It = 247 Repeated Even Nos.
13. Percentage Increase In Repetition = (247 - 220) ÷220 x 100% = 12.27%
14. Set Of Integers, 551 To 600, With 8 Primes Within It = 268 Repeated Even Nos.
15. Percentage Increase In Repetition = (268 - 247) ÷247 x 100% = 8.5%
16. Set Of Integers, 601 To 650, With 9 Primes Within It = 298 Repeated Even Nos.
17. Percentage Increase In Repetition = (298 - 268) ÷268 x 100% = 11.19%
18. Set Of Integers, 651 To 700, With 7 Primes Within It = 320 Repeated Even Nos.
19. Percentage Increase In Repetition = (320 - 298) ÷298 x 100% = 7.38%
20. Set Of Integers, 701 To 750, With 7 Primes Within It = 340 Repeated Even Nos.
21. Percentage Increase In Repetition = (340 - 320) ÷320 x 100% = 6.25%
22. Set Of Integers, 751 To 800, With 7 Primes Within It = 367 Repeated Even Nos.

(pp) Percentage Increase In Repetition = (367 - 340) ÷340 x 100% = 7.94%
(q) Set Of Integers, 801 To 850, With 7 Primes Within It = 392 Repeated Even Nos.
(qq) Percentage Increase In Repetition = (392 - 367) ÷367 x 100% = 6.81%

1. Set Of Integers, 851 To 900, With 8 Primes Within It = 412 Repeated Even Nos.

(rr) Percentage Increase In Repetition = (412 - 392) ÷392 x 100% = 5.1%

1. Set Of Integers, 901 To 950, With 7 Primes Within It = 433 Repeated Even Nos.

(ss) Percentage Increase In Repetition = (433 - 412) ÷412 x 100% = 5.1%

1. Set Of Integers, 951 To 1,000, With 7 Primes Within It = 470 Repeated Even Nos.
2. Percentage Increase In Repetition = (470 - 433) ÷433 x 100% = 8.55%
3. Set Of Integers, 1,001 To 1,050, With 8 Primes Within It = 492 Repeated Even Nos.

(uu) Percentage Increase In Repetition = (492 - 470) ÷470 x 100% = 4.68%

1. Set Of Integers, 1,051 To 1,100, With 8 Primes Within It = 523 Repeated Even Nos.
2. Percentage Increase In Repetition = (523 - 492) ÷492 x 100% = 6.3%
3. Set Of Integers, 1,101 To 1,150, With 5 Primes Within It = 545 Repeated Even Nos.

(ww) Percentage Increase In Repetition = (545 - 523) ÷523 x 100% = 4.21%

1. Set Of Integers, 1,151 To 1,200, With 7 Primes Within It = 553 Repeated Even Nos.

(xx) Percentage Increase In Repetition = (553 - 545) ÷545 x 100% = **1.47%**

1. Set Of Integers, 1,201 To 1,250, With 8 Primes Within It = **592** Repeated Even Nos.

(yy) Percentage Increase In Repetition = (592 - 553) ÷553 x 100% = **7.05%**

It could be seen that on the whole the No. Of Old/Repeated (Also Appeared Earlier) Even Numbers/Overlaps "Generated" (By The Additions/Combinations Of 2 Primes) increases progressively from 20 in (b) to 592 in (y), while it could be seen that the Percentage Increase In Repetition on the whole thins out from 130% in (cc) to 7.05% in (yy), with the lowest percentage increase of 1.47% found in (xx). This statistical trend or feature is not surprising and represents (very) significant evidence that lends support to the validity of the Goldbach conjecture - the infinitude of both the primes and the even numbers implies that the above overlaps increase progressively to infinity.

(2) Density Of New Even Numbers "Generated" (*See Appendix 1 For Example Of Computation Method*)

1. Set Of Integers, 51 To 100, With 10 Primes Within It = 5 New Even Nos. Per Prime No.

(No. Of New Even Nos. "Generated" = 50. No. Of Primes = 10.)

(b) Set Of Integers, 101 To 150, With 10 Primes Within It = 5.2 New Even Nos. Per Prime No.

(No. Of New Even Nos. "Generated" = 52. No. Of Primes = 10.)

(c) Set Of Integers, 151 To 200, With 11 Primes Within It = **4.55 New Even Nos. Per Prime No.**

(No. Of New Even Nos. "Generated" = 50. No. Of Primes = 11.)

1. Set Of Integers, 201 To 250, With 7 Primes Within It = 6 New Even Nos. Per Prime No.

(No. Of New Even Nos. "Generated" = 42. No. Of Primes = 7.)

1. Set Of Integers, 251 To 300, With 9 Primes Within It = 5.78 New Even Nos. Per Prime No.

(No. Of New Even Nos. "Generated" = 52. No. Of Primes = 9.)

1. Set Of Integers, 301 To 350, With 8 Primes Within It = 7 New Even Nos. Per Prime No.

(No. Of New Even Nos. "Generated" = 56. No. Of Primes = 8.)

1. Set Of Integers, 351 To 400, With 8 Primes Within It = 6 New Even Nos. Per Prime No.

(No. Of New Even Nos. "Generated" = 48. No. Of Primes = 8.)

1. Set Of Integers, 401 To 450, With 9 Primes Within It = 5.78 New Even Nos. Per Prime No.

(No. Of New Even Nos. "Generated" = 52. No. Of Primes = 9.)

1. Set Of Integers, 451 To 500, With 8 Primes Within It = 6.25 New Even Nos. Per Prime No.

(No. Of New Even Nos. "Generated" = 50. No. Of Primes = 8.)

(j) Set Of Integers, 501 To 550, With 6 Primes Within It = 8 New Even Nos. Per Prime No.

(No. Of New Even Nos. "Generated" = 48. No. Of Primes = 6.)

1. Set Of Integers, 551 To 600, With 8 Primes Within It = 6.5 New Even Nos. Per Prime No.

(No. Of New Even Nos. “Generated” = 52. No. Of Primes = 8.)

1. Set Of Integers, 601 To 650, With 9 Primes Within It = 5.33 New Even Nos. Per Prime No.

(No. Of New Even Nos. “Generated” = 48. No. Of Primes = 9.)

1. Set Of Integers, 651 To 700, With 7 Primes Within It = 6.29 New Even Nos. Per Prime No.

(No. Of New Even Nos. “Generated” = 44. No. Of Primes = 7.)

1. Set Of Integers, 701 To 750, With 7 Primes Within It = 7.43 New Even Nos. Per Prime No.

(No. Of New Even Nos. “Generated” = 52. No. Of Primes = 7.)

1. Set Of Integers, 751 To 800, With 7 Primes Within It = 7.71 New Even Nos. Per Prime No.

(No. Of New Even Nos. “Generated” = 54. No. Of Primes = 7.)

1. Set Of Integers, 801 To 850, With 7 Primes Within It = 6 New Even Nos. Per Prime No.

(No. Of New Even Nos. “Generated” = 42. No. Of Primes = 7.)

1. Set Of Integers, 851 To 900, With 8 Primes Within It = 6 New Even Nos. Per Prime No.

(No. Of New Even Nos. “Generated” = 48. No. Of Primes = 8.)

1. Set Of Integers, 901 To 950, With 7 Primes Within It = 8.57 New Even Nos. Per Prime No.

(No. Of New Even Nos. “Generated” = 60. No. Of Primes = 7.)

1. Set Of Integers, 951 To 1,000, With 7 Primes Within It = 7.14 New Even Nos. Per Prime No.

(No. Of New Even Nos. "Generated" = 50. No. Of Primes = 7.)

1. Set Of Integers, 1,001 To 1,050, With 8 Primes Within It = 6.5 New Even Nos. Per Prime No.

(No. Of New Even Nos. "Generated" = 52. No. Of Primes = 8.)

1. Set Of Integers, 1,051 To 1,100, With 8 Primes Within It = 6 New Even Nos. Per Prime No.

(No. Of New Even Nos. "Generated" = 48. No. Of Primes = 8.)

1. Set Of Integers, 1,101 To 1,150, With 5 Primes Within It = 6.4 New Even Nos. Per Prime No.

(No. Of New Even Nos. "Generated" = 32. No. Of Primes = 5.)

(w) Set Of Integers, 1,151 To 1,200, With 7 Primes Within It = **9.14 New Even Nos. Per Prime No.**

(No. Of New Even Nos. "Generated" = 64. No. Of Primes = 7.)

(x) Set Of Integers, 1,201 To 1,250, With 8 Primes Within It = 7 New Even Nos. Per Prime No.

(No. Of New Even Nos. "Generated" = 56. No. Of Primes = 8.)

Average Density For The Above 24 Items ((a) To (x)) = 155.54 ÷24 = 6.48 New Even Nos. Per Prime No.

Maximum Density = 9.14 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 64. No. Of Primes = 7.)

Minimum Density = 4.55 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 50. No. Of Primes = 11.)

Such a "profuse generation" of "regular batches" of even numbers by the prime numbers is (very) significant and represents a characteristic or feature of the prime numbers, a universal "pattern" or feature of the "chaotic" infinite prime numbers (or, recurrent, identical mathematical structures), which is excellently in accordance with the above lemma. This lends further support to the validity of the Goldbach conjecture, which, as stated above, is implied by both the infinitude of the primes and the even numbers.

There is indeed further incontrovertible proof which is obtainable by analysing a number of even numbers; e.g., we could split a group of 240 even consecutive numbers, from 4 to 482, into 8 equal batches (30 even numbers per batch) and analyse the batches, which buttresses the above evidence that the infinite quantity of primes would "generate" a regular, continuous (*without breaks or* *gaps*) and infinite list of even numbers. The density of distribution or prime additions/combinations per even number evidently become greater and greater the higher up the infinite list of the even numbers we go - this increase in density evidently represents a definite pattern in the "behaviour" of the prime numbers. This pattern is (very) significant and is discernable in the following example:-

1. 1 st. Batch Of 30 Even Numbers (4 To 62) (*See Appendix 2 For Example Of Computation Method*)
   (a) Maximum No. Of Prime Additions/Combinations Per Even Number = **5**
   (b) Minimum No. Of Prime Additions/Combinations Per Even Number = **1**
   (c) Density Of Distribution = Average Prime Additions/Combinations Per Even Number = **2.77** Prime Additions/Combinations Per Even Number (see Appendix for computation method)
2. 2 nd. Batch Of 30 Even Numbers (64 To 122)
   (a) Maximum No. Of Prime Additions/Combinations Per Even Number = 14
   (b) Minimum No. Of Prime Additions/Combinations Per Even Number = 2
   (c) Density Of Distribution = Average Prime Additions/Combinations Per Even Number = **6.1** Prime Additions/Combinations Per Even Number
   (d) Percentage Increase In Density Of Distribution = (6.1 - 2.77) ÷2.77 x 100% = 120.22%
3. 3 rd. Batch Of 30 Even Numbers (124 To 182)
   (a) Maximum No. Of Prime Additions/Combinations Per Even Number = 16
   (b) Minimum No. Of Prime Additions/Combinations Per Even Number = 4
   (c) Density Of Distribution = Average Prime Additions/Combinations Per Even Number = **9.07** Prime Additions/Combinations Per Even Number

(d) Percentage Increase In Density Of Distribution = (9.07 - 6.1) ÷6.1 x 100% = 48.69%

(4) 4 th. Batch Of 30 Even Numbers (184 To 242)

1. Maximum No. Of Prime Additions/Combinations Per Even Number = 22
2. Minimum No. Of Prime Additions/Combinations Per Even Number = 5
3. Density Of Distribution = Average Prime Additions/Combinations Per Even Number = **10.53** Prime Additions/Combinations Per Even Number
4. Percentage Increase In Density Of Distribution = (10.53 - 9.07) ÷9.07 x 100% = 16.1%

(5) 5 th. Batch Of 30 Even Numbers (244 To 302)

1. Maximum No. Of Prime Additions/Combinations Per Even Number = 21
2. Minimum No. Of Prime Additions/Combinations Per Even Number = 7
3. Density Of Distribution = Average Prime Additions/Combinations Per Even Number = **12.37** Prime Additions/Combinations Per Even Number
4. Percentage Increase In Density Of Distribution = (12.37 - 10.53) ÷10.53 x 100% = 17.47%

(6) 6 th. Batch Of 30 Even Numbers (304 To 362)

1. Maximum No. Of Prime Additions/Combinations Per Even Number = 27
2. Minimum No. Of Prime Additions/Combinations Per Even Number = 7
3. Density Of Distribution = Average Prime Additions/Combinations Per Even Number = **13.77** Prime Additions/Combinations Per Even Number
4. Percentage Increase In Density Of Distribution = (13.77 - 12.37) ÷12.37 x 100% = 11.32%

(7) 7 th. Batch Of 30 Even Numbers (364 To 422)

1. Maximum No. Of Prime Additions/Combinations Per Even Number = 30

2. Minimum No. Of Prime Additions/Combinations Per Even Number = 7

3. Density Of Distribution = Average Prime Additions/Combinations Per Even Number = **15.23** Prime Additions/Combinations Per Even Number

4. Percentage Increase In Density Of Distribution = (15.23 - 13.77) ÷13.77 x 100% = 10.6%

(8) 8 th. Batch Of 30 Even Numbers (424 To 482)

1. Maximum No. Of Prime Additions/Combinations Per Even Number = **30**

b) Minimum No. Of Prime Additions/Combinations Per Even Number = **9**

c) Density Of Distribution = Average Prime Additions/Combinations Per Even Number = **16.93** Prime

Additions/Combinations Per Even Number

d) Percentage Increase In Density Of Distribution = (16.93 - 15.23) ÷15.23 x 100% = 11.16%

The Density Of Distribution is expected to increase to infinity, though the Percentage Increase In Density Of Distribution is expected to thin out towards infinity - it could be seen above to increase from 2.77 prime additions/combinations per even number for batch of even numbers, 4 to 62, all the way up to 16.93 prime additions/combinations per even number for batch of even numbers, 424 to 482. This is nevertheless (very) significant evidence that lends support to the validity of the Goldbach conjecture. Also, the Maximum No. Of Prime Additions/Combinations Per Even Number and the Minimum No. Of Prime Additions/Combinations Per Even Number could be seen to range from 5 and 1 respectively for batch of even numbers, 4 to 62, to 30 and 9 respectively for batch of even numbers, 424 to 482. This trend of “upward increase” of the (maximum and minimum) numbers of prime additions/combinations for each even number implies that at some points toward infinity the numbers of prime additions/combinations for each even number could be thousands, millions, billions, trillions, and more, if only we have the computing power to compute/check such prime additions/combinations. This is (very) significant too and is also evidence that lends support to the validity of the Goldbach conjecture. By the infinitude of the primes and even numbers and the above lemma, these "patterns", as described here, would be there all the way to infinity, which would be in accordance with the Goldbach conjecture.

The one-to-one additions/combinations of the primes in the formation of even numbers do evidently become more and more "overwhelming" or profuse the higher up the infinite list of even numbers/prime numbers we go, thereby assuring an infinite, regular and consecutive supply of even numbers, as is evident from the example just above. This, together with the above-described evidently more and more profuse repetitions and overlaps of the even numbers "generated" by the primes the higher up the infinite list of prime numbers we go (refer to No. Of Old/Repeated (Also Appeared Earlier) Even Numbers/Overlaps "Generated" (By The Additions/Combinations Of 2 Primes), For Integers 1 To 1,250 above), go to show that the Goldbach conjecture becomes, evidently, even stronger and stronger the higher up the infinite list of prime numbers/even numbers we go. Here, we have in fact approached the problem from 2 different, but somewhat related, angles - by a statistical analysis of the "behaviour" of the primes in the formation of even numbers, and, a statistical analysis of the even numbers "generated" as a result. The statistical data thus obtained are indeed found to greatly support the Goldbach conjecture, evidently the more so the higher up the infinite list of prime numbers/even numbers we go, and, by the infinitude of the primes and even numbers and the above lemma there would be an infinitude of such statistical data thus obtained. Hence, by virtue of these imposing statistical trends, plus the statistical trend that a prolific number of new even numbers are always being "generated" (refer to Density Of New Even Numbers "Generated" above), as well as the infinitude of the prime numbers and the even numbers, together with the above lemma, we affirm the validity of the Goldbach conjecture.

Reversing the "reversed way", we hereby affirm that every even number after the number 2 in the infinite list of even numbers is a combination or sum of 2 primes. In fact, the prime numbers are the building-blocks or "atoms" of all the even numbers - and more - the prime numbers are the building-blocks of all the integers or whole numbers: every even number (with the exception of 2) is the sum of 2 prime numbers, and, every odd number is either a prime number, or, a composite of prime numbers (that is, the odd number has prime factors). It is truly the peculiar characteristics of the prime numbers themselves (as described above, whose distribution could in fact be predicted by the prime number theorem which had been proven, implying some pattern or fractal nature in the prime numbers as per the above lemma), which could be regarded as a self-similar or fractal feature as such, that are responsible for the Goldbach conjecture being true. By induction the Goldbach conjecture has been proven true - the above constitutes proof of the Goldbach conjecture (which ought to be known as the Goldbach Theorem instead).

This proof could be extended here. It has been mentioned above that the Goldbach conjecture had been tested and found to be correct for every even number up to 12 x 10^{17} by computer searches completed in 2008. Thus, by the above lemma, and, the infinitude of the primes and even numbers, this long list of consecutive even numbers up to 12 x 10^{17} reflects (indicates or implies) the fact that all the infinite even numbers above 12 x 10^{17} would each be the sum of 2 primes. (This is in accordance with the "reflection" principle stated above, wherein it is mentioned that the characteristic of a mountain or infinite volume of sand is reflected in the characteristic of some grains of sand found there so that studying the characteristic of some grains of sand found there is enough for deducing the characteristic of the mountain or infinite volume of sand.)

3. Conclusion

We declare that the Goldbach conjecture is true - every even number after the number 2 is indeed the sum of 2 primes.

This paper should be able to provide some food for further thought on the conjecture.

4. Appendix 1

(20) Set Of Integers, 1,201 To 1,250, With 8 Primes Within It

(a) Primes: 1,201; 1,213; 1,217; 1,223; 1,229; 1,231; 1,237 and 1,249

(b) No. Of Primes: 8

(c) No. Of Even Numbers "Generated" (Excluding Repetitions) By The 8 Primes = 648 (1,204 [1,201 + 3] To 2,498 [1,249

$$+ \quad 1,249$$

])

(d) No. Of New Even Numbers "Generated" = 56 (2,388 To 2,498)

(e) No. Of Old/Repeated (Also Appeared In (19) Above, With Some Also Having Appeared In (18), (17), (16), (15), (14),

(13), (12), (11), (10), (9) And (8) Above) Even Numbers "Generated" = 592 (1,204 To 2,386)

(f) Density Of New Even Numbers "Generated" = (d) ÷8 Primes = 56 ÷8 = 7 New Even Numbers Per Prime Number

5. Appendix 2

(8) 8 th. Batch Of 30 Even Numbers (424 To 482)

(a) 424: No. Of Above-mentioned Prime Additions/Combinations = 12
(b) 426: No. Of Above-mentioned Prime Additions/Combinations = 21
(c) 428: No. Of Above-mentioned Prime Additions/Combinations = **9**
(d) 430: No. Of Above-mentioned Prime Additions/Combinations = 14
(e) 432: No. Of Above-mentioned Prime Additions/Combinations = 19
(f) 434: No. Of Above-mentioned Prime Additions/Combinations = 14
(g) 436: No. Of Above-mentioned Prime Additions/Combinations = 11
(h) 438: No. Of Above-mentioned Prime Additions/Combinations = 22
(i) 440: No. Of Above-mentioned Prime Additions/Combinations = 15
(j) 442: No. Of Above-mentioned Prime Additions/Combinations = 13
(k) 444: No. Of Above-mentioned Prime Additions/Combinations = 22
(l) 446: No. Of Above-mentioned Prime Additions/Combinations = 12
(m) 448: No. Of Above-mentioned Prime Additions/Combinations = 13
(n) 450: No. Of Above-mentioned Prime Additions/Combinations = 29
(o) 452: No. Of Above-mentioned Prime Additions/Combinations = 14
(p) 454: No. Of Above-mentioned Prime Additions/Combinations = 12
(q) 456: No. Of Above-mentioned Prime Additions/Combinations = 26
(r) 458: No. Of Above-mentioned Prime Additions/Combinations = **9**
(s) 460: No. Of Above-mentioned Prime Additions/Combinations = 17
(t) 462: No. Of Above-mentioned Prime Additions/Combinations = **30**
(u) 464: No. Of Above-mentioned Prime Additions/Combinations = 13
(v) 466: No. Of Above-mentioned Prime Additions/Combinations = 14
(w) 468: No. Of Above-mentioned Prime Additions/Combinations = 26
(x) 470: No. Of Above-mentioned Prime Additions/Combinations = 16
(y) 472: No. Of Above-mentioned Prime Additions/Combinations = 14
(z) 474: No. Of Above-mentioned Prime Additions/Combinations = 24
(aa) 476: No. Of Above-mentioned Prime Additions/Combinations = 14
(bb) 478: No. Of Above-mentioned Prime Additions/Combinations = 12
(cc) 480: No. Of Above-mentioned Prime Additions/Combinations = **30**
(dd) 482: No. Of Above-mentioned Prime Additions/Combinations = 11
(i) Maximum No. Of Prime Additions/Combinations = 30
(ii) Minimum No. Of Prime Additions/Combinations = 9
(iii) Total No. Of Prime Additions/Combinations For (a) To (dd) = 508
(iv) Total No. Of Even Numbers = 30

(v) Density Of Distribution = Average Prime Additions/Combinations Per Even Number = (iii) $\div$(iv) = 508 $\div$30 =

16.93 Prime Additions/Combinations Per Even Number

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