MALAYSIAN STOCK CORRELATION NETWORKS: BULL MARKET AND BEAR MARKET BEHAVIORS

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Abstract: In this paper, the Malaysian stock market is analyzed using network theory on 782 stocks traded between the period of Jan 2007 to Dec 2010. The correlation networks that capture the price fluctuation pattern during the up trend, and down trend market are constructed. These networks are then characterized with clustering coefficient, maximum size of component, assortativity and unbiased local assortativity. We then compare how these network properties vary with respect to edge density. It is shown that there are more edges among the stocks in the giant component of correlation network constructed from the bear market period, and the clustering coefficient is also higher than the network constructed from bull market period. Regardless of the market behavior, the stock correlation network can be mainly classified as disassortative network with disassortative hubs. However, at lower edge densities, the correlation network can becomes assortatively mixed with disassortative hubs.

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1. Introduction

Real world physical systems such as the World Wide Web [1], social communities [2], biological systems, financial markets, transportation and traffic infrastructure are typical examples of complex systems [3]. It is well known that different types of results can be derived by analyzing these systems using networks, and one of the approaches taken is by performing structural analysis [4]. Due to the fact that the same framework can be applied to multiple types of complex systems, it has attracted attention from diverse sectors of the scientific community in the past decade [5]. Among these, is the complex system of stock market whose behavior is of great interest of many. For example, it is important to analyze the stock market for advancement of better investment planning, and this has been studied from the networks perspective [6]. Most of these networks share the common construction method which is based on the correlation coefficients of stock prices return. The minimal spanning tree method, which was first adapted by Mantegna [7] to study the clustering of stocks, has been widely used to represent the structure of stock market [8]. Later, Tumminello et al. [9] introduced a filtering procedure, that allows one to construct a filtered graph, which contains more information in their internal structure while maintaining the hierarchical organization of the minimal spanning tree.

Recently, Huang et al. [10] have adopted the threshold method to analyze the Chinese stock market, and they showed that the topological change of network that arises from different thresholds have important implication in portfolio construction and risk management. However, correlation network that constructed from different dataset tends to exhibit variability of edge density at the common threshold value. Intuitively, this factor can result in the change of network properties. Therefore, in this study, we adopt the asset graphs approach [11], by fixing the edge density when comparisons are made between different correlation network structures.

To the best of our knowledge, such network analysis has not been carried out on the Malaysian stock market. Based on this motivation, we perform the analysis on 782 stocks traded in Bursa Malaysia for the period of Jan 2007 to Dec 2010. We then extend the analysis by classifying the correlation network with unbiased local assortativity measures [12]. It is suggested by Piraveenan
et al. (2010) that this measures can be related to robustness of network against preferential attack. In correlation network, the preferential attack is analogous to bankruptcy and delisting of a listed company [10], where removal of stocks (of large degree) can result in topological change on the correlation network. Hence, it is also useful to understand the connectivity at the level of local structure of a correlation network, and classify it with local assortativity measures. Through this classification, the topological difference between between two visually indistinguishable networks can be highlighted. Thus, it is an additional good tool that should be considered in analyzing diverse types of stock correlation network.

This paper is organized as follows. In Section 2, we describe the construction of stock correlation network, and the quantities that will be used to characterize the network are defined. In Section 3, the empirical results are presented. Lastly, Section 4 summarizes the results and potential ideas for future research are given.

2. Network Construction and Topological Quantities

2.1. Network Construction

Let $C_i(t)$ be the closing price of stock $i$ at time $t$, and the time interval $\Delta t$ for each successive return be 1 day. Then, the logarithmic return of stock $i$ at time $t$ is given as [13],

$$R_i(t) = \ln C_i(t) - \ln C_i(t - \Delta t)$$  \hspace{1cm} (1)

and the correlation coefficient between stock $i$ and $j$ is

$$\rho_{ij} = \frac{\sum_{t=1}^{T}(R_i(t) - \mu_i)(R_j(t) - \mu_j)}{T\sigma_i\sigma_j}$$  \hspace{1cm} (2)

where $\mu_i$ and $\mu_j$, $\sigma_i$ and $\sigma_j$ are respectively the sample mean and standard deviation of $R_i(t)$ and $R_j(t)$ [14]. Since there is no delayed response taken on this correlation coefficient, we have $\rho_{ij} = \rho_{ji}$. Thus the correlation matrix $\rho$ with elements $\rho_{ij}$ is a symmetric matrix. If $\rho_{ij} = 1$, then the return of stock $i$ and stock $j$ are said to be completely correlated. Otherwise, if $\rho_{ij} = -1$, the return of both stocks are said to be anti-correlated. They are uncorrelated if $\rho_{ij} = 0$. In the stock correlation network, the stocks are represented by a set of vertices, and the correlation between the stocks will determine the edges. By specifying a threshold value $-1 \leq \theta \leq 1$, with $i \neq j$, if $\rho_{ij} > \theta$, an undirected
edge is connected between vertices $i$ and $j$. The edge density of the correlation network at threshold $\theta$ is computed as

$$\chi = \frac{2|E|}{|V|(|V| - 1)}$$

(3)

where $|E|$ denotes the number of undirected edges that are formed in the correlation network, and $|V| = 782$ is the number of stocks under study. We denote, $G_\chi$ to be the stock correlation network constructed with edge density $\chi$.

Note that the market behavior is determined based on the KLCI (Kuala Lumpur Composite Index), by noting the up trend behavior during the bull market, and the down trend behavior during the bear market. The correlation coefficient of stocks $i$ and $j$ during the bear market period is denoted as $\rho_{ij}^D$, and it is computed in the same way as in Eq.(2) based on the return of closing price in the period of 2008 Jan 3 to 2008 Dec 26, consisting of 241 daily returns. Similarly, the correlation coefficient during the bull market period is denoted as $\rho_{ij}^U$, and is computed from the return of stocks $i$ and $j$ between 2008 Dec 2 to 2010 Dec 3, consisting 494 data points. The correlation coefficient $\rho_{ij}^N$ incorporates both the bull and bear market behavior and is computed from the return of closing price between the period of 2007 Jan 3 to 2010 Dec 3, with the total of 969 data points. The correlation networks constructed from $\rho_{ij}^N$, $\rho_{ij}^U$ and $\rho_{ij}^D$ with edge density $\chi$ are respectively denoted as $G_\chi^N$, $G_\chi^U$ and $G_\chi^D$.

In summary, the correlation network $G_\chi^U$ ($G_\chi^D$) capture the price fluctuation correlation behavior during the bull (bear) market, whereas $G_\chi^N$ capture the price fluctuation behavior for both the bull and bear market. The data of this study can be downloaded from [23].

### 2.2. Network Topological Quantities

#### 2.2.1. Clustering Coefficient

The definition of local clustering coefficient for a vertex $v_i$ was proposed by Watts and Strogatz [15], which is

$$C_i = \frac{2.m_{v_i}}{n_{v_i}(n_{v_i} - 1)}$$

(4)

where $m_{v_i}$ denotes the number of edges connected between the neighbors of vertex $v_i$, and $n_{v_i}$ denotes the number of neighbors for vertex $v_i$. Note that if $n_{v_i} \leq 1$, $C_i$ is considered to be 0 [5]. The clustering coefficient for correlation
network $G_\chi$ is given as

$$C(G_\chi) = \frac{1}{|V|} \sum_{i=1}^{|V|} C_i$$

(5)

where $|V|$ is the number of connected vertices in $G_\chi$. If a stock $v_i$ has high local clustering coefficient, then this means that the neighbors of stock $v_i$ tend to correlate with each other. Likewise, if the local clustering coefficient of a stock is 0, this means that it is only correlated with only one stock. Such stocks have very little influence to the overall network structure $G_\chi$ in terms of price fluctuation correlation.

2.2.2. Connected Components

Two vertices $u, v \in G_\chi$ are said to be connected if there is a path between $u$ to $v$ in $G_\chi$ [16]. Otherwise, $u$, and $v$ are disconnected. We follow Huang et al. (2009) to define the size of the connected component $|CO|$ as the number of connected vertices in the component, and the maximum size of connected component in the correlation network $G_\chi$ is denoted as $|CO_{max}|$. In stock correlation network, stocks from the same component structure can influence each other directly or indirectly through price fluctuation [10].

2.2.3. Assortativity and Unbiased Local Assortativity

The assortativity mixing was originally introduced by Newman (2002) to characterize the correlation between the degree of vertices in real world networks. Let $q_k$ be the remaining degree distribution, where the remaining degree is denoted as the number of remaining edges of a vertex that one encounters when traversing a randomly chosen edge. Then, $e_{j,k}$ be the joint probability distribution of the remaining degrees of two vertices that attached at the opposite end of a randomly chosen edge. The assortativity of a network is then given as [17]

$$r = \frac{1}{\sigma_q^2} \left( \sum_{jk} jk(e_{j,k} - q_j q_k) \right)$$

(6)

where $\sigma_q$ denotes the standard deviation of the remaining degree distribution $q_k$. The sums $\sum_j j q_j$ or $\sum_k k q_k$ understood to be the mean $\mu_q$ of the remaining degree distribution. The assortativity of correlation network $G_\chi$ can then be
expressed as
\[
r(G_\chi) = \frac{1}{\sigma_q^2} \left[ \sum_{jk} jk(e_{j,k}) - \mu_q^2 \right]
\] (7)

Network that is assortatively mixed \( r(G_\chi) > 0 \), implies that vertices with similar degree tend to connect with each other. If the network is disassortatively mixed \( r(G_\chi) < 0 \), then vertices with dissimilar degree tend to connect with each other in the network. There is no assortative mixing in the correlation network if \( r(G_\chi) = 0 \).

The unbiased local assortativity was proposed by Piraveenan et al. (2010) [12] [18] [19]. In the stock correlation network of \( G_\chi \), the unbiased local assortativity of a connected vertex \( v_i \) is defined as
\[
q_i(G_\chi) = \frac{j(j+1)(\overline{k} - \mu_q)}{2M\sigma_q^2}
\] (8)

where \( j \) is the remaining degree of vertex \( v_i \), \( \overline{k} \) is the average remaining degree of vertex \( v_i \)'s neighbors, \( M \) is the number of undirected edges in the correlation network, \( \mu_q \) and \( \sigma_q \) are respectively the mean and standard deviation of the remaining degree distribution \( q_k \) [12] [18]. The unbiased local assortativity sum to the global assortativity
\[
r(G_\chi) = \sum_{i=1}^{N_c} q_i(G_\chi)
\] (9)

computed using eq.(7) and \( N_c \) is the total number of connected vertices in \( G_\chi \). The sign of local assortativity \( q_i(G_\chi) \) for a vertex \( v_i \) is determined by the term \((\overline{k} - \mu_q)\), where if neighbors of vertex \( v_i \) have higher average remaining degree than the global average remaining degree, then vertex \( v_i \) is assortative, meaning that the neighbors of vertex \( v_i \) have comparatively larger degree compared to other vertices in the network. If the global average remaining degree is higher, then \( v_i \) is disassortative. From here onwards, we will neglect the “unbiased” term when defining local assortativity measure.

3. Empirical Findings and Results

3.1. Overview on the Empirical Data

Figure (1a) shows the density plots for correlation coefficients \( \rho_{ij}^D \), \( \rho_{ij}^U \) and \( \rho_{ij}^N \) that are computed between every possible pairs of considered stocks. Our first
finding is that the distribution of correlation coefficients \( \rho_{ij}^U \), \( \rho_{ij}^D \) and \( \rho_{ij}^N \) are respectively distributed with means 0.0685, 0.046 and 0.069. Meanwhile, the median for \( \rho_{ij}^D \), \( \rho_{ij}^U \) and \( \rho_{ij}^N \) are respectively 0.0685, 0.0388 and 0.0561. Figure (1b) shows the variation of edge density of constructed correlation network with respect to threshold \( \theta \). From figure (1b), it can be noted that the edge density of correlation network constructed from \( \rho_{ij}^D \) are relatively higher than \( \rho_{ij}^U \) when \( \theta > 0.02 \), and are higher than both \( \rho_{ij}^U \) and \( \rho_{ij}^N \) when \( \theta > 0.06 \). This indicates that there are larger number pairs of stocks tend to have higher correlation (above said threshold) during the bear market than the other trend. The stock prices are more correlated because they tend to fall with a collective behavior that is triggered by market instability [20] [14].

### 3.2. Network Analysis

In this section, the network properties mentioned in Section 2 for different correlation networks \( G_X^U \), \( G_X^D \), and \( G_X^N \) are compared. As shown in figure (1b) that edge densities for each correlation coefficients \( \rho_{ij}^N \), \( \rho_{ij}^U \) and \( \rho_{ij}^D \) tend to be different even at the same threshold value, thus it is understood that different correlation network with the same edge density are generated from different threshold values.
3.2.1. Clustering Coefficient

Figure (2a) shows the variation of clustering coefficient at each market behavior. When the network of distinct market behaviors are considered separately, $G_D^\chi$ tends to have larger clustering coefficient in comparison to $G_U^\chi$ at $0.00327 \leq \chi \leq 0.03275$, showing higher clustering strength during the bear market than the bull market. It is also worthwhile to remark that the clustering property exhibited by these networks are large even though the edge density of the network is relatively low. Such behavior is in contrast with the Chinese stock market network studied by Huang et al. (2009). For reference, the network properties for $G_N^\chi$ are summarized in Table 1. The scatter plot of local clustering coefficient for $G_N^\chi$ at $\chi = 0.00082$ is illustrated in figure (2b). It can be seen that the clustering coefficient of larger degree vertices appear to be bigger than 0.25, indicating the existence of correlation between the neighbors of large degree stocks. We remark that this behavior is also exhibited in $G_U^\chi$ and $G_D^\chi$. Therefore, it appears that there is always a group of highly correlated stocks that clustered together in the stock correlation network.

3.2.2. Component and Edge Density

Figure (3) shows the variation on the maximum size of the components between $G_N^\chi$, $G_U^\chi$ and $G_D^\chi$ at a fixed edge density. From the figure, it can be observed that network of $G_U^\chi$ is fully connected at a lower edge density ($\chi = 0.111$) when compared to $G_D^\chi$ (fully connected at edge density 0.178). Meanwhile, $G_N^\chi$ is
fully connected at \( \chi = 0.249 \). Alternatively, the critical thresholds of network structures \( G^D_\chi \), \( G^U_\chi \) and \( G^N_\chi \) to be fully connected are respectively at 0.133, 0.156 and 0.105. To measure the number of edges in the giant component, we let \( \chi_g(G_\chi) \) be the edge density of giant component in \( G_\chi \) which can be computed through

\[
\chi_g(G_\chi) = \frac{2|E_g|}{|V_g|(|V_g| - 1)}
\]

where \( |E_g| \) and \( |V_g| \) respectively denote the number of undirected edges and vertices in the giant component of correlation network \( G_\chi \). Figure (4) illustrates the edge density of the giant component in \( G^U_\chi \) and \( G^D_\chi \) under different \( \chi \). Comparing figures (3) and (4), we can observe that before saturation, the network of \( G^D_\chi \), with relatively smaller giant component than \( G^U_\chi \), tends to have higher edge density. This suggests that in comparison to the bull market period, the collective behavior of stocks in the giant component during the bear market period can result in more correlations with greater correlation strength among each other.

### 3.2.3. Assortativity and Local Assortativity Analysis

In this section, we will classify the stock correlation network with local assortativity profiles. We begin the classification by first analyzing the assortativity at the global network level. Figures (5) and (6) illustrate the global assortativity of network \( G^U_\chi \), \( G^D_\chi \) and \( G^N_\chi \) with respect to \( \chi \). It can be seen that for \( 0.00164 \leq \chi \leq 0.03275 \), all the correlation network are disassortatively mixed. In particular, \( G^D_\chi \) appeared to be more disassortative in comparison to \( G^U_\chi \), showing that the tendency of vertices connecting to vertices with dissimilar degree is higher in the correlation network of \( G^D_\chi \) than \( G^U_\chi \). However, when \( \chi \leq 0.00164 \), both correlation network structures are assortatively mixed at certain \( \chi \).

The local assortative profiles for \( G^D_\chi \), \( G^U_\chi \) and \( G^N_\chi \) are respectively illustrated in figures (7), (8) and (9). It is shown from these figures, that the correlation networks \( G^N_\chi \), \( G^U_\chi \) and \( G^D_\chi \) share the same type of local assortative profiles, where the curves start to decrease in the middle region of \( k \approx 60 \) for \( \chi = 0.00819 \) and \( k \approx 120 \) for \( \chi = 0.0246 \), leading to larger degree vertices that are locally disassortative. Such a decreasing behavior appears to be unchanged under the change of edge density in the correlation network. Thus, it can be inferred that it is the contribution of local disassortativeness of the larger degree hubs results in the overall network structure to become disassortatively mixed. However, at lower edge densities, when the correlation network is assortatively mixed, for
Figure 3: The variation of maximum size of component with respect to $\chi$ for $G^N_\chi$, $G^U_\chi$, and $G^D_\chi$. The last point in each plot denotes the network structure is fully connected.

Figure 4: The edge density in the giant component of $G^U_\chi$ and $G^D_\chi$ with respect to $\chi$.

Figure 5: The variation of assortativity for correlation network $G^N_\chi$, $G^U_\chi$ and $G^D_\chi$ with respect to $\chi$ at interval $0.00164 \leq \chi \leq 0.03275$. 
instance when $\chi = 0.00049$ with the local assortativity profiles shown in figure (10a). One can observe that the larger degree hub is still locally disassortative, and it can be inferred that it is the contribution of local assortativeness that exhibited by the smaller degree hubs ($k = 6$ to $k = 13$) makes the overall network structure becomes assortatively mixed.

The disassortativeness of the largest degree hub in $G^D_\chi$, $G^U_\chi$ and $G^N_\chi$ with respect to edge density are illustrated in figure (10b) where the largest degree hub remained disassortative through out the variation of edge density ($\chi \leq 0.03275$). Piraveenan et al. [12] proposed that the complex network can be categorized into four classes, namely (i) assortative networks with assortative hubs, (ii) assortative networks with disassortative hubs, (iii) disassortative networks with disassortative hubs and (iv) disassortative networks with assortative hubs. We therefore can classify the correlation network as following. For $0.00164 \leq \chi \leq 0.0327$, the stock correlation network can be classified into class (iii) networks. This indicates that the correlation network share the same type of local assortative profiles with some of the real world networks such as the food webs, internet and collaboration networks [12], which are also of class (iii) networks. At lower edge density ($\chi < 0.00164$), when the correlation networks are assortatively mixed, they can be classified into class (ii) networks.

### 4. Conclusion

In this paper, the empirical study of the Malaysian stock correlation network constructed from three different trading periods are presented via asset graph approach. We found that when $0.00327 \leq \chi \leq 0.03275$, the clustering coefficient exhibited by the correlation network constructed from the bear market...
Figure 7: Scatter plot of local assortativity $\rho_i(G^N\chi)$ with respect to degree $k$ of vertex $v_i$ at a) $\chi = 0.00819$ and b) $\chi = 0.0246$.

Figure 8: Scatter plot of local assortativity $\rho_i(G^D\chi)$ with respect to degree $k$ of vertex $v_i$ at a) $\chi = 0.00819$ and b) $\chi = 0.0246$.

Figure 9: Scatter plot of local assortativity $\rho_i(G^U\chi)$ with respect to degree $k$ of vertex $v_i$ at a) $\chi = 0.00819$ and b) $\chi = 0.0246$. 
period return is larger than the bull market period. Through scatter plot of local clustering coefficient, we observe that there is always a group of highly correlated stocks that cluster together in the correlation network. Component structure analysis shows that the collective behavior during the bear market period can result in more correlation relationship with higher correlation strength among the stocks in the giant component. As estimation of correlation between the price changing of different stocks in portfolios is an essential topic in risk management [21], thus understanding the relationship of the correlation pattern among the highly correlated stocks that arises from different $\chi$ at different market behavior will also be useful for portfolio construction and risk management [22].

We have also classified the stock correlation network with unbiased local assortativity profiles, and identified that within the edge density interval of $0.00164 \leq \chi \leq 0.03275$, the constructed stock correlation networks belong to the class of disassortative network with disassortative hubs. However, at lower edge density ($\chi \leq 0.00164$), when the networks are assortatively mixed, they can be categorized as assortative network with disassortative hubs. These classification is important as it highlights the difference of topological properties among diverse of complex networks. Thus, it is a good tool that should be considered in analyzing the market structure.

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| $\chi$ | $r(G_N^\chi)$ | $|CO_{max}|$ | $C(G_N^\chi)$ |
|-------|---------------|-------------|---------------|
| 0.000164 | -0.1358 | 24 | 0.3487 |
| 0.000328 | -0.0561 | 36 | 0.3844 |
| 0.000491 | -0.1584 | 47 | 0.3556 |
| 0.000655 | -0.1998 | 59 | 0.4697 |
| 0.000819 | -0.1750 | 65 | 0.4803 |
| 0.000982 | -0.2023 | 69 | 0.5448 |
| 0.001115 | -0.2303 | 75 | 0.5956 |
| 0.001310 | -0.2501 | 79 | 0.5988 |
| 0.001470 | -0.2404 | 83 | 0.5828 |

Table 1: Properties of network of $G_N^\chi$ with respect to edge density $\chi$, $r(G_N^\chi)$ denotes the global assortativity, $|CO_{max}|$ denotes the maximum size of the connected components and $C(G_N^\chi)$ denotes the clustering coefficient of the correlation network.

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[23] *www.klsecod.com*, Data accessed on 2010 Sept 20. For this set of data, stock splits had been manually adjusted.