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POSSIBILITY VAGUE SOFT SET AND ITS APPLICATION IN DECISION MAKING

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Abstract: We introduce the concept of possibility vague soft sets and its operations. We give applications of this theory in solving a decision-making problem. We also introduce the similarity measure of two possibility vague soft sets and discuss their application in a medical diagnosis problem.

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Key Words: possibility fuzzy soft set, soft set, vague soft set

1. Introduction

Fuzzy set was introduced by Zadeh in [1] as a mathematical tool to solve the problem and vagueness in everyday life. Molodtsov [2] mentioned a soft set as a mathematical way to represent and solve these problems with uncertainties which traditional mathematical tools cannot handle. He has proved several applications of his theory in solving problem in economics, engineering, environment, social science, medical science and business management. Roy and Maji used this theory to solve some decision-making problems [3]. Alkhazaleh et al [4] introduced the concept of soft multiset as a generalization of Molodtsov's soft set. They [5] also introduced possibility fuzzy soft set and some propositions of possibility fuzzy soft set with application in decision making. Salleh et

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al. [6] introduced multiparameterized soft set and Alkhazaleh et al [7] further introduced the concept of fuzzy parameterized interval-valued fuzzy soft set. Vague set theory was proposed by Gau et al. [8] and then Chen [9] provides measure of similarity between vague sets. Chaudhuri et al. [10] introduced the concept of soft relation and fuzzy soft relation and solved some problems in decision-making. Zhu et al.[11] incorporated Molodtsov's soft set theory with the probability theory and proposed the notion of probabilistic soft sets, and also Alhazaymeh et al. [12] who introduced the concept of the soft intuitionistic fuzzy sets.

In this paper, we introduce the concept of possibility vague soft set and some operations with illustrative example of decision making. Finally we give some application of the possibility vague soft set in medical diagnosis.

2. Preliminaries

In this section, we recall some definitions and properties of possibility fuzzy soft set required for our proposition of possibility vague soft sets.

Definition 2.1. (see [5]) Let $U = \{x_1, x_2, ...x_n\}$ be the universal set of elements and let $E = \{e_1, e_2, ..., e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \longrightarrow I^U$ and μ be a fuzzy subset of E, that is, $\mu : E \longrightarrow I^U$, where I^U is the collection of all fuzzy subsets of U. Let $F_\mu : E \longrightarrow I^U \times I^U$ be a function defined as follows:

$$F_\mu(e) = (F(e)(x), \mu(e)(x)), \forall x \in U.$$

Then F_μ called a possibility fuzzy soft set (PFSS in short) over the soft universe (U, E). We can write $F_\mu(e_i)$ as follows:

$$F_\mu(e_i) = \{(\frac{x_1}{F(e_i)(x_1)}, \mu(e_i)(x_1)), (\frac{x_2}{F(e_i)(x_2)}, \mu(e_i)(x_2)), ..., (\frac{x_n}{F(e_i)(x_n)}, \mu(e_i)(x_n))\},$$

F_μ can also be written as (F_μ, E). If $A \subseteq E$ we have a PVSS (F_μ, A).

Definition 2.2. (see [5]) Let F_μ and G_δ be two PFSSs over (U, E). F_μ is said to be a possibility fuzzy soft subset (PFS subset) of G_δ, and $F_\mu \subseteq G_\delta$ if:

(i) $\mu(e)$ is a fuzzy subset of $\delta(e)$, $\forall e \in E$,

(ii) $F(e)$ is a fuzzy subset of $G(e)$, $\forall e \in E$.

Definition 2.3. (see [5]) A PFSS is said to be a possibility null fuzzy soft set , denoted by φ_0, if $\varphi_0 : E \longrightarrow I^U \times I^U$ such that

$$\varphi_0 = (F(e)(x), \mu(e)(x)),\ \forall e \in E,$$

where $F(e) = 0$, and $\mu(e) = 0$, $\forall e \in E$.

Definition 2.4[5] A PFSS is said to be a possibility absolute fuzzy soft set, denoted by A_1, if $A_1 : E \longrightarrow I^U \times I^U$ such that

$$A_1 = (F(e)(x), \mu(e)(x)), \quad \forall e \in E,$$

where $F(e) = 1$, and $\mu(e) = 1$, $\forall e \in E$.

Definition 2.5. (see [5]) Let F_μ be a PFSS over (U,E). Then the complement of F_μ denoted by F^c_μ is defined by $F^c_\mu = c(F(e), \mu(e))$, $\forall e \in E$, where c is a fuzzy complement.

Definition 2.6. (see [5]) Union of two PFSSs F_μ and G_δ, denoted by $F_\mu \tilde{\cup} G_\delta$, is a PFSS $H_\nu : E \longrightarrow I^U \times I^U$ defined by

$$H_\nu(e) \quad = \quad (H(e)(x), \nu(E)(x), \forall e \in E,$$

such that $H(e) = s(F(e), G(e))$ and $\nu(e) = s(\mu, \delta)$, where s is a $s - norm$.

Definition 2.7. (see [5]) The intersection of two PFSSs F_α and G_β is denoted by $F_\alpha \tilde{\cap} G_\beta$ is a PFSS $H_\nu : E \longrightarrow I^U \times I^U$ defined by

$$H_\nu(e) \quad = \quad (H(e)(x), \nu(E)(x), \forall e \in E,$$

such that $H(e) = t(F(e), G(e))$ and $\nu(e) = t(\mu, \delta)$, where t is a $t - norm$.

Definition 2.8. (see [5]) If (F_μ, A) and (G_δ, B) are two PFSSs then "$(F_\mu, A) AND (G_\delta, B)$" denoted by $(F_\mu, A) \wedge (G_\delta, B)$ is defined by

$$(F_\mu, A) \wedge (G_\delta, B) \quad = \quad (H_\lambda, A \times B)$$

where $H_\lambda(\alpha, \beta) = (H(\alpha, \beta)(x), \nu(\alpha, \beta)(x))$, $\forall(\alpha, \beta) \in (A \times B)$, such that $H(\alpha, \beta) = t(f(\alpha), G(\beta))$ and $\nu(\alpha, \beta) = t(\mu(\alpha), \delta(\beta))$, $\forall(\alpha, \beta) \in (A \times B)$.

Definition 2.9. (see [5]) If (F_μ, A) and (G_δ, B) are two PFSSs then "$(F_\mu, A) OR (G_\delta, B)$" denoted by $(F_\mu, A) \vee (G_\delta, B)$ is defined by

$$(F_\mu, A) \vee (G_\delta, B) \quad = \quad (H_\lambda, A \times B)$$

where $H_\lambda(\alpha, \beta) = (H(\alpha, \beta)(x), \nu(\alpha, \beta)(x))$, $\forall(\alpha, \beta) \in (A \times B)$, such that $H(\alpha, \beta) = s(f(\alpha), G(\beta))$ and $\nu(\alpha, \beta) = s(\mu(\alpha), \delta(\beta))$, $\forall(\alpha, \beta) \in (A \times B)$.

Definition 2.10. (see [5]) Similarity between two PFSSs F_μ and G_δ denoted by $S(F_\mu, G_\delta)$, is defined as follows:

$$S(F_\mu, G_\delta) = M(F(e), G(e)) \cdot M(\mu(e), \delta(e))$$

such that

$M(F(e), G(e)) = max_i M_i(F(e), G(e))$ and $M(\mu(e), \delta(e)) = max_i M_i(\mu(e), \delta(e))$,

where

$$M_i(F(e), G(e)) = 1 - \frac{\sum_{j=1}^{n} |F_{ij} - Gij|}{\sum_{j=1}^{n} |F_{ij} + Gij|},$$

$$M_i(\mu(e), \delta(e)) = 1 - \frac{\sum_{j=1}^{n} |\mu_{ij} - \delta ij|}{\sum_{j=1}^{n} |\mu_{ij} + \delta ij|}.$$

3. Possibility Vague Soft Sets

In this section we introduce the concept of possibility vague soft sets as an extension of the vague soft sets introduced by Xu et al. [13]. A possibility of each element in the universe is attached with the parameterization of vague sets while defining a vague soft set.

Definition 3.1. Let $U = \{x_1, x_2, ...x_n\}$ be the universal set of elements and let $E = \{e_1, e_2, ..., e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $\tilde{F} : E \longrightarrow [0, 1]^2$ and μ be a vague subset of E, that is $\mu : E \longrightarrow I^U$ where I^U is the collection of all vague subsets of U. Let $\tilde{F}_\mu : E \longrightarrow [0, 1]^2 \times I^U$ be a function defined as follows:

$$\tilde{F}_\mu(e) = (\tilde{F}(e)(x), \mu(e)(x)), \forall x \in U.$$

Then $\tilde{F}_\mu$ called a possibility vague soft set (PVSS in short) over the soft set universe (U, E). For each parameter e_i, $\tilde{F}_\mu(e_i) = (\tilde{F}(e_i)(x), \mu(e_i)(x))$ indicates not only the degree of belongingness of the elements of U in $\tilde{F}(e_i)$ but also the degree of possibility of belongingness of the elements of U in $\tilde{F}(e_i)$, which is represented by $\mu(e_i)$. Thus we can write $\tilde{F}_\mu(e_i)$ as follows:

$$\tilde{F}_\mu(e) = \{(\frac{x_1}{\tilde{F}(e_i)(x_1)}, \mu(e_i)(x_1)), (\frac{x_2}{\tilde{F}(e_i)(x_2)}, \mu(e_i)(x_2)), ..., (\frac{x_n}{\tilde{F}(e_i)(x_n)}, \mu(e_i)(x_n))\}.$$

$\tilde{F}_\mu$ can also be written as $(\tilde{F}_\mu, E)$. If $A \subseteq E$, we have a PVSS $(\tilde{F}_\mu, A)$.

Example 3.1. Let $U = \{x_1, x_2, x_3\}$ be a set of three chairs. Let $E = \{e_1, e_2, e_3\}$ be a set of types of chairs where e_1= office chair, e_2= bath chair, e_3= beach chair, and let $\mu : E \longrightarrow I^U$. We define a function $\tilde{F}_\mu : E \longrightarrow [0,1]^2 \times I^U$ as follows:

$\tilde{F}_\mu(e_1) = \{(\frac{x_1}{[0.4,0.6]}0.3), (\frac{x_2}{[0.5,0.5]}, 0.9), (\frac{x_3}{[0.9,0]}, 1)\}$,
$\tilde{F}_\mu(e_2) = \{(\frac{x_1}{[0.6,0.7]}0.5), (\frac{x_2}{[0.1,0.1]}, 0.2), (\frac{x_3}{[0.9,0.9]}, 0)\}$,
$\tilde{F}_\mu(e_3) = \{(\frac{x_1}{[0,0.7]}0.6), (\frac{x_2}{[0.7,0.8]}, 0.4), (\frac{x_3}{[0.3,0.3]}, 8)\}$,

Then $(\tilde{F}_\mu, A)$ is a PVSS over (U, E). In the matrix notation, we write

$$\tilde{F}_\mu = \begin{pmatrix} [0.4, 0.6], 0.3 & [0.5, 0.5], 0.9 & [0.9, 0], 1 \\ [0.6, 0.7], 0.5 & [0.1, 0.1], 0.2 & [0.9, 0.9], 0 \\ [0, 0.7], 0.6 & [0.7, 0.8]0.4 & [0.3, 0.3], 0.8 \end{pmatrix}$$

Thus the entry of the second row and first column of the matrix $\tilde{F}_\mu$ that is, $[0.6, 0.7], 0.5$ implies $\tilde{F}_\mu(e_2)(x_1)$.

Definition 3.2. Let $\tilde{F}_\mu$ and $\tilde{G}_\delta$ be two PVSSs over (U, E). $\tilde{F}_\mu$ is said to be a possibility vague soft subset (PVS subset) of $\tilde{G}_\delta$, and $\tilde{F}_\mu \subseteq \tilde{G}_\delta$ if:

(i) $\mu(e)$ is a vague subset of $\delta(e)$, $\forall e \in E$,

(ii) $\tilde{F}(e)$ is a vague subset of $\tilde{G}(e)$, $\forall e \in E$.

Definition 3.3. Let $\tilde{F}_\mu$ and $\tilde{G}_\delta$ be two PFSSs over (U, E). Then $\tilde{F}_\mu$ and $\tilde{G}_\delta$ are said to be equal if if:

(i) $\mu(e)$ is equal to $\delta(e)$, $\forall e \in E$,

(ii) $\tilde{F}(e)$ is is equal to $\tilde{G}(e)$, $\forall e \in E$.

In other words, $\tilde{F}_\mu = \tilde{G}_\delta$ if $\tilde{F}_\mu$ is a PVS subset of $\tilde{G}_\delta$ and $\tilde{G}_\delta$ is a PVS subset of $\tilde{F}_\mu$.

Definition 3.4. A PVSS is said to be a possibility null vague soft set , denoted by $\tilde{\varphi}_0$, if $\tilde{\varphi} : E \longrightarrow [0,1]^2 \times I^U$ such that

$$\tilde{\varphi}(e) = (\tilde{F}(e)(x), \mu(e)(x)), \forall e \in E,$$

where $\tilde{F}(e) = 0$, and $\mu(e) = 0$, $\forall e \in E$.

Definition 3.5. A PVSS is said to be a possibility absolute vague soft set, denoted by $\tilde{\psi}$, if $\tilde{\psi} : E \longrightarrow I^U \times I^U$ such that

$$\tilde{\psi}(e) = (\tilde{F}(e)(x), \mu(e)(x)), \forall e \in E,$$

where $\tilde{F}(e) = 1$, and $\mu(e) = 1$, $\forall e \in E$.

Definition 3.6. Let $\tilde{F}_\mu$ be a PVSS over (U,E). Then the complement of $\tilde{F}_\mu$ denoted by $\tilde{F}^c_\mu$ is defined by $\tilde{F}^c_\mu = c(\tilde{F}(e), \mu(e))$, $\forall e \in E$, where c is a vague complement.

Definition 3.7. Union of two PVSSs $\tilde{F}_\mu$ and $\tilde{G}_\delta$, denoted by $\tilde{F}_\mu \tilde{\cup} \tilde{G}_\delta$, is a PVSS $\tilde{H}_\nu : E \longrightarrow [0,1]^2 \times I^U$ defined by

$$\tilde{H}_\nu(e) \quad = \quad (\tilde{H}(e)(x), \nu(E)(x), \forall e \in E,$$

such that $\tilde{H}(e) = max(\tilde{F}(e), G(e))$ and $\nu(e) = max(\mu(e), \delta(e))$.

Definition 3.8. The intersection of two PVSSs $\tilde{F}_\alpha$ and $\tilde{G}_\beta$ is denoted by $\tilde{F}_\alpha \tilde{\cap} \tilde{G}_\beta$ is a PVSS $\tilde{H}_\nu : E \longrightarrow [0,1]^2 \times I^U$ defined by

$$\tilde{H}_\nu(e) \quad = \quad (\tilde{H}(e)(x), \nu(E)(x), \forall e \in E,$$

such that $\tilde{H}(e) = min(\tilde{F}(e), G(e))$ and $\nu(e) = min(\mu(e), \delta(e))$.

Proposition 3.1. Let $\tilde{F}_\mu$, $\tilde{G}_\delta$ and $\tilde{H}_\nu$ be any three PVSSs over (U, E). Then the following results hold:

(i) $\tilde{F}_\mu \tilde{\cup} \tilde{G}_\delta = \tilde{G}_\delta \tilde{\cup} \tilde{F}_\mu$,

(ii) $\tilde{F}_\mu \tilde{\cap} \tilde{G}_\delta = \tilde{G}_\delta \tilde{\cap} \tilde{F}_\mu$,

(iii)$\tilde{F}_\mu \tilde{\cup} (\tilde{G}_\delta \tilde{\cup} \tilde{H}_\nu) = (\tilde{F}_\mu \tilde{\cup} (\tilde{G}_\delta) \tilde{\cup} \tilde{H}_\nu$,

(iv) $\tilde{F}_\mu \tilde{\cap} (\tilde{G}_\delta \tilde{\cap} \tilde{H}_\nu) = (\tilde{F}_\mu \tilde{\cap} \tilde{G}_\delta) \tilde{\cap} \tilde{H}_\nu$.

Proof. The proofs are straightforward by using the fact that vague sets are commutative and associative. □

4. AND and OR Operations on PVSS with Applications in Decision Making

In this section, we define the state of AND and OR operations on possibility vague soft sets. Applications of possibility vague soft sets in decision-making problem are given.

Definition 4.1. If $(\tilde{F}_\mu, A)$ and $\tilde{G}_\delta, B)$ are two PVSSs then "$(\tilde{F}_\mu, A)$ AND $(\tilde{G}_\delta, B)$" denoted by $(\tilde{F}_\mu, A) \wedge (\tilde{G}_\delta, B)$ is defined by

$$(\tilde{F}_\mu, A) \wedge (\tilde{G}_\delta, B) \quad = \quad (\tilde{H}_\lambda, A \times B),$$

where $\tilde{H}_\lambda(\alpha, \beta) = (\tilde{H}(\alpha, \beta)(x), \nu(\alpha, \beta)(x))$, $\forall(\alpha, \beta) \in A \times B$, such that $\tilde{H}(\alpha, \beta) = min(\tilde{F}(\alpha), \tilde{G}(\beta))$ and $\nu(\alpha, \beta) = min(\mu(\alpha), \delta(\beta))$, $\forall(\alpha, \beta) \in A \times B$.

Example 4.1. Suppose that the universe set consists of three televisions x_1, x_2, x_3, that is $U = \{x_1, x_2, x_3\}$, and there are three parameters $E = \{e_1, e_2, e_3\}$ which describe their performances according to certain specific task. Suppose a company wants to buy a television depending on any two parameters. Let two observations $\tilde{F}_\mu$ and $\tilde{G}_\delta$ by two experts be as follows:

$$\tilde{F}_\mu(e_1) = \{(\frac{x_1}{[0.4, 0.6]}0.3), (\frac{x_2}{[0.5, 0.5]}, 0.9), (\frac{x_3}{[0.9, 0]}, 1)\},$$

$$\tilde{F}_\mu(e_2) = \{(\frac{x_1}{[0.6, 0.7]}0.5), (\frac{x_2}{[0.1, 0.1]}, 0.2), (\frac{x_3}{[0.9, 0.9]}, 0)\},$$

$$\tilde{F}_\mu(e_3) = \{(\frac{x_1}{[0, 0.7]}0.6), (\frac{x_2}{[0.7, 0.8]}, 0.4), (\frac{x_3}{[0.3, 0.3]}, 8)\},$$

$$\tilde{G}_\delta(e_1) = \{(\frac{x_1}{[0.2, 0.5]}0.1), (\frac{x_2}{[0.6, 0.7]}, 0.5), (\frac{x_3}{[0.8, 0.9]}, 0.6)\},$$

$$\tilde{G}_\delta(e_2) = \{(\frac{x_1}{[0.7, 0.8]}0.7), (\frac{x_2}{[0.2, 0.2]}, 0.2), (\frac{x_3}{[0.5, 0.5]}, 0.7)\},$$

$$\tilde{G}_\delta(e_3) = \{(\frac{x_1}{[0.3, 0.7]}0.2), (\frac{x_2}{[0.1, 0.6]}, 0.9), (\frac{x_3}{[0.5, 0.5]}, 3)\}.$$

Then $(\tilde{F}_\mu, A) \wedge (\tilde{G}_\delta, B) = (\tilde{H}_\lambda, A \times B)$ where

$$\begin{aligned}\tilde{H}_\lambda(e_1, e_1) = \{(&\frac{x_1}{min(0.4, 0.2), min(0.6, 0.5)}, min(0.3, 0.1)),\\ &(\frac{x_2}{min(0.5, 0.6), min(0.5, 0.7)}, min(0.9, 0.5))\\ &, (\frac{x_3}{min(0.9, 0.8), min(0, 0.9)}, min(0, 0.6))\}\\ &= \{(\frac{x_1}{[0.2, 0.5]}0.1), (\frac{x_2}{[0.5, 0.5]}, 0.5), (\frac{x_3}{[0.8, 0]}, 0)\}.\end{aligned}$$

Similarly we get

$$\tilde{H}_\lambda(e_1, e_2) = \{(\frac{x_1}{[0.4, 0.6]}0.3), (\frac{x_2}{[0.2, 0.2]}, 0.2), (\frac{x_3}{[0.5, 0]}, 0.7)\},$$

$$\tilde{H}_\lambda(e_1, e_3) = \{(\frac{x_1}{[0.3, 0.6]}0.2), (\frac{x_2}{[0.1, 0.5]}, 0.9), (\frac{x_3}{[0.5, 0]}, 0.3)\},$$

$$\tilde{H}_\lambda(e_2, e_1) = \{(\frac{x_1}{[0.2, 0.5]}0.1), (\frac{x_2}{[0.1, 0.1]}, 0.2), (\frac{x_3}{[0.8, 0.9]}, 0)\},$$

$$\tilde{H}_\lambda(e_2, e_2) = \{(\frac{x_1}{[0.6, 0.7]}0.5), (\frac{x_2}{[0.1, 0.1]}, 0.2), (\frac{x_3}{[0.5, 0.5]}, 0)\},$$

$$\tilde{H}_\lambda(e_2,e_3)=\{(\frac{x_1}{[0.3,0.7]}0.2),(\frac{x_2}{[0.1,0.1]},0.2),(\frac{x_3}{[0.5,0.5]},0)\},$$

$$\tilde{H}_\lambda(e_3,e_1)=\{(\frac{x_1}{[0,0.5]}0.1),(\frac{x_2}{[0.6,0.7]},0.4),(\frac{x_3}{[0.3,0.3]},0.9)\},$$

$$\tilde{H}_\lambda(e_3,e_2)=\{(\frac{x_1}{[0,0.7]}0.6),(\frac{x_2}{[0.2,0.2]},0.2),(\frac{x_3}{[0.3,0.3]},0.7)\},$$

$$\tilde{H}_\lambda(e_3,e_3)=\{(\frac{x_1}{[0,0.7]}0.2),(\frac{x_2}{[0.1,0.6]},0.9),(\frac{x_3}{[0.3,0.3]},0.3)\}.$$

In the matrix notation, we get

$$(\tilde{F}_\mu,A)\wedge(\tilde{G}_\delta,B) = \begin{pmatrix} [0.2,0.5],0.1 & [0.5,0.5],0.5 & [0.8,0],0.6 \\ [0.4,0.6],0.3 & [0.2,0.2],0.2 & [0.5,0],0.7 \\ [0.3,0.6],0.2 & [0.1,0.5],0.9 & [0.5,0],0.3 \\ [0.2,0.5],0.1 & [0.1,0.1],0.2 & [0.5,0.5],0 \\ [0.6,0.7],0.5 & [0.1,0.1],0.2 & [0.5,0.5],0 \\ [0.3,0.7],0.2 & [0.1,0.1],0.2 & [0.5,0.5],0 \\ [0,0.5],0.1 & [0.2,0.2],0.2 & [0.3,0.3],0.6 \\ [0,0.7],0.6 & [0.2,0.2],0.2 & [0.3,0.3],0.7 \\ [0,0.7],0.2 & [0.1,0.6],0.9 & [0.3,0.3],0.3 \end{pmatrix}$$

We subtract the false-membership function from the truth-membership function as follows:

$$= \begin{pmatrix} -0.3,0.1 & 0,0.5 & \underline{0.8},0.6 \\ -0.2,0.3 & 0,0.2 & \underline{0.5},0.7 \\ -0.3,0.2 & -0.4,0.9 & \underline{0.5},0.3 \\ -0.3,0.1 & \underline{0},0.2 & \underline{0},0 \\ -0.1,0.5 & \underline{0},0.2 & \underline{0},0 \\ -0.4,0.2 & \underline{0},0.2 & \underline{0},0 \\ -0.5,0.1 & \underline{0},0.2 & \underline{0},0 \\ -0.7,0.6 & \underline{0},0.2 & \underline{0},0.7 \\ -0.7,0.2 & -0.5,0.9 & \underline{0},0.3 \end{pmatrix}$$

To determine the best television to be bought, we first mark the highest numerical grade in each row. Table 1 display the highest numerical grade for each pair. The score of each television is calculated by taking the sum of products of these numerical grades with the corresponding possibility λ. The television with the highest score is the desired television. We do not consider the numerical grades of the television against the pairs $(e_i,e_i), i=1,2,3$ as

$\tilde{H}$	x_i	$Highestnumericalgrade$	λ_i
(e_1, e_1)	x_3	X	X
(e_1, e_2)	x_3	0.5	0.7
(e_1, e_3)	x_3	0.5	0.3
(e_2, e_1)	x_2, x_3	0	0.2,0
(e_2, e_2)	x_2, x_3	X	X
(e_2, e_3)	x_2, x_3	0	0.2,0
(e_3, e_1)	x_2, x_3	0	0.2,0.6
(e_3, e_2)	x_2, x_3	0	0.2,0.7
(e_3, e_3)	x_3	X	X

Table 1: Record of highest numerical grade for each pair

both the parameters are the same. score(x_1) = 0, score (x_2) = $(0 \times 0.2) + (0 \times 0.2) + (0 \times 0.2) + (0 \times 0.2) = 0$, score ($x_3$) = $(0.5 \times 0.7) + (0.5 \times 0.3) = 0.5$.

The television with the highest score is x_3. Hence, television x_3 is highly recommended to be bought.

Definition 4.2. If $(\tilde{F}_\mu, A)$ and $\tilde{G}_\delta, B)$ are two PVSSs then "$(\tilde{F}_\mu, A)$ OR $(\tilde{G}_\delta, B)$" denoted by $(\tilde{F}_\mu, A) \vee (\tilde{G}_\delta, B)$ is defined by

$$(\tilde{F}_\mu, A) \vee (\tilde{G}_\delta, B) \quad = \quad (\tilde{H}_\lambda, A \times B)$$

where $\tilde{H}_\lambda(\alpha, \beta) = (\tilde{H}(\alpha, \beta)(x), \nu(\alpha, \beta)(x))$, $\forall(\alpha, \beta) \in A \times B$, such that $\tilde{H}(\alpha, \beta) = max(\tilde{F}(\alpha), \tilde{G}(\beta))$ and $\nu(\alpha, \beta) = max(\mu(\alpha), \delta(\beta))$, $\forall(\alpha, \beta) \in A \times B$.

Example 4.2. Let $U = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2, e_3\}$, with $\tilde{F}_\mu$ and $\tilde{G}_\nu$ as in example 4.1. Suppose now the company wants to buy a television based on two parameters. Then we have $(\tilde{F}_\mu, A) \vee (\tilde{G}_\delta, B) = (\tilde{H}_\lambda, A \times B)$ where

$$\begin{aligned} \tilde{H}(e_1, e_1) = \{(&\frac{x_1}{max(0.4, 0.2), max(0.6, 0.5)}, max(0.3, 0.1)), \\ &(\frac{x_2}{max(0.5, 0.6), max(0.5, 0.7)}, max(0.9, 0.5)) \\ &, (\frac{x_3}{max(0.9, 0.8), max(0, 0.9)}, max(0, 0.6))\} \\ &= \{(\frac{x_1}{[0.4, 0.6]}0.3), (\frac{x_2}{[0.6, 0.7]}, 0.9), (\frac{x_3}{[0.9, 0.9]}, 1)\}. \end{aligned}$$

Similarly we get
$\tilde{H}_\lambda(e_1, e_2) = \{(\frac{x_1}{[0.7,0.8]}0.7), (\frac{x_2}{[0.5,0.5]}, 0.9), (\frac{x_3}{[0.9,0.5]}, 1)\}$

$\tilde{H}_\lambda(e_1, e_3) = \{(\frac{x_1}{[0.4,0.7]}0.3), (\frac{x_2}{[0.5,0.6]}, 0.9), (\frac{x_3}{[0.9,0.5]}, 1)\}$
$\tilde{H}_\lambda(e_2, e_1) = \{(\frac{x_1}{[0.6,0.7]}0.5), (\frac{x_2}{[0.6,0.7]}, 0.5), (\frac{x_3}{[0.9,0.9]}, 0.6)\}$
$\tilde{H}_\lambda(e_2, e_2) = \{(\frac{x_1}{[0.7,0.8]}0.7), (\frac{x_2}{[0.2,0.2]}, 0.2), (\frac{x_3}{[0.9,0.9]}, 0.7)\}$
$\tilde{H}_\lambda(e_2, e_3) = \{(\frac{x_1}{[0.6,0.7]}0.5), (\frac{x_2}{[0.7,0.8]}, 0.9), (\frac{x_3}{[0.5,0.5]}, 0.8)\}$
$\tilde{H}_\lambda(e_3, e_1) = \{(\frac{x_1}{[0.2,0.7]}0.6), (\frac{x_2}{[0.7,0.8]}, 0.5), (\frac{x_3}{[0.8,0.9]}, 0.8)\}$
$\tilde{H}_\lambda(e_3, e_2) = \{(\frac{x_1}{[0.7,0.8]}0.7), (\frac{x_2}{[0.7,0.8]}, 0.4), (\frac{x_3}{[0.5,0.5]}, 0.8)\}$
$\tilde{H}_\lambda(e_3, e_3) = \{(\frac{x_1}{[0.3,0.7]}0.6), (\frac{x_2}{[0.7,0.8]}, 0.9), (\frac{x_3}{[0.5,0.5]}, 0.8)\}$
In matrix notation, we have

$$(\tilde{F}_\mu, A) \vee (\tilde{G}_\delta, B) = \begin{pmatrix} [0.4, 0.6], 0.3 & [0.6, 0.7], 0.9 & [0.9, 0.9], 0.1 \\ [0.7, 0.8], 0.7 & [0.5, 0.5], 0.9 & [0.9, 0.5], 0.1 \\ [0.4, 0.7], 0.3 & [0.5, 0.6], 0.9 & [0.9, 0.5], 0.1 \\ [0.6, 0.7], 0.5 & [0.6, 0.7], 0.5 & [0.9, 0.9], 0.6 \\ [0.7, 0.8], 0.7 & [0.2, 0.2], 0.2 & [0.9, 0.9], 0.7 \\ [0.6, 0.7], 0.5 & [0.7, 0.8], 0.9 & [0.5, 0.5], 0.8 \\ [0.2, 0.7], 0.6 & [0.7, 0.8], 0.5 & [0.8, 0.9], 0.8 \\ [0.7, 0.8], 0.7 & [0.7, 0.8], 0.4 & [0.5, 0.5], 0.8 \\ [0.3, 0.7], 0.6 & [0.7, 0.8], 0.9 & [0.5, 0.5], 0.8 \end{pmatrix}$$

We again subtract the false-membership function from the truth-membership function as follows:

$$(\tilde{F}_\mu, A) \vee (\tilde{G}_\delta, B) = \begin{pmatrix} -0.2, 0.3 & -0.1, 0.9 & \underline{0}, 1 \\ -0.1, 0.7 & 0, 0.9 & \underline{0.4}, 1 \\ -0.3, 0.3 & -0.1, 0.9 & \underline{0.4}, 0.1 \\ -0.1, 0.5 & -0.1, 0.5 & \underline{0}, 0.6 \\ -0.1, 0.7 & \underline{0}, 0.2 & \underline{0}, 0.7 \\ -0.1, 0.5 & -0.1, 0.9 & \underline{0}, 0.8 \\ -0.5, 0.6 & \underline{-0.1}, 0.5 & \underline{-0.1}, 0.8 \\ -0.1, 0.7 & -0.1, 0.4 & \underline{0}, 0.8 \\ -0.4, 0.6 & -0.1, 0.9 & \underline{0}, 0.8 \end{pmatrix}$$

As before, we first determine the highest score in each row and the result is displayed in Table 2. Now the score of each television is calculated by taking the sum of products of these numerical grades with the corresponding possibility λ. The television with the highest score is the desired television. We have not taken the numerical grades of the television against the pairs (e_i, e_i), $i = 1, 2, 3$, as both the parameters are the same. score$(x_1) = 0$, score $(x_2) = (-0.1 \times 0.5) = -0.5$, score $(x_3) = (0.4 \times 0.1) + (0.4 \times 0.1) + (0 \times 0.6) + (0 \times 0.8) + (-0.1 \times 0.8) + (0 \times 0.8) = 0$.

$\tilde{H}$	x_i	$Highest numerical grade$	λ_i
(e_1, e_1)	x_3	X	X
(e_1, e_2)	x_3	0.4	1
(e_1, e_3)	x_3	0.4	1
(e_2, e_1)	x_3	0	0.6
(e_2, e_2)	x_2, x_3	X	X
(e_2, e_3)	x_3	0	0.8
(e_3, e_1)	x_2, x_3	-0.1	0.5,0.8
(e_3, e_2)	x_3	0	0.8
(e_3, e_3)	x_3	X	X

Table 2: Record of highest numerical grade for each pair

Hence, the suitable televisions for the company are televisions x_1 and x_3.

5. Similarity between Two Possibility Vague Soft Set

In this section, we introduce a measure of similarity between two PVSSs. The set theoretical approach has been taken in this regard because it is easier for calculation and is a very popular method too[5, 16, 17, 18].

Many researchers have studied the problem of similarity measurement between fuzzy sets, fuzzy numbers, vague set and possibility fuzzy soft set. Majumdar and Samanta [16, 17, 18] have studied the similarity measure of soft sets, fuzzy soft sets, and generalized fuzzy soft set. Also Alkhazaleh et al. [5] studied similarity measurement between two possibility fuzzy sets.

Definition 5.1. Similarity between two PVSSs $\tilde{F}_\mu$ and $\tilde{G}_\delta$, denoted by $\tilde{S}(\tilde{F}_\mu, \tilde{G}_\delta)$, is defined as follows:

$$\tilde{T}(\tilde{F}_\mu, \tilde{G}_\delta) = M(\tilde{F}(e), \tilde{G}(e)) \cdot M(\mu(e), \delta(e))$$

such that

$$M(\tilde{F}(e), \tilde{G}(e)) = max_i M_i(\tilde{F}(e), \tilde{G}(e)),$$

$$M(\mu(e), \delta(e)) = max_i M_i(\mu(e), \delta(e)),$$

where

$$M_i(\tilde{F}(e), \tilde{G}(e)) = \frac{1}{n}\sum_{i=1}^{n}(1 - |\frac{\tilde{S}(\tilde{F}(e)(x_i)) - \tilde{S}(\tilde{G}(e)(x_i))}{2}|),$$

$$M_i(\tilde{F}(e),\tilde{G}(e)) = 1 - \frac{\sum_{j=1}^{n}|\mu_{ij|}-\delta_{ij}|}{\sum_{j=1}^{n}|\mu_{ij|}-\delta_{ij}|}.$$

Let $f^{\square}(x_i) = 1-f(x_i)$, then $x_i = [t_{x_i}, 1-f_{x_i}] = [t_{x_i}, f^{\square}_{x_i}]$. In this case we can see that $\tilde{S}(\tilde{F}(x_i)) = t(x_i) + f^{\square}(x_i) - 1$ where $\tilde{S}(\tilde{F}(e)(x_i)) \in [-1,1]$, and similarly of $\tilde{G}(e)(x_i))$.

Definition 5.2. Let $\tilde{F}_\mu$ and $\tilde{G}_\delta$ be two PVSSs over (U,E). We say that $\tilde{F}_\mu$ and $\tilde{G}_\delta$ are significantly similar if $\tilde{T}(\tilde{F}_\mu,\tilde{G}_\delta) \geq \frac{1}{2}$.

Example 5.1. Consider example 4.1 where $\tilde{F}_\mu$ and $\tilde{G}_\delta$ are defined as follows:
$\tilde{F}_\mu(e_1) = \{(\frac{x_1}{[0.4,0.6]}0.3),(\frac{x_2}{[0.5,0.5]},0.9),(\frac{x_3}{[0.9,0]},1)\}$,
$\tilde{F}_\mu(e_2) = \{(\frac{x_1}{[0.6,0.7]}0.5),(\frac{x_2}{[0.1,0.1]},0.2),(\frac{x_3}{[0.9,0.9]},0)\}$,
$\tilde{F}_\mu(e_3) = \{(\frac{x_1}{[0,0.7]}0.6),(\frac{x_2}{[0.7,0.8]},0.4),(\frac{x_3}{[0.3,0.3]},8)\}$,
$\tilde{G}_\delta(e_1) = \{(\frac{x_1}{[0.2,0.5]}0.1),(\frac{x_2}{[0.6,0.7]},0.5),(\frac{x_3}{[0.8,0.9]},0.6)\}$,
$\tilde{G}_\delta(e_2) = \{(\frac{x_1}{[0.7,0.8]}0.7),(\frac{x_2}{[0.2,0.2]},0.2),(\frac{x_3}{[0.5,0.5]},0.7)\}$,
$\tilde{G}_\delta(e_3) = \{(\frac{x_1}{[0.3,0.7]}0.2),(\frac{x_2}{[0.1,0.6]},0.9),(\frac{x_3}{[0.5,0.5]},3)\}$.
Here

$$M_1(\mu(e),\delta(e)) = 1 - \frac{\sum_{j=1}^{3}|\mu_{1j}(e)-\delta_{1j}(e)|}{\sum_{j=1}^{3}|\mu_{1j}(e)+\delta_{1j}(e)|}$$

$$= 1 - \frac{|0.3-0.1|+|0.5-0.5|+|1-0.6|}{|0.3+0.1|+|0.5+0.5|+|1+0.6|} = 0.8$$

Similarly we get $M_2 = (\mu(e),\delta(e)) = 0.61$ and $M_3 = (\mu(e),\delta(e)) = 0.56$. Then

$$M(\mu(e),\delta(e)) = max(M_1(\mu(e),\delta(e)), M_2(\mu(e),\delta(e)), M_3(\mu(e),\delta(e))) = 0.8.$$

$$\begin{aligned} M_1(\tilde{F}(e),\tilde{G}(e)) &= \frac{1}{n}(\sum_{j=1}^{n}(1-|\frac{\tilde{S}(\tilde{F}(e)(x_j)) - \tilde{S}(\tilde{G}(e)(x_j))}{2}|)) \\ &= \frac{1}{3}[(1-|\frac{0-(-0.3)}{2}|) + (1-|\frac{0-0.3}{2}|) \\ &\quad +(1-|\frac{-0.1-0.7}{2}|)] \\ &= \frac{1}{3}(0.85+0.85+0.6) = 0.77. \end{aligned}$$

Similarly we get $M_2(\tilde{F}(e),\tilde{G}(e)) = 0.8$ and $M_3(\tilde{F}(e),\tilde{G}(e)) = 0.75$. Then

$$M(\tilde{F}(e),\tilde{G}(e)) = max\{M_1(\tilde{F}(e),\tilde{G}(e)), M_2(\tilde{F}(e),\tilde{G}(e)), M_3(\tilde{F}(e),\tilde{G}(e))\} = 0.8.$$

Hence, the similarity between two PVSSs $\tilde{F}_\mu$ and $\tilde{G}_\delta$ is given by $\tilde{T}(\tilde{F}_\mu,\tilde{G}_\delta) = M(\tilde{F}(e),\tilde{G}(e)) \cdot M(\mu(e),\delta(e)) = 0.8 \times 0.8 = 0.64$.

6. Application of Similarity Measure in Medical Diagnosis

In the following example we will try to approximate the possibility that an ill person having certain visible symptoms is suffering from flu. For this we first build a possibility vague soft set model for flu and the possibility vague soft set of symptoms for the sick person. Next we find the similarity measure of these two sets. If they are significantly similar then we conclude that the person is possibly suffering from flu.

Let the universal set consists of only two elements "yes" and "no" that is U =(yes, no). Here the set of parameters E is the set of certain visible symptoms. Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$, where e_1 is fever, e_2 is aches, e_3 is chills, e_4 is tiredness, e_5 is sudden onset, e_6 is coughing, e_7 is sneezing and e_8 is sore throat.

Our model possibility vague soft set for flu M_μ is given in Table 3, which can be prepared with the help of a physician.

M_μ	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
y	1	0	0	1	1	1	1	0
μ_y	1	1	1	1	1	1	1	1
n	0	1	1	0	0	0	0	1
μ_n	1	1	1	1	1	1	1	1

Table 3: Model PVSS for flu

After interviewing the sick person, we can build the PVSS $\tilde{G}_\delta$ as in Table 4. Now we find the similarity measure of these two sets as in the previous example. It can be seen that $\tilde{T}(\tilde{F}_\mu, \tilde{G}_\delta) \cong 0.12 \leq \frac{1}{2}$.

Hence the measure of the two PVSSs are not significantly similar. Therefore, we conclude that the person is not suffering from flu.

$\tilde{F}_\alpha$	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
y	[0.7,0.7]	[0.3,0.9]	[0.9,0.9]	[0.5,0.5]	[0.3,0.3]	[0.6,0.6]	[0,0]	[1,1]
μ_y	0.3	0.5	0.2	0.4	0.8	0.9	1	0
n	[0.9,1]	[0.8,0.9]	[0,0]	[1,1]	[0.3,0.7]	[0.3,0.3]	[0.1,0.1]	[0.6,0.8]
μ_n	0.2	0.1	0.5	0.8	0.7	0.6	0.3	0.4

Table 4: PVSS for the ill person

7. Conclusions

In this paper, the basic concepts of a possibility fuzzy soft set are reviewed, and some basic operations on possibility fuzzy soft sets are given. We introduce the concept of possibility vague soft set and give application of this theory to solve a decision-making problem. Similarity measure of two possibility vague soft sets is discussed and an application of this to medical diagnosis has been shown. It is also desirable to further explore the applications of using the possibility vague soft set approach to solve real world decision making problems. In addition, this new extension not only provides a significant addition to existing theories for handling uncertainties, but also leads to potential areas of further research and pertinent applications. A potential area of research involves extending our work to study the relationship between soft set, rough set and vague soft set.

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