

ON STABILITY OF SWITCHED LINEAR SYSTEMS

Manlika Ratchagit

Department of Mathematics and Statistics

Maejo University

Chiangmai, 50290, THAILAND

Abstract: This paper is concerned with stability of a class of switched discrete time-delay systems. The system to be considered is subject to interval time-varying delays, which allows the delay to be a fast time-varying function and the lower bound is not restricted to zero. Based on the discrete Lyapunov functional, a switching rule for the asymptotic stability for the system is designed via linear matrix inequalities.

AMS Subject Classification: 93C30, 93D20, 37C75

Key Words: switching design, discrete system, asymptotic stability, Lyapunov function, linear matrix inequality

1. Introduction

As an important class of hybrid systems, switched systems arise in many practical processes that cannot be described by exclusively continuous or exclusively discrete models, such as manufacturing, communication networks, automotive engineering control and chemical processes (see, e.g., [1, 2, 3] and the references therein). On the other hand, time-delay phenomena are very common in practical systems. A switched system with time-delay individual subsystems is called a switched time-delay system; in particular, when the subsystems are linear, it is then called a switched time-delay linear system. During the last decades, the stability analysis of switched linear continuous/discrete time-delay systems has attracted a lot of attention [4, 5, 6]. The main approach for stability analysis relies on the use of Lyapunov-Krasovskii functionals and linear matrix inequality (LMI) approach for constructing a common Lyapunov func-

tion [7-13]. Although many important results have been obtained for switched linear continuous-time systems, there are few results concerning the stability of switched linear discrete systems with time-varying delays. It was shown in [5, 6, 14] that when all subsystems are asymptotically stable, the switching system is asymptotically stable under an arbitrary switching rule. The asymptotic stability for switching linear discrete time-delay systems has been studied in [15], but the result was limited to constant delays. In [16], a class of switching signals has been identified for the considered switched discrete-time delay systems to be stable under the average dwell time scheme.

This paper studies asymptotic stability problem for switched linear discrete systems with interval time-varying delays. Specifically, our goal is to develop a constructive way to design switching rule to asymptotically stabilize the system. By using improved Lyapunov-Krasovskii functionals combined with LMIs technique, we propose new criteria for the asymptotic stability of the system. Compared to the existing results, our result has its own advantages. First, the time delay is assumed to be a time-varying function belonging to a given interval, which means that the lower and upper bounds for the time-varying delay are available, the delay function is bounded but not restricted to zero. Second, the approach allows us to design the switching rule for stability in terms of LMIs.

The paper is organized as follows: Section 2 presents definitions and some well-known technical propositions needed for the proof of the main results. Switching rule for the asymptotic stability is presented in Section 3.

2. Preliminaries

The following notations will be used throughout this paper. R^+ denotes the set of all real non-negative numbers; R^n denotes the n -dimensional space with the scalar product of two vectors $\langle x, y \rangle$ or $x^T y$; $R^{n \times r}$ denotes the space of all matrices of $(n \times r)$ -dimension. A^T denotes the transpose of A ; a matrix A is symmetric if $A = A^T$.

Matrix A is semi-positive definite ($A \geq 0$) if $\langle Ax, x \rangle \geq 0$, for all $x \in R^n$; A is positive definite ($A > 0$) if $\langle Ax, x \rangle > 0$ for all $x \neq 0$; $A \geq B$ means $A - B \geq 0$. $\lambda(A)$ denotes the set of all eigenvalues of A ; $\lambda_{\min}(A) = \min\{Re\lambda : \lambda \in \lambda(A)\}$.

Consider a discrete systems with interval time-varying delay of the form

$$\begin{aligned} x(k+1) &= A_\sigma x(k) + D_\sigma x(k-h(k)), \quad k = 0, 1, 2, \dots \\ x(k) &= v_k, \quad k = -h_2, -h_2 + 1, \dots, 0, \end{aligned} \tag{1}$$

where $x(k) \in R^n$ is the state, $\text{sig}(\cdot) : R^n \rightarrow \mathcal{N} := \{1, 2, \dots, N\}$ is the switching rule, which is a function depending on the state at each time and will be designed. A switching function is a rule which determines a switching sequence for a given switching system. Moreover, $\sigma(x) = i$ implies that the system realization is chosen as the i -th system, $i = 1, 2, \dots, N$. It is seen that the system (1) can be viewed as an autonomous switched system in which the effective subsystem changes when the state $x(k)$ hits predefined boundaries. $A_i, D_i, i = 1, 2, \dots, N$ are given constant matrices. The time-varying function $h(k)$ satisfies the following condition:

$$0 < h_1 \leq h(k) \leq h_2, \quad \forall k = 0, 1, 2, \dots$$

Remark 1. It is worth noting that the time delay is a time-varying function belonging to a given interval, in which the lower bound of delay is not restricted to zero.

Definition 1. The switched system (1) is asymptotically stable if there exists a switching function $\sigma(\cdot)$ such that the zero solution of the system is asymptotically stable.

3. Main Results

Theorem 1. *The switched system (1) is asymptotically stable if there exist symmetric positive definite matrices $P > 0, Q > 0$ and a matrix R satisfying the following condition*

$$(i) \quad \mathcal{M}_i(P, Q, R) = \begin{pmatrix} (h_2 - h_1 + 1)Q - P & -A_i^T R & 0 \\ -R^T A_i & P + R + R^T & -R^T D_i \\ 0 & -D_i^T R & -Q \end{pmatrix} < 0,$$

$i = 1, 2, \dots, N$.

The switching rule is chosen as $\sigma(x(k)) = i$.

Proof. Consider the following Lyapunov-Krasovskii functional for any i th system (1)

$$V(k) = V_1(k) + V_2(k) + V_3(k),$$

where

$$V_1(k) = x^T(k)Px(k), \quad V_2(k) = \sum_{i=k-h(k)}^{k-1} x^T(i)Qx(i),$$

$$V_3(k) = \sum_{j=-h_2+2}^{-h_1+1} \sum_{l=k+j+1}^{k-1} x^T(l)Qx(l),$$

We can verify that

$$\lambda_1 \|x(k)\|^2 \leq V(k). \tag{2}$$

Let us set $z(k) = [x(k) \ x(k + 1) \ x(k - h(k))]^T$, and

$$E = \begin{pmatrix} 0 & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad F = \begin{pmatrix} P & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Then, the difference of $V_1(k)$ along the solution of the system is given by

$$\begin{aligned} \Delta V_1(k) &= x^T(k + 1)Px(k + 1) - x^T(k)Px(k) \\ &= z^T(k)Ez(k) - 2z^T(k)F^T \begin{pmatrix} 0.5x(k) \\ 0 \\ 0 \end{pmatrix}. \end{aligned} \tag{3}$$

Using the expression of system (1)

$$0 = -x(k + 1) + A_i x(k) + D_i x(k - h(k)),$$

we have

$$\begin{aligned} &-2z^T(k)F^T \begin{pmatrix} 0.5x(k) \\ -x(k + 1) + A_i x(k) + D_i x(k - h(k)) \\ 0 \end{pmatrix} z(k) \\ &= -z^T(k)F^T \begin{pmatrix} 0.5I & 0 & 0 \\ A_i & -I & D_i \\ 0 & 0 & 0 \end{pmatrix} z(k) - z^T(k) \begin{pmatrix} 0.5I & A_i^T & 0 \\ 0 & -I & 0 \\ 0 & D_i^T & 0 \end{pmatrix} Fz(k). \end{aligned}$$

Therefore, from (3) it follows that

$$\Delta V_1(k) = z^T(k)M_i z(k), \tag{4}$$

where

$$M_i = \begin{pmatrix} 0 & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & 0 \end{pmatrix} - F^T \begin{pmatrix} 0.5I & 0 & 0 \\ A_i & -I & D_i \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0.5I & A_i^T & 0 \\ 0 & -I & 0 \\ 0 & D_i^T & 0 \end{pmatrix} F.$$

The difference of $V_2(k)$ is given by

$$\begin{aligned} \Delta V_2(k) &= \sum_{i=k+1-h(k+1)}^k x^T(i)Qx(i) - \sum_{i=k-h(k)}^{k-1} x^T(i)Qx(i) \\ &= \sum_{i=k+1-h(k+1)}^{k-h_1} x^T(i)Qx(i) + x^T(k)Qx(k) \\ &\quad - x^T(k-h(k))Qx(k-h(k)) \\ &\quad + \sum_{i=k+1-h_1}^{k-1} x^T(i)Qx(i) - \sum_{i=k+1-h(k)}^{k-1} x^T(i)Qx(i). \end{aligned} \tag{5}$$

Since $h(k) \geq h_1$ we have

$$\sum_{i=k+1-h_1}^{k-1} x^T(i)Qx(i) - \sum_{i=k+1-h(k)}^{k-1} x^T(i)Qx(i) \leq 0,$$

and hence from (5) we have

$$\begin{aligned} \Delta V_2(k) \leq \sum_{i=k+1-h(k+1)}^{k-h_1} x^T(i)Qx(i) + x^T(k)Qx(k) \\ - x^T(k-h(k))Qx(k-h(k)). \end{aligned} \tag{6}$$

The difference of $V_3(k)$ is given by

$$\begin{aligned} \Delta V_3(k) &= \sum_{j=-h_2+2}^{-h_1+1} \sum_{l=k+j}^k x^T(l)Qx(l) - \sum_{j=-h_2+2}^{-h_1+1} \sum_{l=k+j+1}^{k-1} x^T(l)Qx(l) \\ &= \sum_{j=-h_2+2}^{-h_1+1} \left[\sum_{l=k+j}^{k-1} x^T(l)Qx(l) + x^T(k)Qx(k) \right. \\ &\quad \left. - \sum_{l=k+j}^{k-1} x^T(l)Qx(l) - x^T(k+j-1)Qx(k+j-1) \right] \\ &= \sum_{j=-h_2+2}^{-h_1+1} [x^T(k)Qx(k) - x^T(k+j-1)Qx(k+j-1)] \\ &= (h_2 - h_1)x^T(k)Qx(k) - \sum_{j=k+1-h_2}^{k-h_1} x^T(j)Qx(j). \end{aligned} \tag{7}$$

Since $h(k) \leq h_2$, and

$$\sum_{i=k=1-h(k+1)}^{k-h_1} x^T(i)Qx(i) - \sum_{i=k+1-h_2}^{k-h_1} x^T(i)Qx(i) \leq 0,$$

we obtain from (6) and (7) that

$$\begin{aligned} \Delta V_2(k) + \Delta V_3(k) &\leq (h_2 - h_1 + 1)x^T(k)Qx(k) - x^T(k - h(k))Qx(k - h(k)). \end{aligned} \tag{8}$$

Therefore, combining the inequalities (4), (8) gives

$$\Delta V(k) \leq z^T(k)M_i z(k), \tag{9}$$

where

$$M_i = \begin{pmatrix} (h_2 - h_1 + 1)Q - P & -A_i^T R & 0 \\ -R^T A_i & P + R + R^T & -R^T D_i \\ 0 & -D_i^T R & -Q \end{pmatrix}.$$

Therefore, we finally obtain from (9) and the condition (i) that

$$\Delta V(k) \leq z^T(k)M_i z(k) < 0, \quad \forall i = 1, 2, \dots, N, k = 0, 1, 2, \dots,$$

which, combining the condition (2) and the Lyapunov stability theorem (see [17]), concludes the proof of the theorem.

4. Conclusion

This paper has proposed a switching design for the asymptotic stability of switched linear discrete-time systems with interval time-varying delays. Based on the discrete Lyapunov functional, a switching rule for the asymptotic stability for the system is designed via linear matrix inequalities.

Acknowledgments

This work was supported by Faculty of Science, Maejo University, Thailand.

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