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THE M-N-HOMOMORPHISM AND M-N-ANTI HOMOMORPHISM OVER M-N-FUZZY SUBGROUPS

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Abstract: The paper [13] introduces the concept of M-fuzzy groups. In the present paper, the concept of M-N-fuzzy subgroups are given and some elementary properties are discussed. We will also extend some results on this subject.

AMS Subject Classification: 20N25, 22F05
Key Words: fuzzy subset, M-N-fuzzy subgroup, M-N-normal fuzzy subgroups, M-N-fuzzy Abelian subgroups, M-N-homomorphism, M-N-antihomomorphism, M-N-fuzzy characteristic

1. Introduction

Zadeh's classical paper [6] of 1965 introduced the concepts of fuzzy sets and fuzzy set operations. Fuzzy subgroups and its important properties were defined and established by Rosenfeld [1] of 1971. A. Solairaju and R. Nagarajan [2] introduced the notion of $Q-$fuzzy groups. In [9], Biswas introduced the concept of anti-fuzzy subgroup of groups. Chandrasekhara and Gopalakrish [4, 5] defined the anti homomorphism in groups near rings and obtained some results. Also A. sheik Abdullah and k. Jeyaraman [3] defined the concept of anti homomorphism in fuzzy subgroup and normal fuzzy subgroups. On the other hand R. Mutharaj, M. Rajinikanuan and M.S.Muthuraman [10] discussed the $M-$ homomorphism and $M-$ anti homomorphism of $M-$ fuzzy subgroup.

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P. Pandiammal, R. Nutarjan and N. Palaniappan [7] discussed the anti-L-fuzzy normal M-subgroups.

In this paper, a new concept of M-N-homomorphism and M-N-anti homomorphism between two M-N-groups G_1 and G_2 is defined, many results analogous to homomorphism and anti homomorphism of groups established.

2. Preliminaries

Definition 2.1. (see [13]) Let M, N be left and right operator sets of group G respectively, if $(mx)n = m(xn)$ for all $x \in G, m \in M, n \in N$. Then G is said to be an M-N-group.

Definition 2.2. (see [13]) If a subgroup of M-N-group is also M-N-group, then it is said to be an M-N-subgroup of G.

Definition 2.3. (see [13]) If M-N-subgroup is also normal subgroup, then is said to be M-N-normal subgroup of G.

Definition 2.4. (see [6]) Let X be a non empty set, a fuzzy set μ is just a function from X onto $[0, 1]$.

Definition 2.5. Let μ be a fuzzy subset of a set G. For $t \in [0, 1]$, the level subset of μ is the set, $\mu_t = \{x \in G; \mu(x) \geq t\}$. This is called a fuzzy level subset of μ.

Definition 2.6. Let μ be a fuzzy subgroup of a group G. The subgroup μ_t of G, for $t \in [0, 1]$ such that $\mu(e) \geq t$ is called a level subgroup of μ.

Definition 2.7. (see [1]) Let G be a group and μ be a fuzzy set on G. μ is said to be a fuzzy subgroup of G, if for $x, y \in G$

1. $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$
2. $\mu(x^{-1}) = \mu(x)$.

Definition 2.8. (see [12]) A fuzzy subgroup μ of a group G is called normal fuzzy subgroup if $\mu(x^{-1}yx) \geq \mu(y)\ \forall x, y \in G$.

Definition 2.9. (see [9]) Let Φ be a function from X to Y and let μ be fuzzy subset on X and λ be a fuzzy subset on Y, define the fuzzy subset $\Phi(\mu)$ on Y and $\Phi^{-1}(\lambda)$ on X by, for all $y \in Y$

$$\Phi(\mu)(y) = \begin{cases} \sup\{\mu(x); \Phi(x) = y\}, & \text{if } \Phi^{-1} \neq \phi, \\ 0; & \text{Otherwise,} \end{cases}$$

And for all $x \in X, \phi^{-1}(\lambda)(x) = \lambda(\Phi(x))$. Then $\Phi(\mu)$ is called the image of μ under Φ and $\Phi^{-1}(\lambda)$ is called the pre image (or the inverse of λ under Φ).

Definition 2.10. (see [11]) Let A, B be general sets, $\Phi : A \to B$ a surjective mapping and μ a fuzzy set of A. If $\Phi(x) = \Phi(y)$ follows $\mu(x) = \mu(y)$, then μ is called $\Phi-$ invariant.

3. M-N-Fuzzy and Normal Fuzzy Subgroups

Definition 3.1. (see [13]) Let G be an M-N-group and μ be a fuzzy subgroup of G. If

1. $\mu(mx) \geq \mu(x)$,
2. $\mu(xn) \geq \mu(x)$

hold for any $x \in G$, $m \in M$ and $n \in N$, then μ is said to be an M-N-fuzzy subgroup of G.

Definition 3.2. (see [13]) Let G be an M-N-group. μ is said to be an M-N-normal fuzzy subgroup of G if μ is not only an M-N-fuzzy subgroup of G, but also normal fuzzy subgroup of G.

Proposition 3.3. *Let G be an M-N-group. μ, λ both be M-N-fuzzy subgroups of G, then the intersection of μ, λ is an M-N-fuzzy subgroup of G.*

Proof. Straight forward. □

Corollary 3.4. *The intersection of to M-N-normal fuzzy subgroups μ, λ is an M-N-normal fuzzy subgroup of G.*

Proof. Straight forward. □

Corollary 3.5. *If μ is an M-N-fuzzy subgroup of an M-N-group G, then the following statement hold for all $x, y \in G, m \in M$ and $n \in N$.*

1. $\mu((m(xy)n) \geq \min\{\mu(x), \mu(y)\}$
2. $\mu((mx^{-1})n) \geq \mu(x)$

Theorem 3.6. *Let G be an M-N-group, λ be a fuzzy set of G. Then λ is M-N-fuzzy subgroup of G iff for any $t \in [0, 1], \lambda_t$ is an M-N-subgroup of G, when $\lambda_t \neq \phi$.*

Proof. Straight forward. □

Corollary 3.7. *Let μ be a fuzzy set of the M-N-group G, then μ is an M-N-normal fuzzy subgroup iff μ_t is an M-N-normal- subgroup of G for any $t \in [0, 1]$, when $\mu_t \neq \phi$.*

Proof. Straight forward. □

Definition 3.8. Let μ be an M-N-fuzzy subgroup of an M-N-group G and let $H = \{x \in G, m \in M, n \in N; \mu(mxn) = \mu(e)\}$. Then μ is an M-N-fuzzy abelian subgroup of G.

4. M-N-Homomorphism and M-N-Anti-Homomorphism

Definition 4.1 (8)**.** Let G_1 and G_2 both be M-N-groups and Ψ be a homomorphism from G_1 onto G_2. If $\Psi(mx) = m\Psi(x)$ and $\Psi(xn) = \Psi(x)n$ for all $x \in G_1, m \in M$ and $n \in N$. Then Ψ is called an M-N-homomorphism.

Proposition 4.2. *Let G_1 and G_2 both be M-N-groups and Ψ be a homomorphism from G_1 onto G_2. If μ is an M-N-fuzzy subgroup of G_2, then $\Psi^{-1}(\mu)$ is an M-N-fuzzy subgroup of G_1.*

Corollary 4.3. *Let G_1 and G_2 both be M-N-groups and Ψ be a homomorphism from G_1 onto G_2. If μ is an M-N-normal fuzzy subgroup of G_2, then $\Psi^{-1}(\mu)$ is an M-N-normal fuzzy subgroup of G_1.*

Proposition 4.4. *Let G_1 and G_2 both be M-N-groups and Ψ be a homomorphism from G_1 onto G_2. If μ is an M-N-fuzzy subgroup of G_1, then $\Psi(\mu)$ is an M-N-fuzzy subgroup of G_2.*

Corollary 4.5. *Let G_1 and G_2 both be M-N-groups and Ψ be a homomorphism from G_1 onto G_2. If μ is an M-N-normal fuzzy subgroup of G_1, then $\Psi(\mu)$ is an M-N-normal fuzzy subgroup of G_2.*

Definition 4.6. Let G_1 and G_2 both be M-N-groups, then the function Ψ from G_1 onto G_2 is said to be M-N-anti homomorphism. If $\Psi(m(xy)) = m\Psi(y)\Psi(x)$ and $\Psi((xy)n) = \Psi(y)\Psi(x)n$ for all $x \in G_1, m \in M$ and $n \in N$.

Definition 4.7. μ be an M-N-fuzzy characteristic subgroup of M-N-group G if $\mu(\Psi(m(xy)n) = \mu(m(xy)n)$.

5. Main Results

Theorem 5.1. *Let $\Psi : G_1 \to G_2$ be an M-N-anti homomorphism. If λ is an M-N-fuzzy subgroup of G_2, then $\Psi^{-1}(\lambda)$ is an M-N-fuzzy subgroup of G_1.*

Proof. Let $x, y \in G_1, m \in M, n \in N$

$$\begin{aligned}\Psi^{-1}(\lambda)(m(xy)) =& \lambda(\Psi(m(xy)) = \lambda(m(\Psi(y)(x))\\ \geq& \min\{\lambda(m(\Psi(y)), \lambda(\Psi(x))\}\\ \geq& \min\{m\Psi^{-1}(\lambda(y)), \Psi^{-1}(\lambda(x))\}\end{aligned}$$

And

$$\begin{aligned}\Psi^{-1}(\lambda)((xy)n) =& \lambda(\Psi((xy)n) = \lambda((\Psi(y)\Psi(x)n)\\ \geq& \min\{\lambda((\Psi(y)), \lambda(\Psi(x)n)\}\\ \geq& \min\{\Psi^{-1}(\lambda(y)), \Psi^{-1}(\lambda(x)n)\}\end{aligned}$$

Also $\Psi^{-1}(\lambda(mx^{-1})n) = \lambda(m\Psi((x^{-1})n) = \lambda(m\Psi((x)n) = \lambda(\Psi(x)) = \Psi^{-1}(\lambda(x))$. Therefore $\Psi^{-1}(\lambda)$ is M-N-fuzzy subgroup of G_1.

□

Corollary 5.2. *Let $\Psi : G_1 \to G_2$ be an M-N-anti homomorphism. If λ is an M-N-normal fuzzy subgroup of G_2, then $\Psi^{-1}(\lambda)$ is an M-N-normal fuzzy subgroup of G_1.*

Proof. Let $x, y \in G_1, m \in M, n \in N$ By theorem 5.1 $\Psi^{-1}(\lambda)$ is an M-N-fuzzy subgroup of G_1. $\Psi^{-1}(\lambda)((xy)) = \lambda(\Psi(xy)) = \lambda((\Psi(y)\Psi(x)) = \lambda(\Psi(yx)) = \Psi^{-1}(\lambda)((yx))$. Which is implies that $\Psi^{-1}(\lambda)$ is an M-N-normal fuzzy subgroup of G_1. □

Theorem 5.3. *An M-N-fuzzy characteristic subgroup on M-N-fuzzy subgroup is an M-N-normal fuzzy subgroup.*

Proof. Let Ψ be an M-N-anti homomorphism of G which is implies that $\Psi(xy) = \Psi(y)\Psi(x)$.

$$\Psi(m(xy)) = m\Psi(y)\Psi(x)$$
$$\Psi((xy)n) = \Psi(y)\Psi(x)n$$

Since $\lambda(m(xy)n) = \lambda(\Psi(m(xy)n)$ and

$$\lambda(m(xy)n) = \lambda(m\Psi(y)\Psi(x)n) = \lambda(\Psi(m(yx)n).$$

Hence λ is an M-N-normal fuzzy subgroup of G. □

Theorem 5.4. *An M-N-Anti homomorphism pre image of an M-N-fuzzy abelian subgroup is an M-N-fuzzy abelian subgroup.*

Proof. Let μ be an M-N-fuzzy subgroup of G_1, we need to prove μ is an M-N-fuzzy abelian subgroup of G_1 suppose that λ is an M-N-fuzzy abelian subgroup of G_2. Then $H_2 = \{y \in G_2, m \in M, n \in N; \lambda(myn) = \lambda(e_2)\}$ is an M-N-fuzzy abelian subgroup of G_2, where e_2 is the identity of G_2. Consider the set $H_1 = \{x \in G1, m \in M, n \in N; \mu(mxn) = \mu(e_1)\}$ where e_1 is the identity of G_1. Let $m(xy)n \in H_1 \subseteq G_1$, then $\mu(mxyn) = \mu(e_1)$

$$\begin{aligned}\lambda(\Psi(m(xy)n) =& \lambda(\Psi(e_1))\\ \lambda(\Psi(m(xy)n) =& \lambda(e_2)\\ \lambda(m(\Psi(y)\Psi(x)n) =& \lambda(e_2)\end{aligned}$$

$m(\Psi(y)\Psi(x)n \in H_2$ and H_2 is abelian, thus

$$\begin{aligned}\lambda(m(\Psi(y)\Psi(x)n) = \lambda(m(\Psi(x)\Psi(y)n) = \lambda(m\Psi(xy)n) =& \lambda(m\Psi(yx)n)\\ \mu(m(xy)n) =& \mu(m(yx)n)\\ \mu(e_1) =& \mu(m(yx)n)\end{aligned}$$

Therefore H_1 is an M-N-abelian subgroup and μ is an M-N-fuzzy abelian subgroup of G_1. □

Theorem 5.5. *An M-N-Anti homomorphism image of an M-N-fuzzy abelian subgroup is an M-N-fuzzy abelian subgroup.*

Proof. Let μ be an M-N-fuzzy subgroup of G_2, we need to prove μ is an M-N-fuzzy abelian subgroup of G_2, suppose that Ψ is an M-N-anti homomorphism from G_1 into G_2 since is an M-N-fuzzy abelian subgroup of G_1. Then $H_1 = \{x \in G_1, m \in M, ninN; \mu(mxn) = \mu(e_1)\}$ is an M-N-abelian subgroup of G_1 where e_1 is the identity of G_1. Let λ be an M-N-fuzzy abelian subgroup of G_2 and $H_2 = \{y \in G_2, m \in M, n \in N; \lambda(myn) = \lambda(e_2)\}$ is an M-N-fuzzy abelian subgroup of G_2, where e_2 is the identity of G_2. If $m(xy)n \in H_2 \subseteq G_2, \lambda(mxyn) = \lambda(e_2)$

$$\begin{aligned}\sup_{z\in\Psi^{-1}(m(xy)n)} \mu(z) =& \sup_{z\in\Psi^{-1}(e_2)} \mu(z)\\ \mu(m(xy)n) =& \mu(e_1)\end{aligned}$$

then $m(xy)n \in H_1$ and H_1 is an M-N-abelian subgroup, thus $\mu(m(xy)n) = \mu(m(yx)n)$

$$\begin{aligned}\sup_{z\in\Psi^{-1}(m(xy)n)} \mu(z) =& \sup_{z\in\Psi^{-1}(m(yx)n)} \mu(z)\\ \lambda(m(xy)n) =&\lambda(m(yx)n)\\ \lambda(e_2) =&\lambda(m(yx)n)\end{aligned}$$

Therefore H_2 is an M-N-abelian subgroup of G_2 and λ is an M-N-fuzzy abelian subgroup of G_2. □

Theorem 5.6. *Let Ψ be an M-N-homomorphism from an M-N-group G_1 onto an M-N-group G_2. If μ is an M-N-fuzzy subgroup of G_1 and μ is an Ψ-invariant, then $\Psi(\mu)$ is an M-N-fuzzy subgroup of G_2.*

Proof. Let $t \in Im\Psi(\mu)$, then for some $y \in G_2$.

$$(\Psi(\mu))(y) = \sup_{x\in\Psi^{-1}} \mu(x) = t;$$

where $t \leq \mu(e)$.

We know μ_t is an M-N-subgroup of G_1, if $t = 1$ then $(\Psi(\mu))_t = G_2$. If $0 < t < 1$, then $(\Psi(\mu))_t = \Psi(\mu_t)$, since $z \in (\Psi(\mu))_t \iff \Psi(\mu)(z) \geq t \iff \sup_{x\Psi^{-1}(z)} \mu(x) \geq t$

Iff there exist $x \in G_1$ such that $\Psi(x) = z$ and $\mu(x) \geq t$ iff $z \in (\Psi(\mu_t))$. Hence $(\Psi(\mu))_t = \Psi(\mu_t)$ and is an M-N-homomorphism,$(\Psi(\mu_t))$ is an M-N-subgroup of G_2 therefore $(\Psi(\mu))_t$ is an M-N-subgroup of G_2 and $\Psi(\mu)$ is an M-N-fuzzy subgroup of G_2. □

Theorem 5.7. *Let Ψ be an M-N-anti homomorphism from an M-N-group G_1 onto an M-N-group G_2. If μ is an M-N-fuzzy subgroup of G_1 and μ is an Ψ-invariant, then $\Psi(\mu)$ is an M-N-fuzzy subgroup of G_2.*

Proof. Let $t \in Im\Psi(\mu)$, then for some $y \in G_2$.

$$(\Psi(\mu))(y) = \sup_{x\in\Psi^{-1}} \mu(x) = t;$$

where $t \leq \mu(e)$.

We know μ_t is an M-N-subgroup of G_1, if $t = 1$ then $(\Psi(\mu))_t = G_2$ If $0 < t < 1$, then $(\Psi(\mu))_t = \Psi(\mu_t)$, since $z \in (\Psi(\mu))_t \iff \Psi(\mu)(z) \geq t \iff \sup_{x\in\Psi^{-1}(z)} \mu(x) \geq t$

Iff there exist $x \in G_1$ such that $\Psi(x) = z$ and $\mu(x) \geq t$ iff $z \in (\Psi(\mu_t))$. Hence $(\Psi(\mu))_t = \Psi(\mu_t)$ and Ψ is an M-N-anti homomorphism, $\Psi((\mu_t))$ is an M-N-subgroup of G_2 therefore $(\Psi(\mu))_t$ is an M-N-subgroup of G_2 and $\Psi(\mu)$ is an M-N-fuzzy subgroup of G_2. □

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