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A MODIFIED TRADING MODEL WITH AN APPLICATION TO THE GERMAN DAX INDEX

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Abstract: In [1] the authors presented a trading model, which enabled the description of typical movements of share values over the time. The basic assumptions are the dependence of temporal share value variation $W(t)$ on the daily buying stimulus, the "stimulus velocity" $dK(t)/dt$, and the distance between $W(t)$ and the basic share value M. The stimulus velocity itself was assumed to be proportional to the deviation of $W(t)$ from its personal expectation S. With these both assumptions an "ordinary" trading process was defined. Additionally, the possibility of "disturbing activities" was introduced, assuming their proportionality to the share value acceleration. The latter model was called a "general" trading process. Here, a modification of this model will be discussed, replacing the difference $W-M$ by its absolute value. In this way, the in [1] not easily explainable case of negative value velocity despite of positive daily buying stimulus can be prevented. Furthermore, it will be shown that the trading model can also be applied for simulations of DAX index time sequences.

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1. Notation

In this paper, the same notation will be used as in [1], with one exception: the quantities M, S, $W(t)$ and $K(t)$ will represent double intentions. Firstly, the description of share values, measured in Euro (€). Secondly, the application to DAX indices, with M, S, $W(t)$ and $K(t)$ as dimensionless numbers in the following slightly changed meanings:

t	time (d); with one day, d, as unit
M	basic share value (€); usually unaffected (material) basis **or** basic DAX index; dimensionless basic value for the DAX index
$W(t)$	share value at time t (€); current price for one share at time t **or** DAX index at time t; current DAX index
$\frac{\mathrm{d}W(t)}{\mathrm{d}t}$	value velocity (€/d) **or** index velocity (1/d); change of $W(t)$ per day
S	saturation value (€); personal expectation for W **or** saturation index
$K(t)$	buying stimulus (€); stimulus, in €, for the share customer **or** buying stimulus; stimulus characterizing commonly all shares which contribute to the DAX index
$\frac{\mathrm{d}K(t)}{\mathrm{d}t}$	stimulus velocity (€/d); daily stimulus for share buying **or** stimulus velocity (1/d); daily common buying stimulus
$\frac{\mathrm{d}^2K(t)}{\mathrm{d}t^2}$	stimulus acceleration (€/d^2); change of the daily stimulus with time **or** stimulus acceleration (1/d^2)
a	trading factor (1/d); part of $S-W(t)$ the customer is willing to spend per share for buying at the given day **or** trading factor (1/d); common factor for all shares which contribute to the DAX index
$g \cdot \frac{\mathrm{d}^2W(t)}{\mathrm{d}t^2}$	disturbing activity (€/d^2); additional buying stimulus acceleration ("distortion factor" $g > 0$, dimensionless) **or** disturbing activity (1/d^2)

The constants a and g are positive numbers.

2. Introduction

In the original mathematical trading model [1], the bargain was taken as the mean over all simultaneous movements in the share trade, generally. Therefore, the share values simulated by this model represented a summarizing result of all coincidental trading activities. The regularly published official DAX index has similar origins. It also represents an average over trading processes, indeed for several and different shares, but nevertheless in the same sense as here for a single kind of shares. From this the idea arose, to use the trading model from [1] also for DAX index simulations. Of course, a modified significance has to be chosen then for the quantities M, S, $W(t)$ and $K(t)$ as basic DAX index, saturation index, DAX index itself and buying stimulus, respectively, according to the above given notation.

Section 3 will be concerned with a problem which additionally appeared in the original model for share values during an ordinary trading process: the possibility of also negative share value velocities at positive stimulus velocities (even with "breakdown" tendencies) and, vice versa, of positive share value velocities during negative stimulus velocities. In the paper [1], some possible explanations for such appearances have been discussed, but the conclusions were not so easy to understand. On the other hand, they could not pass upon their realistic content, because of deficits in suitable examples. But nevertheless, in the original model we decided to choose basic equations for the trading process as simple as possible. Therefore further efforts to enhance the model towards greater nearness to reality may be allowed. Indeed, a very straightforward modification will be presented, only consisting in a small step: the substitution of the factor $W - M$ by its absolute value in equation (2) of [1], describing the relation between value velocity and stimulus velocity. The consequences will be discussed on the basis of Figure 2 from paper [1].

In the fourth section the application to DAX index calculations will be presented. The known DAX index time sequence will be used for attempts of fitting simulations to it, in the interval from 1998, April, to December of 2000. This example will demonstrate the obvious qualification of the trading model for such appropriate simulations. Furthermore, from the fitting procedure some interesting conclusions can be drawn, especially to a possible correlation between variations of the basic value M and the following behaviour of the DAX index, despite the originally assumed "inviolability" of M. Another approach for describing the German stock market index using a switching ARCH model can be found in [2].

3. Modified Trading Model

As mentioned above, the basic idea of the original trading model [1] consisted in the assumptions that the daily buying stimulus for the customer, the "stimulus velocity" $\mathrm{d}K(t)/\mathrm{d}t$, depends on the deviation of $W(t)$ from S, the personal expectation for $W(t)$,

$$\frac{\mathrm{d}}{\mathrm{d}t}K(t) = a \cdot [S - W(t)] \tag{1}$$

and the temporal variation of the share value $W(t)$ itself is given by $\mathrm{d}K(t)/\mathrm{d}t$ as well as the distance between $W(t)$ and basic share value M:

$$\frac{\mathrm{d}}{\mathrm{d}t}[W(t) - M] = \frac{[W(t) - M]}{M} \cdot \frac{\mathrm{d}}{\mathrm{d}t}K(t) \tag{2}$$

With these both assumptions an "ordinary" trading process can be defined.

Thirdly, the possibility of "disturbing activities" (like "greediness", "panic") was introduced in this model, assuming their proportionality to the share value acceleration:

$$\frac{\mathrm{d}^2K(t)}{\mathrm{d}t^2} = a \cdot \frac{\mathrm{d}}{\mathrm{d}t}[S - W(t)] + g \cdot \frac{\mathrm{d}^2W(t)}{\mathrm{d}t^2} \tag{3}$$

instead of equation (1). A process fulfilling the assumptions (2) and (3) is called a *general trading process*.

Here, a slight modification of the model, the replacement of the difference $W - M$ in equation (2) by its absolute value, leads to:

$$\frac{\mathrm{d}}{\mathrm{d}t}W(t) = \frac{|W(t) - M|}{M} \cdot \frac{\mathrm{d}}{\mathrm{d}t}K(t). \tag{2a}$$

Similar to the original model [1], from equations (2a) and (3) a nonlinear second-order differential equation follows for the general trading process:

$$y'' \cdot [\delta \cdot g \cdot (y - M) - M] = \delta \cdot a \cdot (y - M) \cdot y' - \frac{M}{(y - M)} \cdot y'^2 \tag{4}$$

with $y(t) = W(t)$; $y' = \mathrm{d}y/\mathrm{d}t$ and $y'' = \mathrm{d}^2y/\mathrm{d}t^2$.

The factor δ characterizes the sign of $y - M$ at the corresponding places. It is defined as

$$\delta = +1, \quad \text{for } y(t) - M > 0, \tag{5a}$$

$$\delta = -1, \quad \text{for } y(t) - M < 0. \tag{5b}$$

For $\delta = +1$, (4) turns into equation (18) of the original model in [1]. It has been shown in [1] that a given share value time sequence can be analyzed

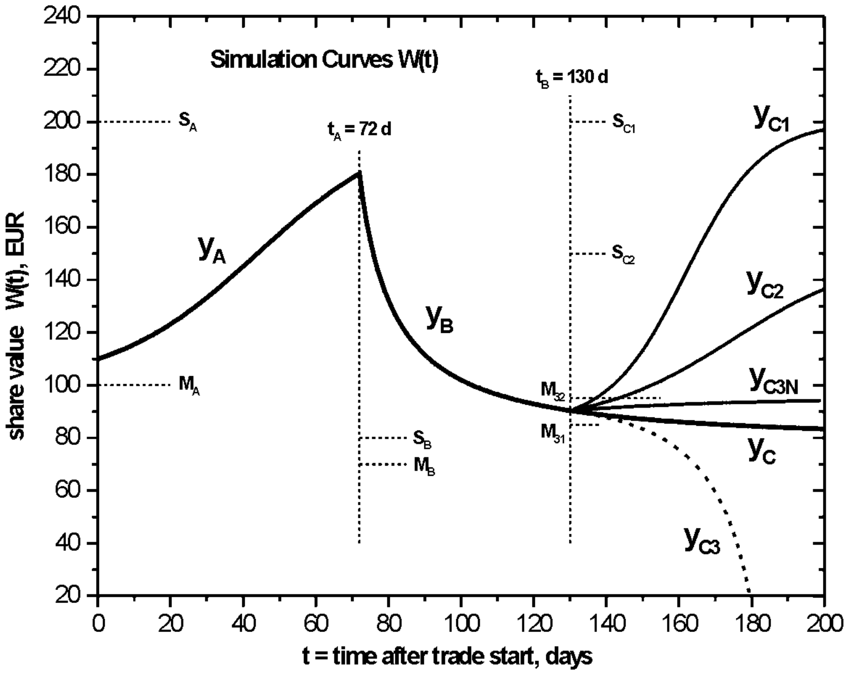

Figure 1: Simulation of an ordinary share trading process over three intervals, with different target prices S and basic values M, according to [1], here completed by the new curve y_{C3N}, due to the use of equation (4) substituting the old curve y_{C3}

as series of initial value problems (IVP), subsequently following one after the other over time, with constant parameters M, S, a, g and the share value $W(t)$ as solution of the differential equation in each problem. The case $g = 0$ stands for an ordinary trading process, even with analytical solutions of the differential equation. For a general trading process with $g > 0$, only numerical solutions are possible. More details can be found in [1]. All of them apply also here, especially to equation (4).

Figure 1 illustrates the effect of modifying the trading model by transition from equation (2) to (2a). There the same simulated share trading process is depicted, as originally presented in Figure 2 of [1]: A process consisting of three initial value problems, denoted as A, B and C with one after the other in succession. The first IVP, "A", describes a buying process, the second, "B", pure selling and "C" three different variants of buying in addition to the continuation of "B". Here, all curves have been calculated from equation (4) numerically, with one exception: curve Y_{C3} (dashed) is the repetition from Figure 2 of [1]. The parameters used for the calculation of every IVP are listed in Table 1.

period (d)	**IVP**	**M** (€)	**S** (€)	**a** (1/d)	**g**	**curve**	**remarks**
0 – 72	A	100	200	0.05	0	Y_A	
72 – 130	B	70	80	0.07	0	Y_B	
130 – 200	C	70	80	0.07	0	Y_C	1)
		85	200	0.07	0	Y_{C1}	
		85	150	0.07	0	Y_{C2}	
		95	150	0.07	0	Y_{C3}	2)
		95	150	0.07	0	Y_{C3N}	3)

Table 1: Parameters used in the simulations for Figure 1; 1) continuation of Y_B, 2) from [1]: "breakdown"!, 3) here: no "breakdown"

The interesting influence of the modified model becomes visible in the change from the original curve Y_{3C} to the new one Y_{3CN}. The tendency to a "breakdown" of the share value $W(t)$, as originally visible in Y_{3C}, now vanishes with Y_{3CN}. The occurence of such a "breakdown", originally also in cases of an ordinary trading process with $g = 0$, was caused for curve Y_{C3} in [1] by the negative sign of the starting quantity $y - M$ for IVP "C", with $Y_{3C}(130) = 90.37$ €, a value lower than $M = 95$ €. Then, according to equation (2), a negative $y - M$ requires a negative $\mathrm{d}W(t)/\mathrm{d}t$, as long as the inequality $W(t) < S$ is fulfilled. This condition leads to the "breakdown" in Y_{3C} finally. The replacement of equation (2) by (2a) has prevented such effects with Y_{C3N} in Figure 1.

All other curves in Figure 1 are identical with these from Figure 2 in [1] which demonstrates identical results from the modified and the original trading model in these cases.

4. Application to the DAX Index

Following the argumentation of the introduction, the trading model must also allow applications to the DAX index. Therefore, in this section, an attempt will be undertaken to fit a part of the well-known DAX index time sequence, with simulations basing on equation (4). For this purpose, the use of the parameters $W(t)$ or $y(t)$, S, M as dimensionless quantities and a in (1/d)-units is required,

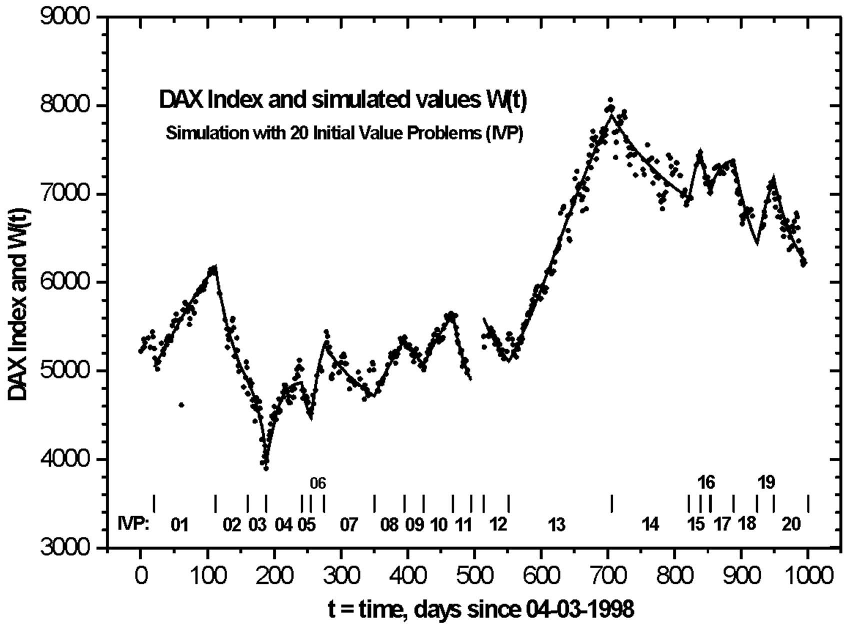

Figure 2: Fitting of the DAX Index by simulations, for the time period 04-03-1998 until 12-28-2000 with 20 Initial Value Problems (IVP); dotted line: DAX index, continuous line: simulations

according to the above given notation. But the procedure of calculations remains the same as before, also with these new interpretations.

Figure 2 presents such an attempt of fitting the DAX index with simulated values $W(t)$ for the period 1998, April, until December of 2000.

Obviously, the graph demonstrates the possibility of DAX index simulations with the trading model. For the chosen time period a fitting has been achieved with a series of 20 initial value problems, denoted by the numbers $01 - 20$ (the interruption near to $t = 500$ d is simply caused by a lack of DAX data).

In Table 2 the parameters M, S, a and g necessary for the calculation of $W(t)$ are listed for every IVP separately. The column "days" indicates the distance to the beginning, $t = 0$ denoting the date 04-03-1998, in the same manner as it is done at the abscissa in Figure 2. In each case the numbers relate to the end of an IVP equal to the start of the next problem. The first IVP 01 is presented with a start 23 days after $t = 0$.

Besides the fact of successful fitting with simulations from the trading model, the chosen example seems to allow additional and interesting conclu-

time	**days**	**IVP**	**M**	**S**	**a**	**g**	**remarks**
04-03-1998	0						
04-23-1998	23	01	4000	6700	$0,03$	0	
	112	02	4000	1700	$0,02$	0	
	160	03	**2000**	2500	$0,005$	$1,09$	1)
	188	04	**3400**	4870	$0,23$	0	2)
	241	05	3000	4000	$0,08$	0	
	255	06	3000	5600	$0,1$	0	
	275	07	3000	4000	$0,012$	0	
	350	08	3000	6000	$0,022$	0	
	395	09	3000	4000	$0,011$	0	
	424	10	3000	6000	$0,026$	0	
	468	11	3000	4000	$0,03$	0	
08-11-1999	495						
08-30-1999	514	12	3000	4500	$0,02$	0	
	552	13	**4000**	9000	$0,013$	0	3)
	706	14	4000	6000	$0,007$	0	
	822	15	4000	8000	$0,05$	0	
	839	16	4000	6300	$0,04$	0	
	854	17	4000	7400	$0,1$	0	
	889	18	4000	5000	$0,02$	0	
	924	19	4000	7900	$0,04$	0	
	949	20	4000	5500	$0,027$	0	
12-29-2000	1001						

Table 2: DAX index fitting parameters for the period 04-23-1998 till 12-29-2000; [1] $W(t)$-breakdown!, [2] $W(t)$ rises again, [3] long distant rise

sions: the obvious influence of changes in the basic DAX index M on the following behaviour of the DAX index itself. The simulated IVP "03" indicates such a behaviour. It starts with a remarkable diminished M in comparison to the former value $M_{02} = 4000$, with $M_{03} = 2000$. Furthermore, IVP "03" can be simulated only assuming additional distortions, with $g = 1.09 > 0$, what means that during the corresponding time period the trading process was not an ordinary but a general one, even leading to a final tendency of "breakdown" for $W(t)$! Actually, a real breakdown was prevented due to the start of the next IVP "04", which on its part was caused by increasing M up to the new value

3400, accompanied by higher personal expectation S and trading factor a. On the other hand, the IVP "13", connected with a step of M from $M_{12} = 3000$ to $M_{13} = 4000$, showes a long term increase of $W(t)$.

The other results look like expected: enhancement or reduction of S corresponds to increasing or diminishing of $W(t)$, respectively, and the trading factor a influences the speed of $W(t)$ changes directly.

Some additional remarks seem to be necessary to the observed changes of basic values M and their effects on the DAX index $W(t)$. Originally in the trading model for share values [1], the quantity M was introduced as a (quasi "material") parameter nearly without external influence. However, some variations of M have been observed also there in the fitting experiment. Here for the DAX index, the quantity M has another meaning than a share value; it is now a dimensionless number characterizing a certain "basic DAX index". But in the presented trading model, the application of M is the same for calculations of a share value as well as of the DAX index, generally: in both cases the parameter M represents an average over a share trading process with a lot of individual activities, in the original model for one kind of shares and here in the DAX index simulations additionally extended to several kinds of shares together.

From this interpretation the wish arises better to understand the origin for such variations of the basic value M in real fitting procedures. Here in the one example of the DAX index during the chosen time period, a diminished M leads to a "disturbed" behaviour of the DAX index $W(t)$, whereas enhancement results in a steadily increase of $W(t)$, obviously. Such an observation suggests the question whether a decrease or increase of M could correspond to an adequate stock change. It is obvious that the answer could affect prognosis investigation on future share or DAX index developments. Of course, more exact conclusions can only be drawn with a higher number of fitting experiments, for the DAX index as well as for single kinds of shares.

5. Summary

A little extension of the original trading model [1] towards the "modified trading model" has been given, which prevented the possible occurrence of "breakdowns" in an ordinary trading process. The modification consists in a simple replacement of the factor $W(t) - M$, between value velocity and stimulus velocity, by its absolute value.

The trading model allows also applications to simulation of the DAX index.

For this purpose, the calculated quantity $W(t)$ as well as its (personally) expected value S stand for an average over all trading activities, but now not only measured in € but also used as dimensionless indices. In this picture the only difference between simulations of share values and DAX indices is the concern with a single or a lot of share kinds, respectively. For the basic value M holds the same, but in the case of DAX index simulations it is an averaged value, generally.

The applicability of the trading model was proofed by a successful fitting experiment with the DAX index from the time period between April of 1998 and 2000, December. Surprisingly, this experiment showed relative great variations of M over the time. Of course, such variations must result from certain shares contributing to the DAX index, but the question arises on reasons for that and on possible correlation to the DAX indices itself. The here chosen example seems to indicate that reduction of M leads to a distortion and enhancement to a stabilization of the trading process. However, this conclusion can only be understood as a hint on the necessity of further more extended experiments.

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