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ROBUST OPTIMAL OUTPUT FEEDBACK SLIDING MODE CONTROL FOR SPACECRAFT ATTITUDE TRACKING MANEUVERS

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Abstract: This research studies the robust optimal output feedback sliding mode control law design for attitude tracking maneuvers of a rigid spacecraft. The proposed control technique, which combines the first-order sliding mode control (SMC) concept and control Lyapunov function (CLF) approach, is applied to quaternion-based attitude tracking with external disturbances and an uncertainty inertia matrix. For the CLF-based optimal controller design the state-dependent Riccati equation (SDRE) method is used in the design procedure to determine a suitable CLF. The Sontag formula is then applied to construct a new optimal controller. The SDRE compensator technique is employed to estimate the system state. The second method of Lyapunov is used to show that the stability of the closed-loop system and external disturbance attenuation can be ensured. An example of multiaxial attitude maneuvers is presented and simulation results are given to verify the usefulness of the proposed controller.

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1. Introduction

Optimal attitude control has been extensively studied by various researchers (see, e.g., [1]-[3]). The main objective of optimal attitude control of a spacecraft is to design a controller that stabilizes the attitude of the spacecraft system to an equilibrium state and minimizes a given performance criterion for the stabilization process. In the past decade SDRE techniques have been developed to offer a systematic and effective means for the design of control systems for nonlinear dynamical systems (Mracek and Cloutier, [4], [5], Cloutier et al. [6], Cloutier and Stansbery [7]). Later, extension of the SDRE methodology to the nonlinear state estimation problem has been presented in Banks et al. [8] and Cimen [9]. The work in reference [8] includes local convergence results for the nonlinear state estimation and a numerical example. Recent applications of the SDRE techniques for implementations of optimal control can be found in the works by Pittner and Simaan [10] and Pittner et al. [11]. On the other hand the CLF-based control methods [12]-[16] can be used to solve the nonlinear optimal control problem without solving the Hamilton-Jacobi-Bellman (HJB) equation. By finding a CLF, which can also be shown to be a value function, an optimal controller that optimizes a derived cost can be designed (Sontag [12], Freeman and Kokotovic [17]).

As extensions of the above studies, optimal control and robust control have been merged to obtain robust optimal control laws. Various nonlinear control methods for developing robust optimal controllers such as nonlinear H_∞ control [18], H_∞ inverse optimal adaptive control [19], minimax control [20], and optimal SMC [21], have been proposed for solving the attitude tracking control problem. However, most of these controller designs require feedback of attitude angle and angular velocity (i.e., they are full-state feedback controllers). In practical attitude controller designs, it may be difficult to know all of the states of the attitude control system.

In this research the CLF approach and the first-order SMC are merged to develop a novel robust optimal control. It is not straightforward to use the concept of CLF as a design tool, since there is no systematic technique for finding CLFs for general nonlinear systems. Here we apply the SDRE method to find a CLF for the tracking system. Furthermore, the SDRE observer is used to estimate the system state in the design procedure. To the author's knowledge, the robust optimal output feedback sliding mode controller design is proposed for the first time in this paper. The desired control objective is that the robust optimal output feedback controller obtained is expected to yield both robustness and optimality.

This paper is organized as follows. In Section 2, the dynamic equation of error rate and the kinematics of attitude error ([21], [22]) are described. In section 3, SDRE approaches for the development of observer-based controller are discussed. The systematic design procedure is described and the control formulation is given. In Section 4, fundamental concepts of the CLF-based control technique are given. The resulting control and first order SMC are then merged to develop a robust optimal feedback controller design. In Section 5, an example of spacecraft attitude maneuvers is presented to verify the usefulness of the proposed controller. In Section 6, we present conclusions.

2. Mathematical Model of Spacecraft Attitude Tracking Control

2.1. Dynamic Equations of the Error Rate

In this section the mathematical model of a rigid spacecraft is briefly restated. The model is the one used in [20], [21] and the reader is referred to these papers for more details. We define here the attitude error $Q_e = [e^T \quad e_4]^T$ with $e = [e_1 \quad e_2 \quad e_3]^T$, the angular velocity $\omega = [\omega_1 \quad \omega_2 \quad \omega_3]^T$ and the error angular velocity rate $\omega_e = [\omega_{1e} \quad \omega_{2e} \quad \omega_{3e}]^T$. The kinematic equation for the attitude error quaternion and the dynamic equation of the error rate are given as

$$\dot{Q}_e = \frac{1}{2}\begin{bmatrix} e^\times + e_4 I_3 \\ -e^T \end{bmatrix}\omega_e \tag{1}$$

and

$$J\dot{\omega}_e = -\omega_e^\times J\omega_e - \omega_e^\times J\omega_r - \omega_r^\times J\omega_e + \tau + \xi, \tag{2}$$

where I_3 is the 3×3 identity matrix and

$$e^\times = \begin{bmatrix} 0 & -e_3 & e_2 \\ e_3 & 0 & -e_1 \\ -e_2 & e_1 & 0 \end{bmatrix} \tag{3}$$

is the 3×3 skew-symmetric matrix.

Here $\tau = [\tau_1 \quad \tau_2 \quad \tau_3]^T$ represents the control vector, $\xi = [\xi_1 \quad \xi_2 \quad \xi_3]^T$ are bounded disturbances, J is the inertia matrix and $\omega_r = [\omega_{1r} \quad \omega_{2r} \quad \omega_{3r}]^T$ denotes the desired reference rate. In (2) ω_e can be obtained from $\omega_e = \omega - \omega_d$ and $\omega_e^\times$ is a skew-symmetric matrix with a formula similar to $e^\times$.

Note that in (1) e_4 is not independent of e_1, e_2, e_3 because of the constraint relation $e^T e + e_4^2 = 1$ among the four unitary quaternions.

To avoid the singularity of the matrix $e^\times + e_4 I_3$ that will occur at $e_4 = 0$, we let the attitude error of the spacecraft be restricted to the workspace W [24] defined by

$$W = \{Q_e | Q_e = [e^T \quad e_4]^T, \|e\| \leq \beta_0 < 1, e_4 \geq \sqrt{1-\beta_0^2} > 0\}, \tag{4}$$

where β_0 is a positive constant. To facilitate the controller design, (1) is utilized to obtain

$$\dot{e} = \frac{1}{2} B_e \omega_e, \tag{5}$$

where $B_e = e^\times + e_4 I_3$. In our design the spacecraft's attitude error is restricted to the workspace W to ensure that B_e is invertible.

Taking the time derivative of (5) and premultiplying both sides of the result by $B_e^{-T} J B_e^{-1}$, one obtains [25]

$$B_e^{-T} J B_e^{-1} \ddot{e} = \frac{1}{2} B_e^{-T} J B_e^{-1} \dot{B}_e \omega_e + \frac{1}{2} B_e^{-T} J B_e^{-1} \dot{\omega}_e. \tag{6}$$

Letting $J_e = B_e^{-T} J B_e^{-1}$ and using (5), equation (6) becomes

$$\begin{aligned} J_e \ddot{e} &= J_e \dot{B}_e B_e^{-1} \dot{e} - B_e^{-T} (2B_e^{-1} \dot{e})^\times J B_e^{-1} \dot{e} - B_e^{-T} (2B_e^{-1} \dot{e})^\times J \omega_r \\ &\quad - B_e^{-T} \omega_r^\times J 2 B_e^{-1} \dot{e} + \frac{1}{2} B_e^{-T} \tau + \frac{1}{2} B_e^{-T} \xi. \end{aligned} \tag{7}$$

Letting $\alpha_i = \sum_{j=1}^{3} J_{ij} \omega_{jr}, \quad i = 1, 2, 3$ and using basic matrix operations, we obtain

$$(B_e^{-1} \dot{e})^\times J \omega_r = -\alpha^\times (B_e^{-1} \dot{e}), \tag{8}$$

where $\alpha^\times$ is a skew-symmetric matrix with a formula similar to $\omega_e^\times$. Substituting (8) into (7), one has

$$\begin{aligned} J_e \ddot{e} &= J_e \dot{B}_e B_e^{-1} \dot{e} - B_e^{-T} (2B_e^{-1} \dot{e})^\times J B_e^{-1} \dot{e} + \alpha^\times (B_e^{-1} \dot{e}) \\ &\quad - B_e^{-T} \omega_r^\times J (B_e^{-1} \dot{e}) + u + d, \end{aligned} \tag{9}$$

where the new control input and external disturbance are defined as

$$u = \frac{1}{2} B_e^{-T} \tau \quad \text{and} \quad d = \frac{1}{2} B_e^{-T} \xi. \tag{10}$$

If we let $x = [e^T \quad \dot{e}^T]^T$ and $\Gamma(e, \dot{e}, \omega_r) = \dot{B}_e B_e^{-1} \dot{e} - J_e^{-1} (2B_e^{-1} \dot{e})^\times J B_e^{-1}$ $+ J_e^{-1} \alpha^\times B_e^{-1} - J^{-1} \omega_r^\times J B_e^{-1}$, then the kinematic and dynamic equations can be transformed into the new coordinates as

$$\dot{x} = f(x) + G(x) u + G(x) d$$

$$y \quad = \quad Hx, \tag{11}$$

where

$$f(x) = \begin{bmatrix} \dot{e} \\ \Gamma(e, \dot{e}, \omega_r)\dot{e} \end{bmatrix} \quad G(x) = \begin{bmatrix} 0_{3\times3} \\ J_e^{-1} \end{bmatrix} \quad \text{and} \quad H = \begin{bmatrix} I_3 & 0_{3\times3} \end{bmatrix}. \tag{12}$$

3. SDRE Approach

3.1. SDRE Control

This section briefly describes the SDRE technique and a systematic procedure for the design of the SDRE controller. The formulation of motion tracking as an optimal control problem of a rigid spacecraft is considered. For the optimal controller design, the difficulty of using the SDRE approach is to find the appropriate state-dependent coefficient (SDC) matrix. In this section it is assumed that external disturbances are not taken into account. The SDRE controller design can be processed as below.

We discuss an optimal control law minimizing the performance index

$$I = \int_0^\infty (x^T Q(x)x + u^T R(x)u)dt, \tag{13a}$$

$$\text{where} \quad \dot{x} = f(x) + G(x)u, \quad x(0) = x_0 \tag{13b}$$

and $Q \in \mathcal{R}^{n\times n}$ is a symmetric positive semidefinite matrix and $R \in \mathcal{R}^{m\times m}$ is a symmetric positive definite matrix. Further, we assume that $f(x)$ can be decomposed as $f(x) = A(x)x$, where $A(x) \in \mathcal{R}^{n\times n}$ is an analytic valued function for every given $x \in \mathcal{R}^n$, and the pair $(A(x), G(x))$ is stabilizable in the linear quadratic regulator sense.

Obviously, it is difficult to obtain the SDC matrix $A(x)$ from the system (13b). Based on the theory in [26], the matrix $A(x)$ is chosen as

$$A(x) = \begin{bmatrix} 0 & I_3 \\ 0 & \Gamma(e, \dot{e}, \omega_r) \end{bmatrix}. \tag{14}$$

Thus, the optimal control υ^* is given as [6]

$$\upsilon^* = -R^{-1}G^T\Pi(x)x, \tag{15}$$

where $\Pi(x)$ is the solution to the generalized SDRE

$$\Pi(x)A(x) + A^T(x)\Pi(x) + Q(x) - \Pi(x)G(x)R^{-1}(x)G^T(x)\Pi(x) = 0. \tag{16}$$

In general the choice of the matrix $A(x)$ is not unique. It has explicitly been selected to make the stability problem analytically tractable. Note that because of the problem of robustness, we have not directly applied the SDRE controller to the attitude tracking system. In Section 4 the control method to design a robust optimal control will be addressed and SDRE concepts will be used in the design process to find a CLF.

3.2. SDRE State Estimator

This section presents the SDRE-estimator-based controller. This control law design uses the SDRE state estimator proposed in [8] to estimate the state feedback in the controller design. We consider systems that can be written as

$$\begin{aligned} \dot{x} &= A(x)x + B(x)u \\ y &= C(x)x, \end{aligned} \tag{17}$$

where $A(x) \in \mathcal{R}^{n\times n}$, $B(x) \in \mathcal{R}^{n\times m}$ and $C(x) \in \mathcal{R}^{p\times n}$ are continuous matrix-valued functions. The SDRE nonlinear estimator can be formulated by mimicking the theory of estimators for linear systems. The control law using estimator compensation is formulated as

$$\begin{aligned} \dot{x} &= A(x)x + B(x)u(\hat{x}) \\ \dot{\hat{x}} &= A(\hat{x})\hat{x} + B(\hat{x})u(\hat{x}) - L(\hat{x})(y - C(\hat{x})\hat{x}), \end{aligned} \tag{18}$$

where

$$L(\hat{x}) = \Gamma(\hat{x})C^T(\hat{x})V^{-1}, \tag{19}$$

and $\Pi(x)$ solves the dual SDRE

$$\Gamma(\hat{x})A(\hat{x}) + A^T(\hat{x})\Gamma(\hat{x}) + U - \Gamma(\hat{x})C^T(\hat{x})V^{-1}(x)C(\hat{x})\Gamma(\hat{x}) = 0. \tag{20}$$

Here, $U \in \mathcal{R}^{n\times n}$ is the symmetric positive semidefinite matrix and $V \in \mathcal{R}^{m\times m}$ is the positive semidefinite matrix that are obtained as the design parameters from the corresponding cost functional for the SDRE state estimator. In the estimator-compensated system, the actual state x can be replaced by the estimated state $\hat{x}$. If the respective pairs $\{A(x), G(x)\}$ and $\{A(x), C(x)\}$ are pointwise stabilizable and have detectable parameterizations, then the compensated closed-loop system is locally asymptotically stable in a neighborhood about the origin (Banks et al. [8]). Banks et al. [8] have shown that the state estimate obtained using the SDRE observer locally asymptotically converges to the actual state.

4. CLF-Based Sliding Mode Control

In this section the attitude tracking problem for a spacecraft is addressed through the combination of optimal control and SMC. A particularly interesting optimal control design is based on the existence of a CLF. Applying the SDRE concept we can find a CLF for the optimal control problem of the attitude tracking. Then the Sontag formula can be used with the CLF obtained to construct a new optimal stabilizing control. Using the proposed technique, we can make optimal or suboptimal control more robust and applicable to spacecraft systems with uncertainties.

4.1. Control Lyapunov Functions

We consider the optimal control problem (13). A suboptimal solution based on CLF is given below.

Definition 1. (Sepulchre et al. [15]) A smooth, positive definite and radially unbounded function V is called a control Lyapunov function for system (13b) if for all $x \neq 0$

$$L_G V = 0 \Longrightarrow L_f V < 0 \quad \forall x \neq 0. \tag{21}$$

Here, the Lie derivative of V with respect to f is defined as the inner product of the gradient of V with f, i.e. $L_f V(x) = \left[\frac{\partial V}{\partial x}\right]^T f(x)$.

A stabilizing optimal control law for the cost of the form (13a) can be selected such that a CLF V becomes the optimal value function. A particular optimizing control law can be designed by the Sontag formula [12]

$$u^* = \begin{cases} -\frac{\psi(x)+\sqrt{(\psi(x))^2+x^T Q x \beta^T(x) R^{-1}(x)\beta(x)}}{\beta(x)} & \text{for} \quad \beta(x) \neq 0 \\ 0 & \text{for} \quad \beta(x) = 0 \end{cases} \tag{22}$$

where $\psi(x) = L_f V(x)$ and $\beta(x) = [L_G V(x)]^T$.

It is noted that the difficulty of employing the Sontag formula is how to find a CLF for the tracking system (13b). The following lemma provides an alternative way to obtain such a CLF.

Lemma 2. *The following positive definite function is a CLF for the spacecraft tracking system (13b)*

$$V = x^T P(x) x, \tag{23}$$

where $P(x)$ is the solution to the following matrix Riccati equation

$$(P + \frac{1}{2}x^T P_x)^T A(x) + A^T(x)(P + \frac{1}{2}x^T P_x) + Q(x)$$

$$-(P+\frac{1}{2}x^TP_x)^TG(x)R^{-1}(x)G^T(x)(P+\frac{1}{2}x^TP_x)=0. \tag{24}$$

Clearly by setting $\Pi(x) = P + \frac{1}{2}x^TP_x$ the matrix Riccati equation (24) is equivalent to the matrix Riccati equation (11). Thus the Lyapunov function above comes from the SDRE design.

Proof. Let the optimal value function for the infinite time problem be given by (23). We obtain

$$\frac{\partial V}{\partial x} = x^TP + Px^T + x^TP_xx. \tag{25}$$

Putting $\Pi(x) = P + \frac{1}{2}x^TP_x$, (25) becomes

$$\frac{\partial V}{\partial x} = 2x^T\Pi(x). \tag{26}$$

We know that $L_GV(x) = \left[\frac{\partial V}{\partial x}\right]^T G$, so it follows that

$$L_GV(x) = 2x^T\Pi(x)G. \tag{27}$$

Clearly, if we set $L_GV(x) = 0$, $\Pi(x)G = 0$ is obtained.

Next, letting $L_GV(x) = 0$ we can show that $L_fV(x) < 0$. First, $L_fV(x)$ can be derived as

$$\begin{aligned} L_fV(x) &= \left[\frac{\partial V}{\partial x}\right]^T A(x)x \\ &= 2x^T\Pi(x)A(x)x. \end{aligned} \tag{28}$$

With the substitution of the matrix Riccati equation (37) one has

$$L_fV(x) = x^T(-Q(x) + \Pi(x)G(x)R^{-1}(x)G^T(x)\Pi(x))x. \tag{29}$$

Substituting $\Pi(x)G = 0$ into (29), we obtain

$$L_fV(x) = -x^TQ(x)x < 0. \tag{30}$$

This implies that we obtain $L_fV(x) < 0$ whenever $L_GV(x) = 0$. Therefore, the function $V(x)$ defined in (23) is the CLF for system (13b). □

4.2. Control Law

Now, we are in a position to address the construction of the optimal sliding mode scheme, which is summarized in the following Theorem.

Theorem 3. *Let* u_o *be the optimal control defined in (22),* κ_i *be the* i^{th} *element of a vector. With* $\rho_i \geq |d_i|, \quad i = 1, 2, \dots m$ *and*

$$sign(\beta^T(x)) = [sign(\kappa_1(\beta^T(x))) \quad sign(\kappa_2(\beta^T(x)) \quad \dots \quad sign(\kappa_m(\beta^T(x))]^T$$

the integrated optimal sliding mode control law

$$u = -u_o - \rho sign(\beta^T(x)) \tag{31}$$

achieves asymptotic stability for system (11).

Proof. Choose the CLF $V = x^T P(x) x$ as the Lyapunov function candidate. Differentiating V yields

$$\begin{aligned} \dot{V}(x) &= \left[\frac{\partial V}{\partial x}\right]^T (f(x) + G(x)u + G(x)d) \\ &= L_f V + L_G V u_0 - L_G V \rho sign(L_G V) + L_G V d \\ &= -\sqrt{(\psi(x))^2 + x^T Q x \beta^T(x) R^{-1}(x) \beta(x)} \\ &\quad -2x^T \Pi(x) G s \left[\rho \text{sign}(\beta^T(x)) - d\right] \\ &\leq -\sqrt{(\psi(x))^2 + x^T Q x \beta^T(x) R^{-1}(x) \beta(x)} < 0 \end{aligned} \tag{32}$$

Clearly, $\dot{V}(x)$ is negative definite and asymptotic stability has been proved. □

4.3. Output Feedback Law

Now the robust optimal output feedback control law design is briefly summarized. In the SDC form the attitude tracking system along with an SDRE compensator can be written as

$$\dot{x} = \begin{bmatrix} 0 & I_3 \\ 0 & \Gamma(e, \dot{e}, \omega_r) \end{bmatrix} x + \begin{bmatrix} 0 \\ J_e^{-1} \end{bmatrix} u(\hat{x}) \tag{33}$$

$$\dot{\hat{x}} = \begin{bmatrix} 0 & I_3 \\ 0 & \Gamma(\hat{e}, \dot{\hat{e}}, \omega_r) \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ J_{\hat{e}}^{-1} \end{bmatrix} u(\hat{x}), \tag{34}$$

where $L(\hat{x})$ is the solution of (20) and $u(\hat{x})$ is given by

$$u(\hat{x}) = u_o(\hat{x}) - \rho sign(\beta^T(\hat{x})) \tag{35}$$

with

$$u_o(\hat{x}) = \begin{cases} -\frac{\psi(\hat{x})+\sqrt{(\psi(\hat{x}))^2+\hat{x}^T Q\hat{x}\beta^T(\hat{x})R^{-1}(\hat{x})\beta(\hat{x})}}{\beta(\hat{x})} & \text{for} \quad \beta(\hat{x}) \neq 0 \\ 0 & \text{for} \quad \beta(\hat{x}) = 0. \end{cases}$$

Here $\psi(\hat{x})$ and $\beta(\hat{x})$ can be determined from $\psi(\hat{x}) = L_f V(\hat{x})$ and $\beta(\hat{x}) = [L_G V(\hat{x})]^T$ with

$$\frac{\partial V(\hat{x})}{\partial \hat{x}} = 2\hat{x}^T \Pi(\hat{x}), \tag{36}$$

where $\Pi(\hat{x})$ is the solution to the generalized SDRE

$$\Pi(\hat{x})A(\hat{x}) + A^T(\hat{x})\Pi(\hat{x}) + Q(\hat{x}) - \Pi(\hat{x})G(\hat{x})R^{-1}(\hat{x})G^T(\hat{x})\Pi(\hat{x}) = 0. \tag{37}$$

Due to the chattering in the sliding mode controller design, the sign function of the control law (35) is replaced by

$$f(s_i) = \tanh(\frac{s_i}{\varepsilon}), \quad i = 1, 2, \ldots m, \tag{38}$$

where $\tanh(\cdot)$ is the hyperbolic tangent function and ε is a positive constant.

5. Simulation Results

An example of a rigid-body satellite [27] is presented with numerical simulations to verify the performance of the developed controller. The spacecraft is assumed to have the inertia matrix

$$J = \begin{bmatrix} 21 & 0.6 & 1.1 \\ 0.6 & 22 & 0.3 \\ 1.1 & 0.3 & 20 \end{bmatrix} \quad \text{kg} \cdot \text{m}^2.$$

Suppose that the workspace W is defined by $\beta^2 = 0.75$. The weighting matrices are chosen to be $Q = \text{diag}(1, 1, 1, 5, 5, 5)$, $R = \text{diag}(1, 1, 1)$, $U = Q$ and $V = 0.2R$. The initial conditions are $Q_e(0) = [0.3 \quad -0.2 \quad -0.3 \quad 0.8832]^T$, $\omega_e(0) = [0.06 \quad -0.04 \quad 0.05]^T$ rad/s for the state and $\hat{Q}_e(0) = [0 \quad -0.5 \quad -0.5 \quad 0.707]^T$,

$\hat{\omega}_e(0) = [0 \quad 0 \quad 0]^T$ rad/s for the estimated state. The desired angular velocity is of the form

$$\omega_d(t) = \begin{bmatrix} 0.1\cos(0.025t) \\ -0.1\sin(0.02t) \\ -0.1\cos(0.0167t) \end{bmatrix} \quad \text{N-m.} \tag{39}$$

The attitude control problem is considered in the presence of external disturbance $d(t)$. The disturbance model [27] is

$$d(t) = 0.01 \times \begin{bmatrix} 3\cos(0.1t) + 1 \\ 1.5\sin(0.1t) + 3\cos(0.1t) \\ 3\sin(0.1t) + 1 \end{bmatrix} \quad \text{N-m} \tag{40}$$

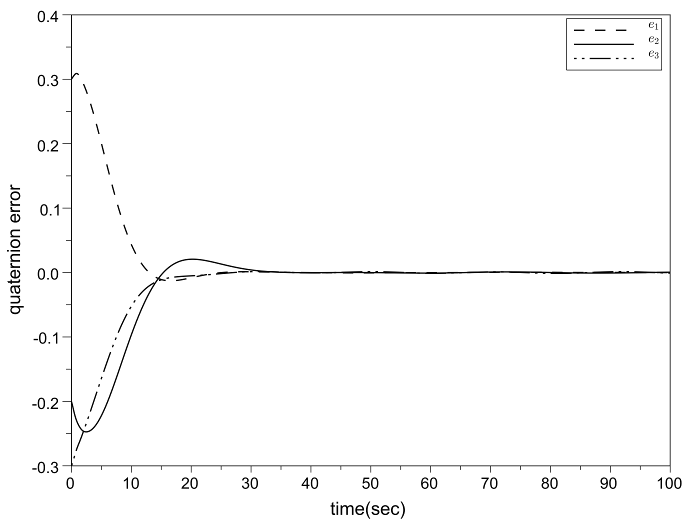

Figure 1: Quaternion tracking errors.

As shown Figs. 1 and 2 responses of the quaternion tracking and angular velocity errors reach zero after 40 seconds. Obviously, the effect of external disturbances on both responses is totally removed. From Fig. 3 it can be seen that the proposed controller stabilizes the closed-loop system and provides smooth control torque responses. As shown in Figs. 4 - 6 it can be seen that the ith component of the estimated disturbance vector properly tracks the ith component of the angular velocity error vector.

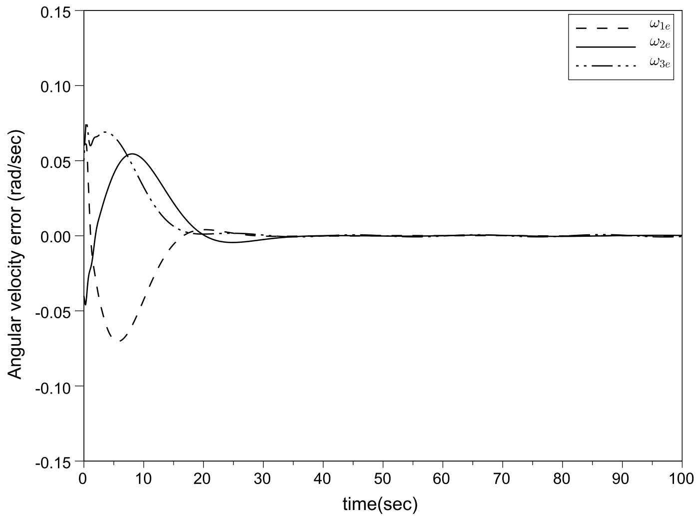

Figure 2: Angular velocity tracking errors.

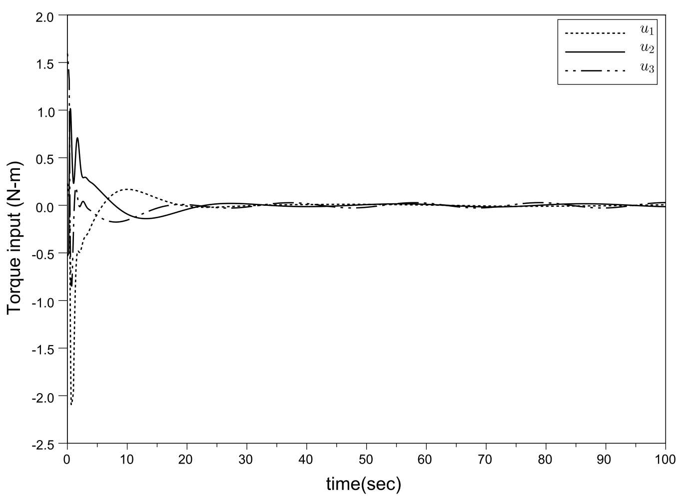

Figure 3: Control torques.

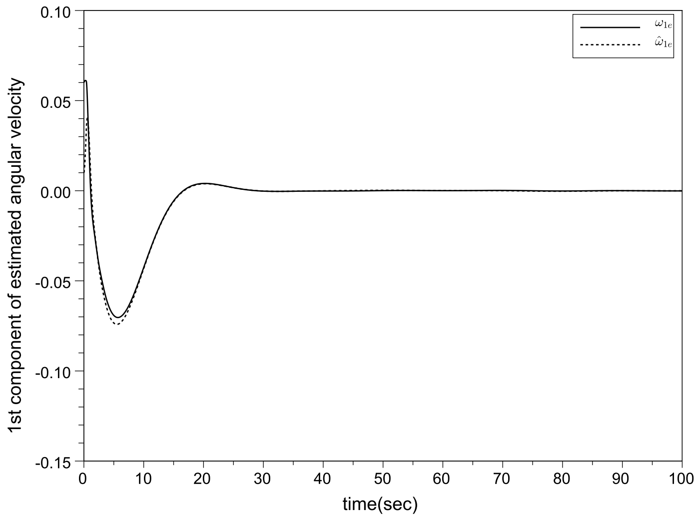

Figure 4: First component of estimated angular velocity error vector.

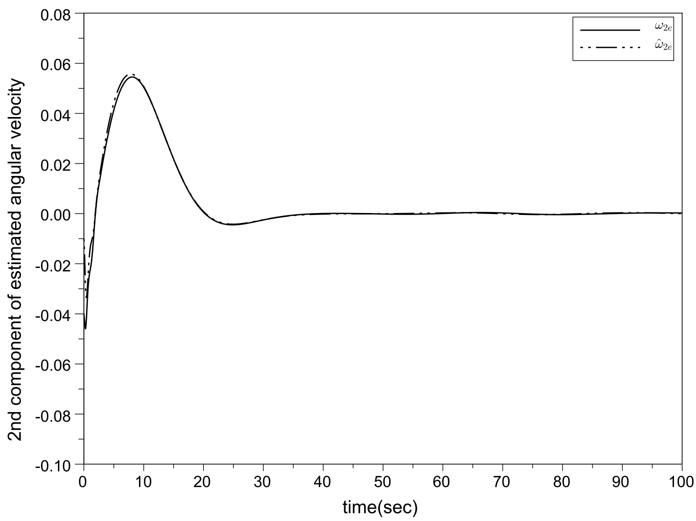

Figure 5: Second component of estimated angular velocity error vector.

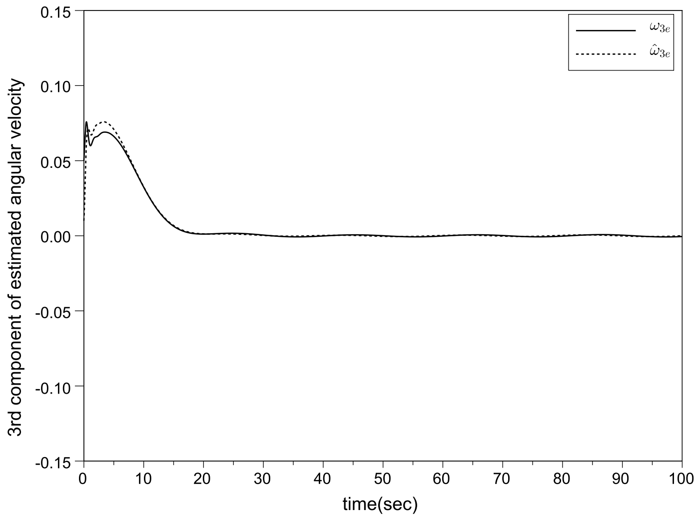

Figure 6: Third component of estimated angular velocity error vector.

6. Conclusion

We have studied a robust optimal output feedback sliding mode controller design to control some spacecraft tracking attitude manoeuvres. The developed control law has been successfully applied to the attitude tracking control problem. To obtain this controller design, first-order SMC combined with the CLF-based optimal control approach has been applied to quaternion-based spacecraft attitude regulation manoeuvres with external disturbances and an uncertain inertia matrix. We have applied the SDRE method to find a suitable CLF in the design procedure. The second method of Lyapunov is used to show that the stability of closed-loop system is achieved. An example of multiaxial attitude manoeuvres is presented and simulation results are included to verify the usefulness of the proposed controller.

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