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VERTEX COVERING AND INDEPENDENT NUMBER ON DIFFERENCE GRAPHS

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Abstract: Let $\alpha(G)$ and $\beta(G)$ be the independent number and vertex covering number of G, respectively. The difference graph, $G_1 \triangle G_2$, of connected graphs G_1 and G_2 has vertex set $V(G_1 \triangle G_2) = V(G_1) = V(G_2)$ and edge set $E(G_1 \triangle G_2) = [E(G_1) - E(G_2)] \cup [E(G_2) - E(G_1)]$. In this paper, we determine generalizations of some graph parameters : independent number and vertex covering number on difference graph.

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Key Words: difference graph, independent number, vertex covering number

1. Introduction

In this paper, graphs must be simple graphs which can be trivial graph. Let G_1 and G_2 be connected graphs. The Difference graph of G_1 and G_2, denote by $G_1 \triangle G_2$, be the graph that $V(G_1 \triangle G_2) = V(G_1) = V(G_2)$ and $E(G_1 \triangle G_2) = [E(G_1) - E(G_2)] \cup [E(G_2) - E(G_1)]$.

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Next, we give the definitions about some graph parameters. A subset U of the vertex set $V(G)$ of G is said to be an independent set of G if the induced subgraph $G[U]$ is a trivial graph. An independent set of G with maximum number of vertices is called a maximum independent set of G . The number of vertices of a maximum independent set of G is called the independent number of G , denoted by $\alpha(G)$.

A vertex of graph G is said to cover the edges incident with it, and a vertex cover of a graph G is a set of vertices covering all the edge of G . The minimum cardinality of a vertex cover of a graph G is called the vertex covering number of G , denoted by $\beta(G)$.

Next we give some properties about difference graph

Proposition 1. *Any difference graphs of connected graphs are always connected.*

Proposition 2. *Let G_1 and G_2 be connected graphs, the difference graph $G_1 \triangle G_2 \cong [G_1 - E(H)] \cup [G_2 - E(H)] = \overline{H_1} \cup \overline{H_2}$ where $G_1 \cap G_2 = H$*

Example.

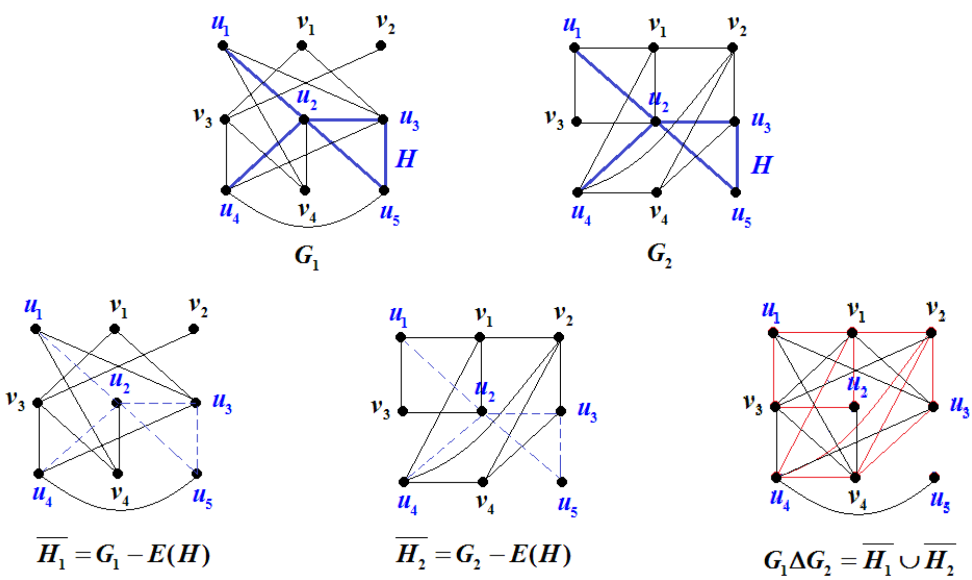

Figure 1: The graph of $G_1 \triangle G_2$

2. Vertex Covering Number of the Difference Graph $G_1 \triangle G_2$

We now prove lemma before stating our main results. We begin this section by giving the lemma 3 which show character of vertex covering number of each $\overline{H_i}$.

Lemma 3. *Let $G_1 \cap G_2 = H$ and $G_1 \triangle G_2 = [G_1 - E(H)] \cup [G_2 - E(H)] = \overline{H_1} \cup \overline{H_2}$. For each $\overline{H_i}$, then*

$$\beta(\overline{H_i}) = \beta(G_i) - \max\{Q_i\}$$

where $\max\{Q_i\}$ be maximum cardinal number of $Q_i = \{u \in V(H) | N_{\overline{H_i}}(u) - C_i = \emptyset\}$; C_i are the minimum vertex covering set of $G_i, i = 1, 2$.

Proof. For each $\overline{H_i} = G_i - E(H)$, we get $\beta(\overline{H_i}) = \beta(G_i - E(H)) \le \beta(G_i)$.

Let $Q_i = \{u \in V(H) | N_{\overline{H_i}}(u) - C_i = \emptyset\}$, we choose the minimum vertex covering set C_i of G_i such that Q_i have the maximum cardinal number $= \max\{Q_i\}$.

Hence, the minimum vertex covering of $\overline{H_i}$ have

$$\beta(\overline{H_i}) = \beta(G_i) - \max\{Q_i\}; \quad i = 1, 2.$$ □

Example.

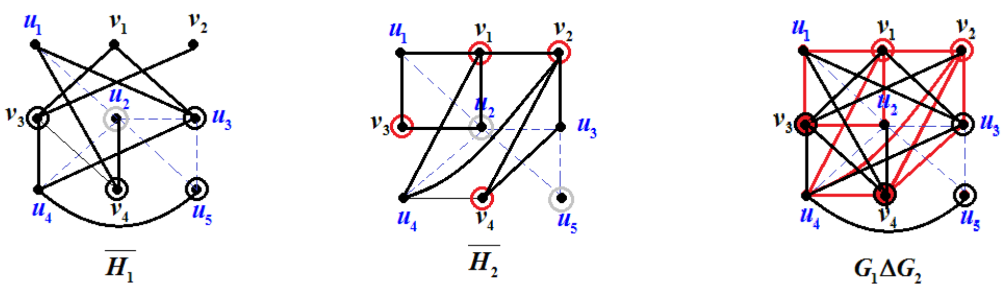

Figure 2: The minimum vertex covering set of $\overline{H_1}, \overline{H_2}$ and $G_1 \triangle G_2$

Next, we establish Theorem 4 for a vertex covering number of $G_1 \triangle G_2$.

Theorem 4. *Let $G_1 \cap G_2 = H$ and $G_1 \triangle G_2 = [G_1 - E(H)] \cup [G_2 - E(H)] = \overline{H_1} \cup \overline{H_2}$, then*

$$\beta(G_1 \triangle G_2) = \beta(G_1) \cup \beta(G_2) - (|Q_1| + |Q_2| + |R|),$$

where $R = \{u|u$ *belong to the minimum vertex covering set of* $\overline{H_1}$ *and* $\overline{H_2}\}$, $\max\{Q_i\}$ *be maximum cardinal number of* $Q_i = \{u \in V(H)|N_{\overline{H_i}}(u) - C_i = \emptyset\}$; C_i *are the minimum vertex covering set of* $G_i, i = 1, 2$.

Proof. Since $G_1 \triangle G_2 = [G_1 - E(H)] \cup [G_2 - E(H)] = \overline{H_1} \cup \overline{H_2}$, we get $\beta(G_1 \triangle G_2) = \beta(\overline{H_1} \cup \overline{H_2}) \leq \beta(\overline{H_1}) + \beta(\overline{H_2})$.

Let $R = \{u|u$ belong to the minimum vertex covering set of $\overline{H_1}$ and $\overline{H_2}\}$, by definition of difference graph, we have $E(\overline{H_1}) \cap E(\overline{H_2}) = \emptyset$, so $\beta(G_1 \triangle G_2) = \beta(\overline{H_1}) + \beta(\overline{H_2}) - |R|$.

By lemma 3, imply that

$$\begin{aligned}\beta(G_1 \triangle G_2) &= \beta(\overline{H_1}) + \beta(\overline{H_2}) - |R| \\ &= \beta(G_1) \cup \beta(G_2) - (|Q_1| + |Q_2| + |R|).\end{aligned}$$

□

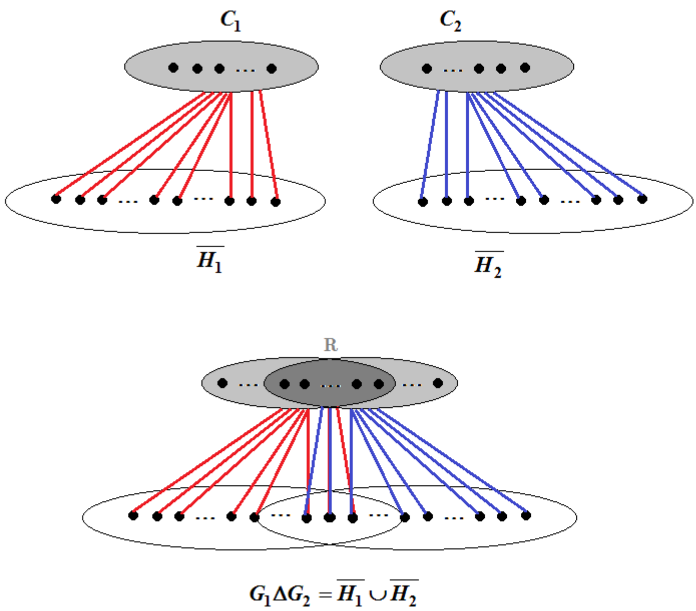

Figure 3: The region of R of $G_1 \triangle G_2$

3. Independent Number of $G_1 \triangle G_2$

We begin this section by giving the lemma 5 that shows a relation of vertex covering number and independent number.

Lemma 5. (see [1]) *Let G be a simple graph with order n. Then $\alpha(G) + \beta(G) = n$.*

Next, we establish Theorem 6 for a independent number of $G_1 \triangle G_2$.

Theorem 6. *Let $G_1 \cap G_2 = H$ and $G_1 \triangle G_2 = [G_1 - E(H)] \cup [G_2 - E(H)] = \overline{H_1} \cup \overline{H_2}$. For each $\overline{H_i}$, then*

$$\alpha(G_1 \triangle G_2) = \alpha(\overline{H_1}) + \alpha(\overline{H_2}) - |S|$$

where $S = \{u|u$ belong to the maximum independent set of $\overline{H_1}$ and $\overline{H_2}\}$

Proof. By Theorem 4 and Lemma 5, we can also show that

$$\begin{aligned} \alpha(G_1 \triangle G_2) + \beta(G_1 \triangle G_2) &= n \\ \alpha(G_1 \triangle G_2) &= n - \beta(\overline{H_1}) - \beta(\overline{H_2}) + |R| \\ &= n - (n - \alpha(\overline{H_1})) - (n - \alpha(\overline{H_2})) + |R| \\ &= \alpha(\overline{H_1}) + \alpha(\overline{H_2}) - (n - |R|) \\ &= \alpha(\overline{H_1}) + \alpha(\overline{H_2}) - |S|. \end{aligned}$$

□

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References

[1] D.B. West, *Introduction to Graph Theory*, Prentice-Hall (2001).