PAIRWISE SEMI STAR GENERALIZED
\(\omega\)-CONTINUOUS FUNCTIONS

D. Narasimhan\(^1\)\(^\S\), K. Kannan\(^2\)
Department of Mathematics
Srinivasa Ramanujan Centre
SASTRA University
Kumbakonam, 612001, INDIA

Abstract: The aim of this short communication is to study some basic properties of pairwise \(s^*gw\)-continuous and pairwise \(s^*gw\)-irresolute mappings in bitopological spaces, that of introduced in topological spaces by K. Chandrasekhara Rao and D. Narasimhan [7] in unital topological spaces.

AMS Subject Classification: 54E55
Key Words: pairwise \(s^*gw\)-continuous, pairwise \(s^*gw\)-irresolute, pairwise \(gw\)-continuous, pairwise \(gw\)-irresolute

1. Introduction

Let \((X, \tau_1, \tau_2)\) or simply \(X\) denote a bitopological space. For any subset \(A \subseteq X\), \(\tau_i\)-\(int(A)\) and \(\tau_i\)-\(cl(A)\) denote the interior and closure of a set \(A\) with respect to the topology \(\tau_i\). The closure and interior with respect to the topology \(\tau_i\) of \(B\) relative to \(A\) is written as \(\tau_i\)-\(cl_B(A)\) and \(\tau_i\)-\(int_B(A)\) respectively. A point \(x \in X\) is called a condensation point of \(A\) if for each \(U \in \tau\) with \(x \in U\), the set \(U \cap A\) is uncountable. \(A\) is called \(w\)-closed if it contains all its condensation points. The complement of an \(w\)-closed set is called \(w\)-open. It is well known that a subset \(A\) of a space \((X, \tau)\) is \(w\)-open if and only if for each \(x \in A\), there exists \(U \in \tau\) such that \(x \in U\) and \(U \cap W\) is countable. The family of all \(w\)-open subsets of a space \((X, \tau)\), by \(\tau_w\) or \(wO(X)\), forms a topology on \(X\) finer than \(\tau\).

Received: April 17, 2012

\(^\S\)Correspondence author

© 2012 Academic Publications, Ltd.
url: www.acadpubl.eu
$w$-closure and $w$-interior with respect to the topology $\tau_i$, that can be defined in a manner similar to $\tau_i$-$cl(A)$ and $\tau_i$-$int(A)$, respectively, will be denoted by $\tau_i$-$cl_w(A)$ and $\tau_i$-$int_w(A)$, respectively. $A^C$ denotes the complement of $A$ in $X$ unless explicitly stated.

We shall require the following known definitions.

**Definition 1.1.** A set $A$ of a bitopological space $(X, \tau_1, \tau_2)$ is called

(a) $\tau_1\tau_2$-semi open if there exists an $\tau_1$-open set $U$ such that $U \subseteq A \subseteq \tau_2$-$cl(U)$,

(b) $\tau_1\tau_2$-semi closed if $X - A$ is $\tau_1\tau_2$-semi open.

equivalently, a set $A$ of a bitopological space $(X, \tau_1, \tau_2)$ is called $\tau_1\tau_2$-semi closed if there exists a $\tau_1$-closed set $F$ such that $\tau_2$-$int(F) \subseteq A \subseteq F$,

(c) $\tau_1\tau_2$-generalized closed ($\tau_1\tau_2$-g closed) if $\tau_2$-$cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\tau_1$-open in $X$,

(d) $\tau_1\tau_2$-generalized open ($\tau_1\tau_2$-g open) if $X - A$ is $\tau_1\tau_2$-g closed,

(e) $\tau_1\tau_2$-semi star generalized closed ($\tau_1\tau_2$-$s^g$ closed) if $\tau_2$-$cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\tau_1$-semi open in $X$,

(f) $\tau_1\tau_2$-semi star generalized open ($\tau_1\tau_2$-$s^g$ open) if $X - A$ is $\tau_1\tau_2$-$s^g$ closed in $X$,

(g) $\tau_1\tau_2$-generalized $w$-closed ($\tau_1\tau_2$-gw closed) if $\tau_2$-$cl_w(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\tau_1$-open in $X$,

(h) $\tau_1\tau_2$-generalized $w$-open ($\tau_1\tau_2$-gw open) if $X - A$ is $\tau_1\tau_2$-gw closed.

(i) $\tau_1\tau_2$-regular generalized $w$-closed ($\tau_1\tau_2$-rgw closed) if $\tau_2$-$cl_w(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\tau_1\tau_2$-regular open in $X$,

(j) $\tau_1\tau_2$-regular generalized $w$-open ($\tau_1\tau_2$-rgw open) if $X - A$ is $\tau_1\tau_2$-rgw closed.

2. **Pairwise Semi Star Generalized $w$-Continuity**

For further study we shall go through the following.

**Definition 2.1.** A map $f : X \to Y$ is called
(a) pairwise $gw$-closed if image of a $\tau_j$-$w$ closed set in $X$ is $\sigma_i\sigma_j$-$gw$ closed in $Y$,

(b) pairwise $rgw$-closed if image of a $\tau_j$-$w$ closed set in $X$ is $\sigma_i\sigma_j$-$rgw$ closed in $Y$,

(c) pairwise pre $w$-closed if image of a $\tau_i$-$w$ closed set in $X$ is $\sigma_i$-$w$ closed in $Y$, $i = 1,2$,

(d) pairwise $rgw$-continuous if inverse image of a $\sigma_j$-$w$ closed in $Y$ is $\tau_i\tau_j$-$rgw$ closed in $X$, $i, j = 1,2$ and $i \neq j$,

(e) pairwise $gw$-continuous if inverse image of a $\sigma_j$-$w$ closed in $Y$ is $\tau_i\tau_j$-$gw$ closed in $X$, $i, j = 1,2$ and $i \neq j$,

(f) pairwise $rgw$-irresolute if the inverse image of $\sigma_i\sigma_j$-$rgw$ closed set $Y$ is $\tau_i\tau_j$-$rgw$ closed in $X$, $i, j = 1,2$ and $i \neq j$.

**Definition 2.2.** A map $f : X \rightarrow Y$ is called

(a) pairwise $s^*gw$-continuous if the inverse image of $\sigma_j$-$w$ closed set in $Y$ is $\tau_i\tau_j$-$s^*gw$ closed in $X$, $i, j = 1,2$, $i \neq j$.

(b) pairwise $s^*gw$-irresolute if the inverse image of $\sigma_i\sigma_j$-$s^*gw$ closed set in $Y$ is $\tau_i\tau_j$-$s^*gw$ closed in $X$, $i, j = 1,2$, $i \neq j$.

Concerning composition of functions, we observe the following results.

**Theorem 2.3.**  
(a) The composition of two pairwise $s^*gw$-irresolute functions is pairwise $s^*gw$-irresolute.

Equivalently, If $f, g$ are pairwise $s^*gw$-irresolute, then $gof$ is also pairwise $s^*gw$-irresolute.

(b) If $f$ is pairwise $s^*gw$-irresolute and $g$ is pairwise $s^*gw$-continuous, then $gof$ is also pairwise $s^*gw$-continuous.

**Proof.** (a) Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ be two pairwise $s^*gw$-irresolute functions. Let $V$ be a $\mu_i\mu_j$-$s^*gw$ closed in $Z$, $i, j = 1,2$, $i \neq j$. Since $g$ is pairwise $s^*gw$-irresolute, we have $g^{-1}(V)$ is $\sigma_i\sigma_j$-$s^*gw$ closed in $Y$. Since $f$ is pairwise $s^*gw$-irresolute, we have $f^{-1}[g^{-1}(V)] = (gof)^{-1}$ is $\tau_i\tau_j$-$s^*gw$ closed in $X$. Therefore, $gof$ is pairwise $s^*gw$-irresolute.

The proof of (b) is similar. \qed
The composition of two pairwise $s^*gw$-continuous functions is not pairwise $s^*gw$-continuous.

**Theorem 2.4.** a) Every pairwise $s^*gw$-continuous function is pairwise $gw$-continuous,

b) Every pairwise $s^*gw$-continuous function is pairwise $rgw$-continuous.

**Proof.** (a) Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a pairwise $s^*gw$-continuous function. Let $V$ be a $\sigma_j$-$w$ closed set in $Y$. Since $f$ is a pairwise $s^*gw$-continuous function, we have $f^{-1}(V)$ is $\tau_i\tau_j$-$s^*gw$ closed in $X$, $i, j = 1, 2, i \neq j$. Since every $\tau_i\tau_j$-$s^*gw$ closed set is $\tau_i\tau_j$-$gw$ closed, we have $f^{-1}(V)$ is $\tau_i\tau_j$-$gw$ closed in $X$. Therefore, $f$ is pairwise $gw$-continuous.

The proof of (b) is similar. \hfill \square

**Definition 2.5.** A space $(X, \tau_1, \tau_2)$ is a pairwise semi star generalized $w$-$T_{1/2}$ [6] (simply, pairwise $s^*gw$-$T_{1/2}$) if every $\tau_i\tau_j$-$s^*gw$ closed set in $(X, \tau_1, \tau_2)$ is $\tau_2$-$w$ closed and $\tau_2\tau_1$-$s^*gw$ closed set in $(X, \tau_1, \tau_2)$ is $\tau_1$-$w$ closed.

The next theorem shows that pairwise $s^*gw$-$T_{1/2}$ spaces are preserved under pairwise $s^*gw$-irresolute map if it is also a pairwise pre $w$-closed map.

**Theorem 2.6.** Let $f : X \to Y$ be onto pairwise $s^*gw$-irresolute and pairwise pre $w$-closed map. If $X$ is pairwise $s^*gw$-$T_{1/2}$ then $Y$ is also pairwise $s^*gw$-$T_{1/2}$.

**Proof.** Let $A$ be $\sigma_i\sigma_j$-$s^*gw$ closed subset of $Y$, $i, j = 1, 2, i \neq j$. Since $f$ is pairwise $s^*gw$-irresolute map, $f^{-1}(A)$ is $\tau_i\tau_j$-$s^*gw$ closed subset of $X$. Since $X$ is a pairwise $s^*gw$-$T_{1/2}$ space, $f^{-1}(A)$ is $\tau_j$-$w$ closed in $X$. Since $f$ is pairwise pre $w$-closed, $f[f^{-1}(A)] = A$ is $\sigma_j$-$w$ closed in $Y$. Therefore, $Y$ is pairwise $s^*gw$-$T_{1/2}$ space. \hfill \square

**Theorem 2.7.** a) Every pairwise $s^*gw$-closed function is pairwise $gw$-closed,

b) Every pairwise $s^*gw$-closed function is pairwise $rgw$-closed.

**Proof.** (a) Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a pairwise $s^*gw$-closed function. Let $V$ be a $\tau_j$-$w$ closed set in $X$. Since $f$ is a pairwise $s^*gw$-closed function, we have $f(V)$ is $\sigma_i\sigma_j$-$s^*gw$ closed in $Y$, $i, j = 1, 2, i \neq j$. Since every pairwise $s^*gw$-closed set is $\sigma_i\sigma_j$-$gw$ closed, we have $f(V)$ is $\sigma_i\sigma_j$-$gw$ closed in $Y$. Therefore, $f$ is pairwise $gw$-closed.

The proof of (b) is similar. \hfill \square
Since every $\tau_j$-$w$ closed set is $\tau_i\tau_j$-$s^*gw$ closed, we have the following theorem.

**Theorem 2.8.** Every pairwise $s^*gw$-irresolute map is pairwise $s^*gw$-continuous map.

**References**


