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PROBABILISTIC ANALYSIS OF TIME TO RECRUIT AND RECRUITMENT TIME IN MANPOWER SYSTEM WITH TWO GROUPS

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Abstract: In this paper, we consider the Manpower System of an organization with two groups. Breakdown occurs in the two groups of the Manpower System due to attrition process. Group A consists of employees other than top management level executives; group B consists of top management level executives. Two models are studied in this paper. In Model-1, group A is exposed to Cumulative Shortage Process (CSP) due to attrition and group B has an exponential life distribution. In Model-2, both the groups A and B are exposed to CSP and have exponential thresholds. Inter occurrence time of shortages to them have exponential distribution. In each model, two cases are discussed. In one case, after the threshold, recruitment policy to compensate the shortages one by one is followed. Joint Laplace transforms of Time to Recruit and Recruitment time have been found. In the second case, recruitment policy of filling vacancies simultaneously is followed. Here, marginal Recruitment time distributions have been obtained.

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1. Introduction

In an organization, the total flow out of the Manpower System (MPS) is termed as shortage. The flow out of the MPS of an organization happens due to resignation, dismissal and death. The shortages that have occurred due to the outflow of manpower should be compensated by recruitment. But recruitment cannot be made frequently since it involves cost. Therefore, the MPS is allowed to undergo Cumulative Shortage Process (CSP). The basic idea is that accumulating random amount of shortages due to successive attritions leads to the breakdown of the system when the total shortage crosses a random threshold level. The breakdown point or the threshold can also be interpreted as that point at which immediate recruitment is necessitated.

The shortage of MPS depends on individual propensity to leave the organization which in turn depends on various factors as discussed before. Such models have been discussed by Grinold and Marshall [7], Bartholomew and Forbes [8]. For statistical approach one may refer to Bartholomew [1]. Lesson [9] has given methods to compute shortages (Resignations, Dismissals, Deaths) and promotion intensities which produce the proportions corresponding to some descried planning proposals. Markovian models are designed for shortage and promotion in MPS by Vassiliou [5]. Subramanian. V [10] has made an attempt to provide optimal policy for recruitment, training, promotion and shortages in manpower planning models with special provisions such as time bound promotions, cost of training and voluntary retirement schemes.

Esary et al. [3] have discussed that any component or decice, when exposed to shocks which cause damage to the device or system, is likely to fail when the total accumulated damage exceeds a level called threshold. R. Ramanarayanan and G. Sankaranarayanan [6], in their paper "On Correlated Life and Repair times" have studied the same using Gaver D.P. [2]. Linton D.G. and Saw J.G. [4] have done Reliability analysis of k-out-of-n : F system. In mapower planning, threshold having SCBZ (setting the clock back to zero) property has been discussed and the expected time to recruitment has been found by Sathiyamoorthy $R.$ and Parthasarathy. S. [11]. For MPS having threshold that follows Exponentiated Exponential distribution, expected time to recruitment and variance have been obtained by S. Parthasarathy and R. Vinoth [12].

In this paper, two groups of the MPS are exposed to shortage process.

Group A consists of employees of MPS other than top level management executives. Group B consists of top level management executives.

We assume that after the threshold of MPS, recruitment takes places using recruitment policies 1) recruitment for vacancies due to shortages one by one (Additive Recruitment Policy) and 2) recruitment for vacancies due to shortages simultaneously (Parallel Recruitment Policy).

Two models are discussed in this paper. In Model-1, group A is exposed to CSP and group B has exponential life distribution. In Model-2, both the groups A and B have exponential thresholds and are exposed to CSP. Inter-occurrence time of shortages to them have exponential distribution.

In models 1 and 2 with additive recruitment policy, joint Laplace Transforms of Time to recruit and Recruitment time are obtained. In both the models with parallel recruitment policy, marginal Recruitment time distributions are obtained.

2. Model-1

2.1. Model Description

In this model, MPS with two groups A and B is considered. Group A has employees of MPS other than top level management executives. Group B consists of top level management executives. Group A is exposed to CSP and group B has exponential life distribution.

The time to breakdown of MPS is $T = min\{T_1, T_2\}$ where T_1 and T_2 are the times to breakdown of groups A and B respectively. We assume that the recruitment time R of a shortage is independent of the shortage magnitude.

2.2. Assumptions

1. Shortages occur in group A in accordance with inter occurrence time distribution $F(x)$ and $\int_0^\infty x dF(x) < \infty$.
2. N is the random number of shortages required for breakdown of group A. The group survives k shortages with an arbitrary but known probability $P_k (= P(N > k))$ for $k = 0, 1, 2, \ldots, \{P_k\}$ is assumed to be a decreasing sequence of numbers in the unit interval subject only to $\sum_{k=0}^{\infty} P_k < \infty$, $P_0 = 1$.

3. Let $p_k(= P_{k-1} - P_k)$, the probability that group A has a breakdown on the k^{th} shortage, be given by the generating function $\phi(S)$, $0 \leq S \leq 1$.

4. T_2 has an exponential distribution with parameter μ.

5. At time 0, the MPS comprising of the two groups A and B is free of shortage due to manpower loss and put into operation. Groups A and B function independently during time to recruit.

6. When the breakdown of MPS occurs, recruitment begins immediately. Productive work does not happen in group B when the system breakdown occurs due to group A. When it occurs due to group B, productive work does not happen in group A and recruitment takes place for group A.

7. Recruitment time of the i^{th} shortage is R_i. R_i's are independent and identically distributed $r.v.$ with distribution function $R(y)$, independent of the shortage magnitudes. Recruitment time for group B is assumed to be R independent of R_i's but with distribution function $R(y)$, $\int_0^\infty y dR(y) < \infty$.

2.3. Model-1.1

In this model, we assume that as per additive recruitment policy, recruitments are done for all the shortages one at a time after time T. $R_T = R_1 + R_2 + \cdots + R_i$ where i is the number of shortages occurred during T. When the MPS breakdown occurs due to group A, we have to recruit for N shortages. Hence, $R_T = R_1 + R_2 + \cdots + R_N$. In the other case, when the MPS breakdown occurs due to group B, we have, $R_T = R_1 + R_2 + \cdots + R_{n+1}$ as we have to rejuvenate group B also with the n $(< N)$ shortages that occurred in group A during T.

The joint distribution of T and R_T is given as

$$\begin{aligned}
P(T \leq x, R_T \leq y) &= P(T_1 \leq x, R_T \leq y, T_2 > T_1) \\
&\quad + P(T_2 \leq x, R_T \leq y, T_1 > T_2) \\
&= \sum_{n=1}^{\infty} \sum_{n=1}^{\infty} p_n R_n(y) \int_0^x e^{-\mu z} dF_n(z) \\
&\quad + \sum_{n=0}^{\infty} P_n R_{n+1}(y) \int_0^x \mu e^{-\mu z} [F_n(z) - F_{n+1}(z)] dz \qquad (2.1)
\end{aligned}$$

where $R_n(y)$ and $F_n(y)$ are the n fold convolutions of $R(y)$ and $F(y)$ respectively. $F_n(y) = 1$ for $y \geq 0$; $F_n(y) = 0$, otherwise.

The term under the first summation symbol of the right side of (2.1) is the joint probability that the MPS breakdown occurs due to group A on the n^{th} shortage during $(0, x)$ and $\sum_{i=1}^{n} R_i \leq y$. The term under the second summation symbol is the joint probability that MPS breakdown occurs due to group B during $(0, x)$, group A survives the n shortages occurred to it before threshold level and $\sum_{i=1}^{n+1} R_i \leq y$.

From (2.1), we get,

$$E(e^{-\epsilon T} e^{-\eta R_T}) = \int_0^{\infty} \int_0^{\infty} e^{-\epsilon x} e^{-\eta y} \sum_{n=1}^{\infty} p_n e^{-\mu x} dR_n(y) dF_n(x)$$

$$+ \int_0^{\infty} \int_0^{\infty} e^{-\epsilon x} e^{-\eta y} \sum_{n=0}^{\infty} P_n \mu e^{-\mu x} [F_n(x) - F_{n+1}(x)] dR_{n+1}(y) dx$$

$$= \sum_{n=1}^{\infty} p_n [R^{*}(\eta) F^{*}(\mu + \epsilon)]^n$$

$$+ \sum_{n=0}^{\infty} P_n [R^{*}(\eta)]^{n+1} [1 - F^{*}(\mu + \epsilon)] [F^{*}(\mu + \epsilon)]^n \frac{\mu}{\mu + \epsilon}$$

where * denotes Laplace-Stieltjes transform.

Using the fact that $P_i = 1 - \sum_{j=1}^{i} p_j$,

$$E(e^{-\epsilon T} e^{-\eta R_T}) = \phi[R^{*}(\eta) F^{*}(\mu + \epsilon)] + \frac{\mu R^{*}(\eta)[1 - F^{*}(\mu + \epsilon)]}{(\mu + \epsilon)[1 - R^{*}(\eta) F^{*}(\mu + \epsilon)]} \{1 - \phi[R^{*}(\eta) F^{*}(\mu + \epsilon)]\} \quad (2.2)$$

The dependence of Recruitment time on the corresponding Time to recruit can be seen easily. From (2.2), for $\eta = 0$, $\epsilon = 0$ we get respectively,

$$E(e^{-\epsilon T}) = \frac{\mu + \epsilon \phi[F^{*}(\mu + \epsilon)]}{(\mu + \epsilon)} \quad (2.3)$$

$$E(e^{-\eta R_T}) = \phi[F^{*}(\mu) R^{*}(\eta)] + \frac{R^{*}(\eta)[1 - F^{*}(\mu)]}{[1 - R^{*}(\eta) F^{*}(\mu)]} \{1 - \phi[R^{*}(\eta) F^{*}(\mu)]\} \quad (2.4)$$

From (2.3) and (2.4), by differentiation,

$$E(T) = \frac{1 - \phi(F^*(\mu))}{\mu} \tag{2.5}$$

$$E(R_T) = \frac{-R^{*\prime}(0)\{1 - \phi(F^*(\mu))\}}{[1 - F^*(\mu)]} \tag{2.6}$$

where $\prime$ denotes differentiation.
Similarly, from (2.2) we get,

$$E(TR) = \frac{-R^{*\prime}(0)}{[1 - F^*(\mu)]} \left\{ F^{*\prime}(\mu)\phi^{\prime}[F^*(\mu)] + \frac{[1 - \phi(F^*(\mu))]}{\mu} \right\} + R^{*\prime}(0) \left\{ \frac{1}{\mu} F^*(\mu)\phi^{\prime}[F^*(\mu)] + \frac{F^{*\prime}(\mu)[1 - \phi(F^*(\mu))]}{[1 - F^*(\mu)]^2} \right\} \tag{2.7}$$

From (2.5), (2.6) and (2.7) covariance can be found.

2.4. Model-1.2

In this model, it is assumed that, after threshold, parallel recruitment policy is adopted and all recruitments are done simultaneously. $R_T = max\{R_1, R_2, \ldots, R_i\}$ where i is the number of shortages occurred during T.

$$P(T \leq x, R_T \leq y) = \sum_{n=1}^{\infty} [R(y)]^n p_n \int_0^x e^{-\mu z} dF_n(z) + \sum_{n=0}^{\infty} P_n [R(y)]^{n+1} \int_0^x \mu e^{-\mu z} [F_n(z) - F_{n+1}(z)] dz \tag{2.8}$$

From (2.8) marginal recruitment time distribution can be obtained as

$$P(R_T \leq y) = \phi[R(y)F^*(\mu)] + \frac{R(y)[1 - F^*(\mu)]}{1 - R(y)F^*(\mu)]} \{1 - \phi[R(y)F^*(\mu)]\} \tag{2.9}$$

3. Model-2

3.1. Model Description

In this model, we consider the MPS when both the groups A and B are exposed to CSP. Recruitment begins immediately after MPS breakdown occurs. Joint transform and marginal recruitment time distributions are obtained for models 2.1 and 2.2 respectively when the groups A and B of the MPS have exponential thresholds and shortages occur to them in accordance with Poisson process.

3.2. Assumptions

1. Shortages occur in group A and group B in accordance with two different and independent Poisson processes with parameters λ and μ respectively.

2. The successive shortages $X_1, X_2, \ldots$ in group A and $Y_1, Y_2, \ldots$ in group B are two sequences of independent and identically distributed positive random variables. X and Y are independent with different distributions.

3. W_1 and W_2 are the thresholds of groups A and B respectively. They are independent and exponentially distributed with different parameters c_1 and c_2 respectively.

4. At time 0, the groups A and B of the MPS are free of shortage and put into operation. Groups A and B function independently during Time to recruit.

5. When there is breakdown in one of the groups, the other group does not do productive function and the two groups are taken for immediate recruitment.

6. For all the shortages occurred in the two groups, recruitment is done. The recruitment times R_i of the shortages are independent and identically distributed $r.v.$ with distribution $R(y)$ independent of the magnitude of the shortage. Let $\int_0^\infty y dR(y) < \infty$.

3.3. Probability of breakdown

$$
\begin{aligned}
&P(\text{group } A \text{ survives } k \text{ shortages}) \\
&\qquad = P(W_1 > X_1 + X_2 + \cdots + X_k) \\
&\qquad = \int_0^\infty P(X_1 + X_2 + \cdots + X_k < y) c_1 e^{-c_1 y} dy \\
&\qquad = a^k
\end{aligned}
\tag{3.1}
$$

where $a = E(e^{-c_1 x})$ and $0 \leq a < 1$.

$$P(\text{group } A \text{ has breakdown on } n^{th} \text{ shortage}) = (1-a)a^{n-1}, \quad n = 1, 2, \ldots \tag{3.2}$$

Similarly

$$P(\text{group } B \text{ survives } k \text{ shortages}) = P(W_2 > Y_1 + Y_2 + \cdots + Y_k) = b^k \quad (3.3)$$

where $b = E(e^{-c_2 y})$ and $0 \leq b < 1$.

$$P(\text{group } B \text{ has breakdown on the } n^{th} \text{ shortage}) = (1-b)b^{n-1}, \quad n = 1, 2, \ldots \quad (3.4)$$

3.4. Model-2.1

We proceed as in model-1.1. $R_T = R_1 + R_2 + \cdots + R_i$ where i is the number of shortages that have occurred during T.
From (3.1) to (3.4),

$$P(T_1 \leq x, R_T \leq y, T_2 > T_1) = \sum_{n=1}^{\infty} \sum_{k=0}^{\infty} R_{n+k}(y) \int_0^x b^k e^{-\mu z} \frac{(\mu z)^k}{k!} (1-a) a^{n-1} \lambda e^{-\lambda z} \frac{(\lambda z)^{n-1}}{(n-1)!} dz \quad (3.5)$$

where $R_n(y)$ is the n fold convolution of $R(y)$ with itself. The term under the double summation is the joint probability that the MPS suffers breakdown during the n^{th} shortage to group A during $(0, x)$, group B survives the k shortages occurred during time to recruit and $\sum_{i=1}^{n+k} R_i \leq y$.

Similarly we can find

$$P(T_2 \leq x, R_T \leq y, T_1 > T_2) = \sum_{n=1}^{\infty} \sum_{k=0}^{\infty} R_{n+k}(y) \int_0^x a^k e^{-\lambda z} \frac{(\lambda z)^k}{k!} (1-b) b^{n-1} \mu e^{-\mu z} \frac{(\mu z)^{n-1}}{(n-1)!} dz \quad (3.6)$$

Using (3.5) and (3.6),

$$\begin{aligned} &E(e^{-\epsilon T} e^{-\eta R_T}) \\ &= \lambda(1-a) R^{\square}(\eta) \int_0^{\infty} e^{-\epsilon x} \sum_{k=0}^{\infty} e^{-\mu x} \frac{[b\mu x R^{\square}(\eta)]^k}{k!} \sum_{n=1}^{\infty} e^{-\lambda x} \frac{[a\lambda x R^{\square}(\eta)]^{n-1}}{(n-1)!} dx \\ &+ \mu(1-b) R^{\square}(\eta) \int_0^{\infty} e^{-\epsilon x} \sum_{k=0}^{\infty} e^{-\lambda x} \frac{[a\lambda x R^{\square}(\eta)]^k}{k!} \sum_{n=1}^{\infty} e^{-\mu x} \frac{[b\mu x R^{\square}(\eta)]^{n-1}}{(n-1)!} dx \end{aligned} \quad (3.7)$$

$$E(e^{-\epsilon T} e^{-\eta R_T}) = R^{\square}(\eta) \frac{[\lambda(1-a) + \mu(1-b)]}{[(\epsilon + \lambda + \mu) - (a\lambda + b\mu) R^{\square}(\eta)]} \quad (3.8)$$

3.5. Model-2.2

We proceed as in model-1.2. $R_T = max\{R_1, R_2, \ldots, R_i\}$ where i is the number of shortages occurred during T. We get,

$$\begin{aligned} &P(T \leq x, R_T \leq y) \\ &\quad = \sum_{n=1}^{\infty}\sum_{k=0}^{\infty}[R(y)]^{n+k}\int_0^x b^k e^{-\mu z}\frac{(\mu z)^k}{k!}\lambda(1-a)a^{n-1}e^{-\lambda z}\frac{(\lambda z)^{n-1}}{(n-1)!}dz \\ &\quad + \sum_{n=1}^{\infty}\sum_{k=0}^{\infty}[R(y)]^{n+k}\int_0^x a^k e^{-\lambda z}\frac{(\lambda z)^k}{k!}\mu(1-b)b^{n-1}e^{-\mu z}\frac{(\mu z)^{n-1}}{(n-1)!}dz \end{aligned} \tag{3.9}$$

Marginal Recruitment time distribution is

$$P(R_T \leq y) = R(y)\frac{[\lambda(1-a)+\mu(1-b)]}{[(\lambda+\mu)-(a\lambda+b\mu)R(y)]} \tag{3.10}$$

where $0 \leq a < 1$ and $0 \leq b < 1$.

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