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EXACT SOLUTIONS FOR KDV-BURGER EQUATIONS WITH AN APPLICATION OF WHITE-NOISE ANALYSIS

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Abstract: In this paper we will give exact solutions of the variable coefficient KdV-Burger equations

$$u_t + \alpha(t)uu_x + \beta(t)u_{xx} + \gamma(t)u_{xxx} = 0,$$

where $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ are bounded measurable or integrable functions on $\mathbb{R}_+$. Moreover, using the Hermite transform and the homogeneous balance principle, the white noise functional solutions for the Wick-type stochastic KdV-Burger equations are explicitly obtained.

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1. Preliminaries

Random waves is an important subject of random PDEs. There are many au-

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thers studied this subject. Wadati first introduced and studied the stochastic KdV equation and gave the diffusion of solition of the KdV equation under Gaussian noise in (see [11]-[13]). Xie firstly researched Wick-type stochastic KdV equation on white noise space and showed the auto-Bachlund transformation and the exact white noise functional solutions in [19], furthermore, Chen and Xie (see [1]-[3]) and Xie (see [17]-[20]) researched some Wick-type stochastic wave equations using white noise analysis method. Recently, Uğurlu and Kaya[10] gave the tanh function method, Wazzan [16] showed the modified tanh-coth method, these methods have been applied to derive nonlinear transformations and exact solutions of nonlinear PDEs in mathematical physics. Many authors considered nonlinear equations of the form

$$P(u, u_t, u_x, u_{xt}, u_{xx}, u_{xxx}, ...) = 0 \tag{1.1}$$

where P is a nonlinear function with respect to the indicated variables. Introducing the one wave variable $\zeta = x - ct$ carry out the two independent partial differential equation (1.1) into an ODE

$$N(u, u', u'', u''', ...) = 0 \tag{1.2}$$

Equation (1.2) is then integrated as long as all terms contain derivatives. The tanh technique is based on the priori assumption that the travelling wave solutions can be expressed in terms of the tanh function [6]. We therefor introduce a new independent variable

$$Y = tanh(\mu\zeta)$$

that leads to the change of derivatives:

$$\frac{d}{d\zeta} = \mu(1 - Y^2)\frac{d}{dY},$$

$$\frac{d^2}{d\zeta^2} = \mu^2(1 - Y^2)(-2Y\frac{d}{dY} + (1 - Y^2\frac{d^2}{dY^2}))$$

The solution can be proposed by the tanh method as a finite power series in Y in the form:

$$u(\mu\zeta) = S(Y) = \sum_{k=0}^{M} a_k Y^k, \tag{1.3}$$

limiting them to solitary and shock wave profiles. However, the extended tanh method admits the use of the finite expansion

$$u(\mu\zeta) = S(Y) = \sum_{k=0}^{M} a_k Y^k + \sum_{k=1}^{M} a_k Y^{-k}, \tag{1.4}$$

where M is a positive integer, in most cases, that will be determined. Expansion (1.4) reduces to the standard tanh method (see [6-8]) for $a_k = 0$, $1 \leq k \leq M$. Substituting (1.3) or (1.4) into the ODE (1.2) results in an algebraic equation in powers of Y. In this paper we will give exact solutions of the variable coefficient KdV-Burger equations

$$u_t + \alpha(t)uu_x + \beta(t)u_{xx} + \gamma(t)u_{xxx} = 0 \tag{1.5}$$

where $\alpha(t), \beta(t)$ and $\gamma(t)$ are bounded measurable or integrable functions on $\mathbb{R}_+$. Also, by using Hermite transform and the homogeneous balance principle the exact solutions for the wick-type stochastic form of the above equation is showed.

2. Exact Solutions of Stochastic Form of (1.5)

In this section, we will give exact solutions of Eqn(1.5). The Wick-type stochastic KdV-Burger equation is the perturbation of equation (1.5) by random force $W(t) \diamond R^{\diamond}(U, U_t, U_x, U_{xx}, U_{xxx})$, which is represented by

$$\begin{aligned} U_t + A(t) \diamond U \diamond U_x + B(t) \diamond U_{xx} + \Gamma(t) \diamond U_{xxx} \\ = W(t) \diamond R^{\diamond}(U, U_t, U_x, U_{xx}, U_{xxx}), \end{aligned} \tag{2.1}$$

where $W(t)$ is Gaussian white noise, i.e., $W(t) = B^{\cdot}(t)$ and B(t) is a Brownian motion. Taking the Hermite transform of equation (2.1), we get

$$\begin{aligned} \widetilde{U}_t(t,x,z) + \widetilde{A}(t,z)\widetilde{U}(t,x,z)\widetilde{U}_x(t,x,z) \\ + \widetilde{B}(t,z)\widetilde{U}_{xx}(t,x,z) + \widetilde{\Gamma}(t,z)\widetilde{U}_{xxx}(t,x,z) = 0, \end{aligned} \tag{2.2}$$

where $z = (z_1, z_2, ...) \in \mathbb{C}^{\mathbb{N}}$ is a parameter. Let $u(t,x,z) = \widetilde{U}(t,x,z), \alpha(t,z) = \widetilde{A}(t,z), \beta(t,z) = \widetilde{B}(t,z), \gamma(t,z) = \widetilde{\Gamma}(t,z)$. Assuming that the solutions of equations (2.1) have the form

$$u(t,x,z) = \psi(\zeta(t,x,z)) \tag{2.3}$$

with

$$\zeta(t,x,z) = kx + s\int_0^t l(\tau,z)d\tau + c$$

where k, s, c are arbitrary constants which satisfy $ks \neq 0$ and $l(t,z)$ nonzero functions of indicated variables to be determined. Substituting (2.3) into (2.2) and integrating the resulting ODE once and setting the constant of integration equal to zero yields

$$sl\psi + \frac{\alpha k}{2}\psi^2 + \beta k^2\psi' + \gamma k^3\psi'' = 0, \tag{2.4}$$

where $\psi' = \frac{d\psi(\zeta)}{d\zeta}$. Considering the homogeneous balance betweenψ'' and ψ^2 in (2.3), gives M=2, hence $\psi(x, t, z)$ can be expressed by the modified tanh-coth method as:

$$\psi(x,t,z) = S(Y) = a_0(t,z) + a_1(t,z)Y(\zeta) + a_2(t,z)Y^2(\zeta) + b_1(t,z)Y^{-1}(\zeta) + b_2(t,z)Y^{-2}(\zeta), \tag{2.5}$$

where $Y(\zeta)$ satisfies the Riccati equation

$$Y' = c_0 + c_1 Y + c_2 Y^2, \tag{2.6}$$

and c_0, c_1, c_2 are constant to be prescribed later. Substituting (2.4) with (2.5) into equation (2.3), taking into account the linear independence of $Y^n(n = -4, -3, ..., 4)$, resulting the following system of nine equations

$$\begin{cases} sla_0 + \frac{1}{2}\alpha k(a_0^2 + 2a_1b_1 + 2a_2b_2) + \beta k^2(a_1c_0 - b_1c_2) \\ \qquad + \gamma k^3(a_1c_1c_0 + 2a_2c_0^2 + 2b_1c_1c_2 + 2b_2c_2^2) = 0, \\ sla_1 + \frac{1}{2}\alpha k(2a_0a_1 + 2a_2b_1) + \beta k^2(a_1c_1 + 2a_2c_0) \\ \qquad + \gamma k^3(a_1c_1^2 + 2a_1c_0c_2 + 6a_2c_1c_0) = 0, \\ sla_2 + \frac{1}{2}\alpha k(a_1^2 + 2a_0a_2) + \beta k^2(a_1c_2 + 2a_2c_1) \\ \qquad + \gamma k^3(3a_1c_1c_2 + 8a_2c_2c_0 + 4a_2c_1^2) = 0, \\ \alpha k a_1a_2 + 2\beta k^2 a_2c_2 + \gamma k^3(2a_1c_2^2 + 10a_2c_1c_2) = 0, \\ \frac{1}{2}\alpha k a_2^2 + 6\gamma k^3 a_2c_2^2 = 0, \\ \frac{1}{2}\alpha k b_2^2 + 6\gamma k^3 b_2c_0^2 = 0. \\ \alpha k b_1b_2 - 2\beta k^2 b_2c_0 + \gamma k^3(2b_1c_0^2 + 10b_2c_0c_1) = 0, \\ slb_1 + \frac{1}{2}\alpha k(2a_0b_1 + 2a_1b_2) - \beta k^2(b_1c_1 + 2b_2c_1) \\ \qquad + \gamma k^3(b_1c_1^2 + 2b_1c_0c_2 + 6b_2c_1c_0) = 0, \\ slb_2 + \frac{1}{2}\alpha k(b_1^2 + 2a_0b_2) - \beta k^2(2b_1c_0 + 2b_2c_1) \\ \qquad + \gamma k^3(3b_1c_1c_0 + 8b_2c_2c_0 + 4b_2c_1^2) = 0, \end{cases} \tag{2.7}$$

In the remaining part of this section we investigate and solve our problem for some particular cases for the Riccati equation (2.6) as follows:

Case 1. $c_0 = c_1 = 1, c_2 = 0$. This choice for the constants implies that, the Riccati equation (2.6) has the solution

$$Y_1(\zeta) = exp(\zeta) - 1. \tag{2.8}$$

Solving the above system of equations (2.7) with the aid of Mathematica, implies: $a_0 = -\frac{2sl}{\alpha k}; a_1 = a_2 = 0; b_1 = 2.4k\alpha^{-1}(\beta - 5\gamma k); b_2 = -\frac{12\gamma}{\alpha}k^2, l = \frac{10k^3(\beta-3\gamma k)}{s(\beta-5\gamma k)} - \frac{k^2}{s}(\beta - \gamma k)$. According to (2.3),(2.4) and (2.8), we get the following particular solution of (2.2):

$$\begin{aligned} u_1(t,x,z) =& \frac{2k}{\alpha(t,z)}(\beta(t,z) - k\gamma(t,z)) - \frac{20k^2(\beta(t,z) - 3k\gamma(t,z))}{\alpha(t,z)(\beta(t,z) - 5k\gamma(t,z))} \\ &+ 2.4k(\frac{\beta(t,z) - 5k\gamma(t,z)}{\alpha(t,z)}) \\ &\times (exp(\zeta(t,x,z)) - 1) - \frac{12k^2\gamma(t,z)}{\alpha(t,z)}(exp(\zeta(t,x,z)) - 1)^2, \end{aligned} \tag{2.9}$$

with

$$\zeta(t,x,z) = kx + s\int_0^t \{\frac{10k^3(\beta(\tau,z) - 3k\gamma(\tau,z))}{s(\beta(\tau,z) - 5k\gamma(\tau,z))} - \frac{k^2}{s}(\beta(\tau,z) - k\gamma(\tau,z))\}d\tau$$

Case 2. $c_1 = c_2 = 1, c_0 = 0$. The Riccati equation (2.6) has the solution

$$Y_2(\zeta) = (exp(-\zeta) - 1)^{-1} \tag{2.10}$$

Solving the above system of equations (2.7) with the aid of Mathematica, implies:

$$a_0 = -\frac{2sl}{\alpha k}; a_1 = 2.4k\alpha^{-1}(\beta + 5\gamma k); a_2 = \frac{-12\gamma k^2}{\alpha}; b_1 = b_2 = 0;$$

$l = \frac{k^2}{s}(\beta + k\gamma)$. According to (2.3),(2.4) and (2.10), we get the following particular solution of (2.2):

$$\begin{aligned} u_2(t,x,z) =& \frac{k}{\alpha(t,z)}(\beta(t,z) + k\gamma(t,z)) + \frac{2.4k}{\alpha(t,z)}(\beta(t,z) + 5k\gamma(t,z)) \\ &\times (exp(-\zeta(t,x,z)) - 1)^{-1} - \frac{12k^2\gamma(t,z)}{\alpha(t,z)}(exp(-\zeta(t,x,z)) - 1)^{-2}, \end{aligned} \tag{2.11}$$

with

$$\zeta(t,x,z) = kx + k^2\int_0^t (\beta(\tau,z) + k\gamma(\tau,z))d\tau$$

Case 3. $c_0 = -c_2 = \frac{1}{2}$, $c_1 = 0$. The Riccati equation (2.6) has the solutions

$$Y_3(\zeta) = coth(\zeta) \pm csch(\zeta), \quad Y_4(\zeta) = tanh(\zeta) \pm isech(\zeta) \tag{2.12}$$

Solving the above system of equations (2.7) with the aid of Mathematica, implies:

$$a_0 = -\frac{1}{2}\{R(\alpha,\beta,\gamma) \mp \sqrt{R^2(\alpha,\beta,\gamma) - 4S(\alpha,\beta,\gamma)}\};$$

$$a_1 = b_1 = \frac{6k}{5\alpha}\beta; \quad a_2 = b_2 = \frac{-3\gamma k^2}{\alpha};$$

$$l = -\frac{\alpha k}{2s}\{R(\alpha,\beta,\gamma) \mp \sqrt{R^2(\alpha,\beta,\gamma) - 4S(\alpha,\beta,\gamma)}\} + T(\alpha,\beta,\gamma),$$

where $R(\alpha,\beta,\gamma) = \frac{2}{k\alpha b_2}\{0.5k\alpha b_1^2 - \beta k^2 b_1 - 2k^3\gamma b_2\}$; $T(\alpha,\beta,\gamma) = \frac{k^3\gamma b_1 - k\alpha a_1 b_2}{2sb_1}$; $S(\alpha,\beta,\gamma) = \frac{2}{k\alpha}\{k\alpha a_1 b_1 + k\alpha a_2 b_2 + 0.5k^2\beta(a_1+b_1) + 0.5k^3\gamma(a_1+a_2+b_2)\}$.

According to (2.3),(2.4) and (2.12), we get the following particular solution of (2.2):

$$u_{3,4}(t,x,z) = -\frac{1}{2}\{R(\alpha,\beta,\gamma) \mp \sqrt{R^2(\alpha,\beta,\gamma) - 4S(\alpha,\beta,\gamma)}\} + \frac{6k}{5\alpha}\beta Y(\zeta(x,y,z))$$
$$-\frac{3\gamma k^2}{\alpha}(Y^2(\zeta(x,y,z)) + Y^{-2}(\zeta(x,y,z))) + \frac{6k}{5\alpha}(\beta + k\gamma)Y^{-1}(\zeta(x,y,z)) \tag{2.13}$$

with

$$\zeta(t,x,z) = kx$$
$$\mp sign\frac{k}{2}\int_0^t \{\alpha(\tau,z)\{R(\tau,z) \mp \sqrt{R^2(\tau,z)) - 4S(\tau,z)}\} + T(\tau,z)\}d\tau.$$

Case 4. $c_2 = 4c_0 = 4, c_1 = 0$. The Riccati equation (2.6) has the solutions

$$Y_5(\zeta) = \frac{1}{2}tan(2\zeta), \quad Y_6(\zeta) = \frac{1}{2}cot(2\zeta) \tag{2.14}$$

Solving the above system of equations (2.7) with the aid of Mathematica, implies:

$$a_0 = \frac{-R_*(\alpha,\beta,\gamma) \pm \sqrt{R_*^2(\alpha,\beta,\gamma) - 4S_*(\alpha,\beta,\gamma)}}{2};$$

$$a_1 = -\frac{48\beta k}{7\alpha}; \; a_2 = -16b_2 = \frac{192\gamma k^2}{\alpha}; \; b_1 = 2.4k\beta\alpha^{-1};$$

$$l = -\frac{\alpha k}{2}\{\frac{-R_*(\alpha,\beta,\gamma) \pm \sqrt{R_*^2(\alpha,\beta,\gamma) - 4S_*(\alpha,\beta,\gamma)}}{2}\} + T_*(\alpha,\beta,\gamma),$$

where

$$R_*(\alpha,\beta,\gamma)=\frac{\alpha b_1^2-4\beta k b_1^2+64\gamma k^2 b_2}{\alpha b_2}; S_*(\alpha,\beta,\gamma)$$
$$=\frac{2\alpha a_1 b_1+2\alpha a_2 b_2+2\beta k(a_1-4b_1)+2\gamma k^2(2a_2+32b_2)}{\alpha};$$

$$T_*(\alpha,\beta,\gamma)=\frac{4\gamma k^3 b_1-\alpha k b_2 a_1-\alpha k a_0 b_1}{s b_1}.$$

According to (2.3),(2.4) and (2.14), we get the following particular solution of (2.2):

$$u_{5,6}(t,x,z)=\frac{-R_*(\alpha,\beta,\gamma)\pm\sqrt{R_*^2(\alpha,\beta,\gamma)-4S_*(\alpha,\beta,\gamma)}}{2}-\frac{48\beta k}{7\alpha}Y(\zeta(t,x,z))$$
$$+\frac{192\gamma k^2}{\alpha}Y^2(\zeta(t,x,z))+2.4k\beta\alpha^{-1}Y^{-1}(\zeta(t,x,z))$$
$$-\frac{12\gamma k^2}{\alpha}Y^{-2}(\zeta(t,x,z)) \quad (2.15)$$

with

$$\zeta(t,x,z)=kx\mp sign\frac{k}{4}\int_0^t\{\alpha(\tau,z)(-R_*(\tau,z)\pm\sqrt{R_*^2(\tau,z)-4S_*(\tau,z)})$$
$$+T_*(\tau,z)\}d\tau$$

Obviously, there are infinitely number of particular solutions for system (2.7) together with Riccati equation (2.6) resulting in many different cases[14-15]. The above mentioned cases are just clarifying how far our proposed technique is applicable. The properties of hyperbolic functions yield that there exists a bounded open set $\mathbf{S}\subset\mathbb{R}_+\times\mathbb{R}, m>0$ and $n>0$ such that $u(x,t,z), u_{xt}(x,t,z)$ are uniformally bounded for all $(t,x,z)\in\mathbf{S}\times\mathbb{K}_m(n)$, continuous with respect to $(t,x)\in\mathbf{S}$ for all $z\in\mathbb{K}_m(n)$ and analytic with respect to $z\in\mathbb{K}_m(n)$ for all $(t,x)\in\mathbf{S}$[4-5]. Using Theorem 2.1 of Xie [20], there exists a stochastic process $U(t,x)$ such that the Hermite transformation of $U(t,x)$ is $u(t,x,z)$ for all $\mathbf{S}\times\mathbb{K}_m(n)$, and $U(t,x)$ is the solution of (2.1). This implies that $U(t,x)$ is the inverse Hermite transformation of $u(t,x,z)$. Hence, for $AB\Gamma\neq 0$ the white noise functional solutions of equations (2.2) as follows:

$$U_1(t,x)=\frac{2k}{A(t)}(B(t)-k\Gamma(t))-\frac{20k^2(B(t)-3k\Gamma(t))}{A(t)(B(t)-5k\Gamma(t))}+2.4k(\frac{B(t)-5k\Gamma(t)}{A(t)})\times$$

$$(exp^{\diamond}(\Xi_1(t,x))-1)-\frac{12k^2\Gamma(t)}{A(t)}(exp^{\diamond}(\Xi_1(t,x))-1)^2 \qquad (2.16)$$

with

$$\Xi_1(t,x)=kx+\int_0^t\{\frac{10k^3(B(\tau)-3k\Gamma(\tau))}{(B(\tau)-5k\Gamma(\tau))}-k^2(B(\tau)-k\Gamma(\tau))\}d\tau$$

$$U_2(t,x)=\frac{k}{A(t)}(B(t)+k\Gamma(t))+\frac{2.4k}{A(t)}(B(t)+5k\Gamma(t))$$

$$\times(exp\diamond(-\Xi_2(t,x))-1)^{-1}-\frac{12k^2\Gamma(t)}{A(t)}(exp\diamond(-\Xi_2(t,x))-1)^{-2} \qquad (2.17)$$

with

$$\Xi_2(t,x)=kx+k^2\int_0^t(B(\tau)+k\Gamma(\tau))d\tau$$

$$U_{3,4}(t,x)=-\frac{1}{2}\{R(A,B,\Gamma)\mp\sqrt{R^2(A,B,\Gamma)-4S(A,B,\Gamma)}\}+\frac{6k}{5A}BY^{\diamond}(\Xi_3(x,y))$$
$$-\frac{3\Gamma k^2}{A}(Y^{\diamond^2}(\Xi_3(x,y))+Y^{\diamond^{-2}}(\Xi_3(x,y)))+\frac{6k}{5A}(B+k\Gamma)Y^{\diamond^{-1}}(\Xi_3(x,y))$$

with

$$\Xi_3(t,x)=kx\mp sign\frac{k}{2}\int_0^t\{A(\tau)\{R(\tau)\mp\sqrt{R^2(\tau))-4S(\tau)}\}+T(\tau)\}d\tau$$

$$U_{5,6}(t,x)=\frac{-R_*(A,B,\Gamma)\pm\sqrt{R_*^2(A,B,\Gamma)-4S_*(A,B,\Gamma)}}{2}-\frac{48Bk}{7A}Y(\Xi_4(t,x))+$$

$$\frac{192\Gamma k^2}{A}Y^2(\Xi(t,x))+2.4kBA^{-1}Y^{-1}(\Xi_4(t,x))-\frac{12\Gamma k^2}{A}Y^{-2}(\Xi_4(t,x)) \quad (2.18)$$

with

$$\Xi_4(t,x)=kx\mp sign\frac{k}{4}\int_0^t\{A(\tau)(-R_*(\tau)\pm\sqrt{R_*^2(\tau)-4S_*(\tau)})+T_*(\tau)\}d\tau$$

3. Examples and Concluding Remarks

For different form of $A(t), B(t)$ and $\Gamma(t)$, we can get different solutions of (2.1) from (2.17)-(2.20). Let $\dot{B_t}$ be the Gaussian white noise, where B_t is Brown motion. We have the Hermite transform $\widetilde{B}^{\cdot}(t,z) = \sum_{k=1}^{\infty} z_k \int_0^t \eta_k(s)ds$. Science $\exp^{\diamond}(B_t) = \exp(B_t - t^2/2)$, we have $tanh^{\diamond}(B_t) = tanh(B_t - t^2/2)$, $coth^{\diamond}(B_t) = cot(B_t - t^2/2)$, $sech^{\diamond}(B_t) = sech(B_t - t^2/2)$ and $csch^{\diamond}(B_t) = csch(B_t - t^2/2)$.

Example 3.1. Suppose $A(t) = \mu B(t)$ and $B(t) = \beta(t) + \nu \dot{B_t}$, where μ, ν are arbitrary constants and $\beta(t)$ is integrable or bounded measurable function on $\mathbb{R}_+$[9]. The white noise functional solutions of (2.1) are as follows: If $A(t)B(t) \neq 0$ and $\Gamma(t) = 0$

$$U_1(t,x) = \frac{2k}{\mu} - \frac{20k^2}{\mu B(t)} + \frac{2.4k}{\mu} \times exp^{\diamond}(\Xi_5(t,x)) - 1) \tag{3.1}$$

with

$$\Xi_5(t,x) = kx + 10k^3 t - k^2\{\int_0^t \beta(\tau)d\tau + \mu B_t - \frac{\mu}{2}t^2\}$$

$$U_2(t,x) = \frac{k}{\mu} + \frac{2.4k}{\mu} \times (exp(-\Xi_6(t,x)) - 1)^{-1} \tag{3.2}$$

with

$$\Xi_6(t,x) = kx + k^2\{\int_0^t \beta(\tau)d\tau + \mu B_t - \frac{\mu}{2}t^2\}$$

$$U_{3,4}(t,x) = -\frac{1}{2}\{R(A,B) \mp \sqrt{R^2(A,B) - 4S(A,B)}\} + \frac{6k}{5\mu}\{Y^{\diamond}(\Xi_7(x,y)) + Y^{\diamond^{-1}}(\Xi_7(x,y))\}$$

with

$$\Xi_7(t,x) = kx \mp sign\frac{k}{2}\int_0^t \{A(s)\{R(s) \mp \sqrt{R^2(s)) - 4S(s)}\} + T(s)\}ds$$

$$U_{5,6}(t,x) = \frac{-R_*(A,B) \pm \sqrt{R_*^2(A,B) - 4S_*(A,B)}}{2}$$

$$-\frac{48k}{7\mu}Y(\Xi_8(t,x)) + 2.4k\mu^{-1}Y^{-1}(\Xi_8(t,x))$$

with

$$\Xi_8(t,x) = kx \mp sign\frac{k}{4}\int_0^t \{A(s)(-R_*(s) \pm \sqrt{R_*^2(s) - 4S_*(s)}) + T_*(s)\}ds.$$

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