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HYDROMAGNETIC TURBULENT FLOW OF A ROTATING SYSTEM PAST A SEMI-INFINITE VERTICAL PLATE WITH HALL CURRENT

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Abstract: In this study we have investigated a turbulent flow of a rotating system past a semi-infinite vertical porous plate. We have considered the flow in the presence of a variable magnetic field. An induced electric current known as Hall current exists due to the presence of both electric field and magnetic field. As the partial differential equations governing this problem are highly non-linear, they are solved numerically using a finite difference scheme. Further we have investigated the effects of various parameters on the velocity, temperature and concentration profile. The skin friction and the rate of mass transfer is calculated using Newton's interpolation formula. We have noted that the Hall current, rotation, Eckert number, injection and Schmidt number affect the velocity, temperature and concentration profiles.

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current, mass transfer

Nomenclature

Roman Symbol	**Quantity**
B	Magnetic flux density, Wb/m2
C^{*}	Dimensional concentration of the injected material, kg/m3
C^{*}_{w}	Concentration of the injected material at the plate, kg/m3
C^{*}_{∞}	Concentration of the injected material in free stream, kg/m3
C_p	Specific heat J/kg K
D	Diffusion coefficient, m2/s
e	electric charge Coulombs/m3
E	The electric field strength, V/m
g	Acceleration due to gravity, m/s2
H	The magnetic field strength, Wb/m2
$H_{x^{*}} H_{y^{*}} H_{z^{*}}$	Components of magnetic field strength
J	The electric current density, A/m2
$j_{x^{*}} j_{y^{*}} j_{z^{*}}$	Components of the current density
k	Thermal conductivity, w/m. K
L	Characteristic length, m
P_e	The electric pressure, N/m2
q	The velocity vector of the fluid, m/s
u^{*}, v^{*}, w^{*}	Components of velocity vector
Q^{*}	Internal heat generation, w/m3
$Q1^{*}$	Coefficient of proportionality for absorption radiation
t^{*}	Dimensional time, s
T^{*}	Dimensional temperature K
T^{*}_{w}	Temperature of the fluid at the plate, K
T^{*}_{∞}	The temperature of the fluid in the free stream, K
u^{*}_{0}	Dimensional Injection velocity, m/s
V_c	electron-atom collision frequency, Hz
x^{*}, y^{*}, z^{*}	Cartesian co-ordinates
Greek Symbols	
β	Coefficient of volumetric expansion
β_c	Coefficient of volumetric expansion due to concentration gradient

δ	Heat source parameter
α	Electrical conductivity, α/m
ρ	Fluid density kg/m3
ρ_e	Space Charge, cbs
τ_e	Collision time of electrons, s
τ_i	Collision time of ions, s
μ_e	Magnetic permeability, H/m
υ	Kinematics viscosity m2/s
ω	Cyclotron frequency, Hz
ω_e	Electron cyclotron frequency, Hz
ω_i	Ion cyclotron frequency, Hz
σ_{ji}	Stress tensor
δ_{ji}	Kronecker delta
η	Dynamic viscosity
Ω	Angular Velocity, m/s
Dimensionless Quantities	
C	Concentration of the injected material, kg/m3
u_0	Injection velocity, m/s
θ	Dimensionless temperature of the fluid, K
t	Dimensionless time, s
u, v, w	Dimensionless velocity components
x, y, z	Dimensionless Cartesian Coordinates
E_c	Eckert number$\left(= \frac{U^2}{c_p(T - T_\infty)}\right)$
E_r	Rotational parameter $\left(= \frac{\Omega \upsilon}{U_0^2}\right)$
G_c	Modified Grashof number $\left(= \frac{\upsilon g \beta_c (C^{\square} - C^{\square}_{\infty})}{U^3}\right)$
G_r	Grashof number $\left(= \upsilon g B \frac{(T^{\square} - T^{\square}_{\infty})}{U^3}\right)$
M	Magnetic parameter
m	Hall parameter
n	Ion-slip parameter
N_u	Nusselt number $\left(= \frac{hL}{\kappa}\right)$
P_r	Prandtl number $\left(= \frac{\mu c_p}{\kappa}\right)$
S_h	Sherwood number$\left(= \frac{hL}{D}\right)$
S_c	Schmidt number $\left(= \frac{v}{D}\right)$

1. Introduction

The theoretical study of MHD flows has been a subject of great interest due to it's widely spread application on designing of cooling systems with liquid metals, petroleum industry, purification of crude oil, separation of matter from fluids and many other applications. Rotation plays an important role in various phenomena like meteorology, geophysical fluids dynamics, gaseous and nuclear reactions.

MHD flows with Hall current effect are encountered in power generators, Magnetohydrodynamics accelerators, refrigeration coils and electric transformers. Study of effects of Hall currents on flows have been done by various people. [1] discussed series solution of hydromagnetic flow heat transfer with Hall effect in a second grade fluid over a stretching sheet. [3] discussed effect of Hall current and heat transfer on rotating flow on a second grade fluid through a porous medium. [8] discussed the behavior of Turbulent Flow with Conjugate Heat Transfer through a Parallel Plate. [6] presented their work on MHD Stokes free convection flow past an infinite vertical porous plate subjected to constant heat flux with ion-slip current and radiation absorption. $H_x□H_y□H_z□$.

[5] studied the MHD Stokes problem for a vertical infinite plate in a dissipative rotating fluid with Hall current.

[7] analysed the Hall effect on the flow in a rotating frame of reference. [2] studied the hydromagnetic Flow Past A Porous Flat Plate With Hall Effect. [4] discussed the effect of Hall current on the MHD boundary layer flow past a semi-infinite plate.

The early works on fluid dynamics is mostly on laminar flows with very little devotion to turbulent flows but most flows of engineering importance are turbulent and hence the reason for this study. In this study the problem of MHD free convection turbulent flow of a rotating fluid past a semi-infinite plate is considered. Flow Analysis and Mathematical Formulation We consider a hydromagnetic turbulent flow of incompressible viscous rotating fluid past an impulsively started semi-infinite porous plate. A variable magnetic field H is applied in a direction normal to the plate. The choice of co-ordinate is such that the $y - axis$ is taken along the plate in the vertical direction and the $x - axis$ is taken normal to the plate.

Initially temperature of the fluid and the plate are assumed to be the same. At $t > 0$, the plate starts moving impulsively on its plane at a velocity U_o. The plate is maintained at a constant temperature T_w higher than the constant temperature T_Ω of the surrounding fluid the flow is turbulent therefore there is large magnetic fields this implies that Hall currents significantly affect the flow.

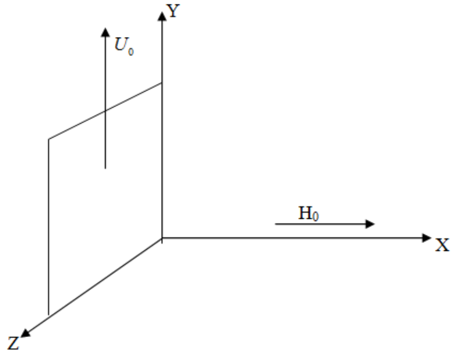


Figure 1: Geometry of the Problem

We let the fluid and the plate to be in a state of rigid rotation with uniform angular velocity Ω about the x- axis taken normal to the plate. For a non-zero rotation rate the velocity vector is of the form

$$q = u(x, y)i + v(x, y)j + w(x, y)k$$

u, is the axial velocity while v and w represent the primary and secondary flows. As the rotation is around it is of the form $\Omega = \Omega_i$

The plate is semi-infinite in extent and the flow is unsteady therefore the physical variables are functions of and x, y and$t$.

The problem is governed by the following set of equations

$$\frac{\partial H}{\partial t^\star} = \frac{1}{\mu_\ell \sigma}(\frac{\partial^2 H}{\partial x^{\star 2}}) - v^\star \frac{\partial H^\star}{\partial y^\star} - H^\star \frac{\partial v^\star}{\partial y^\star}, \tag{1}$$

$$\frac{\partial \bar{v}}{\partial t^\star} + \bar{u}\frac{\partial \bar{v}}{\partial x^\star} + v\frac{\partial \bar{v}}{\partial y^\star} + 2\Omega w^\star$$
$$= v\frac{\partial^2 \bar{v}^\star}{\partial x^{\star 2}} + \beta g(T - T_{\infty}) + \beta_\varsigma(C^\star - C^\star_{\infty}) - \frac{\partial \bar{\mu^\star} v^\star}{\partial x^\star} + \frac{\mu_0 H_0}{\rho} j_z, \tag{2}$$

$$\frac{\partial \bar{w}}{\partial t^\star} = \bar{u_0}\frac{\partial \bar{w^\star}}{\partial x^\star} + v(\frac{\partial \bar{w}}{\partial y^\star}) - 2\Omega v^\star = \nu\frac{\partial^2 \bar{w}^\star}{\partial x^{\star 2}} - \frac{\partial u^\star w^\star}{\partial x^\star} - \frac{\mu_0 H_0}{\rho} J_y, \tag{3}$$

$$\frac{\partial T^\star}{\partial t^\star} = \bar{u_0}\frac{\partial T^\star}{\partial x^\star} + \bar{v}^\star\frac{\partial T}{\partial y^\star}) = \frac{k}{pc_p}\frac{\partial^2 T^\star}{\partial x^{\star 2}} + \frac{Q^\star}{pc_p} - \frac{v}{c_p}[(\frac{\partial v^\star}{dx^\star})^2 + (\frac{\partial w^\star}{\partial x^\star})], \tag{4}$$

$$\frac{\partial C}{\partial t^\star} + \mu^\star\frac{\partial C^\star}{\partial x^\star} + v^\star\frac{\partial C^\star}{\partial y^\star} - D^\star\frac{\partial^2 C^\star}{\partial x^\star}, \tag{5}$$

Initial and boundary conditions are $t^{\square} \leq 0$:

$$\begin{array}{c} H_x^+ = H_x^+(x^+, y^+, t^+) = 0, \\ H_y^+ = 0, H_z^+ = 0 \\ v^+(x^+, y^+, t^+) = 0, w(x^+, y^+, t^+) = 0, \\ T^+(x^+, y^+, t^+) = 0, C(x^+, y^+, t^+) = 0 \end{array}, \tag{6}$$

and $t^{\square} > 0$:

$$\begin{array}{c} H_x^+ = H_x^+(0, y^+, t^+) = H_0, H_y^+ = 0, H_z^+ = 0 \\ v^+(0, y^+, t^+) = U_0, w^+(\infty, y^+, t^+) = 0, \\ T^+(0, y^+, t^+) = T_w, C^+(0, y^+, t^+) = C_w \end{array}, \tag{7}$$

$$\begin{array}{c} v^+(\infty, y^+, t^+) = 0, w^+(\infty, y^+, t^+) = 0, \\ T^+(\infty, y^+, t^+) = T_{\infty}, C^+(\infty, y^+, t^+) = C_{\infty} \end{array}. \tag{8}$$

The generalized Ohm's law including the effect of Hall current is written as

$$\hat{j} + \frac{w_e \tau_e}{H}(\hat{j} \times \hat{H}) = \sigma(\hat{E} + \mu_e \hat{q} \times \hat{H} + \frac{1}{e\eta_e} \nabla p_e). \tag{9}$$

For partially ionized fluids the electron pressure gradient may be neglected. In this case we consider a short circuit problem in which the applied electric field = 0. Thus neglecting electron pressure the x and y components become

$$\hat{j} + \frac{w_e \tau_e}{H}(\hat{j} \times \hat{H}) = \sigma \mu_e (\hat{q} \times \hat{H}). \tag{10}$$

On expanding the equation we get

$$(\hat{J}_{y^+}, \hat{J_{z^+}}) + \frac{\omega_e \tau_e}{H}(j_{z^+} H_0, -j_{y^+} H_0) = \sigma \mu_e (v^\star H_0 - w^\star H_0). \tag{11}$$

Solving the above equation for the current density components j_y and j_z

$$\hat{J}_y = \frac{\sigma \mu_e H_0 (m v^+ w^+)}{1 + m^2}, \tag{12}$$

$$\hat{j_z} = -\frac{\sigma \mu_e H_0 (v^+ - m w^+)}{1 + m^2}, \tag{13}$$

Here $m = \omega_e \tau$ is the hall parameter

Substituting equations 12 and 13 in 2 and 3 respectively we get

$$\frac{\partial \bar{v}}{\partial t^+} + \bar{u_0} \frac{\partial \bar{v}}{\partial x^\star} + v \frac{\partial^2 \bar{v}^+}{\partial y^+} = v \frac{\partial^2 \bar{v}^+}{\partial x^{+2}} + \beta g (T - T_{\infty}) + \beta_\varsigma (C^\star - C^\star_{\infty})$$

$$-\frac{\partial\overline{u^+v^+}}{\partial x^\star}-\frac{\sigma\mu_e^2H_0^2(v^+-mw^+)}{\rho(1+m^2)}, \quad (14)$$

$$\frac{\partial\bar{w}^+}{\partial t^+}+\bar{u_0}\frac{\partial\bar{w}^+}{\partial x^+}+v\frac{\partial\bar{w}^+}{\partial y^+}=v\frac{\partial^2\bar{w}^+}{\partial x^{+2}}-\frac{\partial\overline{\mu^+w^+}}{\partial x^+}-\frac{\sigma\mu_e^2H_0^2(mv^++w)}{\rho(1+m^2)}. \quad (15)$$

In a rotating frame of reference, we include in the equation of motion the Coriolis force $-2\alpha\times q$ to have the equations as

$$\frac{\partial\bar{v}^+}{\partial t^+}+\bar{u_0}\frac{\partial\bar{v}^+}{\partial x^+}+v\frac{\partial\bar{v}^+}{\partial y^+}-2\Omega w^+=v\frac{\partial^2\bar{v}^+}{\partial x^{+2}}+\beta g(T-T_\infty)+\beta_\varsigma g(C^+-C^+_\infty)-\frac{\partial\overline{u^+v^+}}{\partial x^+}$$
$$-\frac{\sigma\mu_e^2H_0^2(v^+-mw^+)}{\rho(1+m^2)}, \quad (16)$$

$$\frac{\partial\bar{w}}{\partial t^+}+\bar{u_0}\frac{\partial\bar{w}}{\partial x^+}+v\frac{\partial\bar{w}^+}{\partial y^+}+2\Omega v^+=v\frac{\partial^2\bar{w}^+}{\partial x^{+2}}$$
$$-\frac{\partial\overline{u^+w^+}}{\partial x^+}-\frac{\sigma\mu_e^2H_0^2(mv^++w^+)}{\rho(1+m^2)}. \quad (17)$$

We adopt the Boussinessque approximation

$$\tau=-\rho\overline{vw}=A\frac{\partial\bar{v}}{\partial z}, \quad (18)$$

From experiment Prandtl deduced that

$$\rho\bar{v}\bar{w}=-\rho l^2(\frac{\partial\bar{v}}{\partial z})^2, \quad (19)$$

Taking that $l=kz$ where k is the von Karman constant so we have

$$\overline{uv}=-k^2x^2(\frac{\partial\bar{v}}{\partial x})^2, \quad (20)$$

$$\bar{u}\bar{w}=-k^2x^2(\frac{\partial\bar{w}}{\partial x})^2, \quad (21)$$

Substituting in equations 16 and 17 we get

$$\frac{\partial\bar{v}^+}{\partial t^+}+\bar{u_0}\frac{\partial\bar{v}^+}{\partial x^+}+v\frac{\partial\bar{v}^+}{\partial y^+}-2\Omega w^+=v\frac{\partial^2\bar{v}^+}{\partial x^{+2}}+\beta g(T-T_\infty)+\beta_\varsigma g(C^+-C^+_\infty)$$
$$-\frac{\partial}{\partial x}(k^2x^2(\frac{\partial v^+}{\partial x^+})^2)-\frac{\sigma\mu_e^2H^2(v^+-mw^+)}{\rho(1+m^2)}, \quad (22)$$

$$\frac{\partial\bar{w}^+}{\partial t^+}+\bar{u_0}\frac{\partial\bar{w}^+}{\partial x^+}+v\frac{\partial\bar{w}^+}{\partial y^+}-2\Omega w^+ = v\frac{\partial^2\bar{w}^+}{\partial x^{+2}}-\frac{\partial}{\partial x}(k^2x^2(\frac{\partial v^+}{\partial x^+})^2)-\frac{\sigma\mu_e^2H^2(v^+-mw^+)}{\rho(1+m^2)}. \quad (23)$$

In this study non-dimensionalization is based on the following non- dimensional quantities

$$\left\{t=\frac{t^+U^2}{\varsigma}\right\}. \quad (24)$$

1.1. Final Set of Non-Dimensional Governing Equations

$$\frac{\partial H}{\partial t}=\frac{1}{R_m}\frac{u^2}{\nu^2}(\frac{\partial^2 H}{\partial x^2})-v\frac{\partial H}{\partial y}-H\frac{\partial v}{\partial y}, \quad (25)$$

$$\frac{\partial v}{\partial t}+u_0\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}-2E_rw=(\frac{\partial^2 v}{\partial x^2})+2k^2x(\frac{\partial v}{\partial x})^2+2k^2x^2(\frac{\partial^2 v}{dx^2})(\frac{\partial v}{\partial x}) +G_r\theta+G_\varsigma C-\frac{M^2H^2(v-mw)}{(1+m^2)}, \quad (26)$$

$$\frac{\partial w}{\partial t}+u_0\frac{\partial w}{\partial x}+v\frac{w}{\partial y}-2E_rw=(\frac{\partial^2 v}{\partial x^2})+2k^2x(\frac{\partial w}{\partial x})^2+2k^2x^2(\frac{\partial^2 w}{dx^2})(\frac{\partial w}{\partial x}) -\frac{M^2H^2(mw+w)}{(1+m^2)}, \quad (27)$$

$$\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{pr}\frac{\partial^2\theta}{\partial x^2}-\frac{\delta}{pr}\theta+Q_1C+E_\varsigma[(\frac{\partial v}{\partial x})^2+(\frac{\partial w}{\partial x})^2], \quad (28)$$

$$\frac{\partial c}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=sc\frac{\partial^2 C}{\partial x^2}. \quad (29)$$

The initial and boundary conditions 6 to 8 in non-dimensional form becomes

$$t\le 0: \begin{array}{c} H_x=H_x(x.y,0)=0, H_y=0, H_z=0 \\ v(x.y,0)=0, w(x.y,0), \theta(x.y,0)=0, C(x.y,o)=0 \end{array}, \quad (30)$$

$$t>0: \begin{array}{c} H_x=H_x(0,y,t)=1, H_y=0, H_z=0 \\ v(0.y,t)=1, w(o.y,t)=0, \theta(0.y,t)=1C(0.y,t)=1 \end{array}, \quad (31)$$

$$\begin{array}{c} H_x = H_x(\infty.y,t) = 0, H_y = 0, H_z = 0 \\ v(\infty.y,t) = 0, w(\infty.y,t) = 0, C(\infty.y,t) = 0 \end{array}, \tag{32}$$

Method of solution Equations 25 to 29 are non-linear thus analytical solution of the governing equations is not possible we therefore solve them by finite difference method. The equations are solved subject to the initial and boundary conditions. (30, 31, 32). In finite difference form Equations 25 to 29

$$v^{k+1}(i,j) = \triangle t \left[\begin{array}{c} \frac{1}{RM} \frac{H^k(i+1,j-2H^k(i,j)+H^k(i-1,j)}{(\Box x^2)} \\ -v^{k(i,j)} \frac{H^k(i+1,j)-H^k(i-1,j)}{2\Box y} \\ -H^k(i,j) \frac{v^k(i+1,j)-v^k(i-1,j)}{2\Box y} \end{array} \right] + H^k(i,j), \tag{33}$$

$$v^{k+1}(i,j) = \triangle t \left[\begin{array}{c} -u_o \frac{v^k(i+1,j)-v^k(i-1,j)}{2\Box x} \\ -v^k(i,j) \frac{v^{k+1}(i,j+)-v^k(i,j-1)}{2\Box y} \\ + \frac{v^k(i+1,j)-2v^k(i,j)+v^k(i-1,j)}{(\Box x)^2} \\ +2k^2 i\triangle x \left(\frac{v^k(i+1,j)-v^k(i-1,j)}{2\Box x} \right) \\ +2k^2(i\triangle x)^2 \left(\frac{v^k(i+1,j)-2v^k(i-1,j)+v^k(i-1,j)}{(\Box x)^2} \right)^2 \\ \left(\frac{v^k(i+1,j)-v^k(i-1,j)}{2\Box x} \right) \\ +2E_r w + Gr\theta^k(i,j) \\ -M^2(H^k(i,j))^2 \left(\frac{v^k(i,j)-mw^k(i,j)}{1+m^2} \right) \end{array} \right] + v^k(i,j), \tag{34}$$

$$w^{k+1}(i,j) = \triangle t \left[\begin{array}{c} -u_0 \frac{w^k(i+1,j)-w^k(i-1,j)}{2\Box x} \\ -v^k(i,j) \frac{w^{k+1}(i,j+1)-w^k(i,j-1)}{2\Box y} \\ + \frac{w^k(i+1,j)-2w^k(i,j)+w^k(i-1,j)}{(\Box x)^2} \\ +2k^2 i\triangle x \left(\frac{w^k(i+1,j)-w^k(i-1,j)}{2\Box x} \right)^2 \\ +2k^2(i\triangle x)^2 \left(\frac{w^k(i+1,j)-2w^k(i,j)+w^k(i-1,j)}{(\Box x)^2} \right) \\ \left(\frac{w^k(i+1,j)-w^k(i-1,j)}{2\Box x} \right) \\ -2E_r v + Gr\theta^k(i,j) \\ -M^2(H^2(i,j))^2 \left(\frac{mv^k(i,j)-w^k(i,j)}{1+m^2} \right) \end{array} \right] + w^k(i,j), \tag{35}$$

$$\theta^{k+1}(i,j) = \triangle t \left[\begin{array}{c} -u_0 \frac{\theta^k(i+1,j)-\theta^k(i-1,j)}{2\Box x} \\ -v^k(i,j)\frac{\theta^{k+1}(i,j+1)-\theta^k(i,j-1)}{2\Box y} \\ +\frac{1}{Pr}\left(\frac{\theta^k(i+1,j)-2\theta^k(i,j)+\theta^k(i-1,j)}{(\Box x)^2}\right) \\ -\frac{\delta}{Pr}\theta^k(i,j) + Ec \\ \left[\left(\frac{v^k(i+1,j)-v^k(i-1,j)}{2\Box x}\right) + \left(\frac{w^k(i+1,j)-w^k(i-1,j)}{2\Box x}\right)\right] \end{array} \right] + \theta^k(i,j), \tag{36}$$

$$C^{k+1}(i,j) = \triangle t \left[\begin{array}{c} -u_0 \frac{C^k(i+1,j)-C^k(i-1,j)}{2\Box x} \\ -v^k(i,j)\frac{C^k(i,j-1)}{2\Box y} \\ +Sc\left(\begin{array}{c} \frac{C^k(i+1,j)-2C^k(i,j)+C^k(i-1,j)}{(\Box x)^2} \\ +\frac{C^k(i,j+1)-2C^k(i,j)+C^k(i,j-1)}{(\Box y)^2} \end{array} \right) \end{array} \right] + C^k(i,j), \tag{37}$$

i, j, k refer to x, y, and t respectively.

The plate is impulsively started and therefore, the velocity at $x = 0$ changes from zero at $t < 0$ to 1.The concentration of the injected substance changes suddenly from its zero value at $t < 0$, 0 to 1. The initial conditions take the form

$$\begin{array}{llllll} Atx = 0 & H^0(0,j) = 1 & v^0(0,j) = 1 & w^0(0,j) = 0 & \theta^0(0,j) = 1 & C^0(0,j) = 1 \\ Aty > 0 & H^0(i,j) = 0 & v^0(i,j) = 0 & w^0(i,j) = 0 & \theta^0(i,j) = 0 & C^0(i,j) = 0 \end{array}$$

for all $i > 0$ and all j.

The boundary conditions are:

$$\begin{array}{lll} Atx = 0H^k(0,j) = 1 & v^k(0,j) = 1w^k(0,j) = 0 & \theta^k(0,j) = 1C^k(0,j) = 1 \\ Aty = 0H^0(i,0) = 0 & v^0(i,j) = 0w^0(i,j) = 0 & \theta^0(i,j) = 0C^0(i,j) = 0 \end{array}. \tag{38}$$

for all k.

Computer is used to generate numerical solutions.

The computations are made with small values of $\triangle t$. In our case we used$\triangle t = 0.000125$ and$\triangle x = \triangle y = 0.05$. We shall regard$x = 1.15$, i.e. $i = 23$ as corresponding to$x = \infty$ and therefore set$v^k(23,j) = w^k(23,j) = \theta^k(23,j)$. This is chosen because v, w and θtend to zero at around $x = 2$. The velocities $v^{k+1}(i,j)$ and $w^{k+1}(i,j)$ at the end of time step$\triangle t$ is computed from equation 34 and 35in terms of velocity and temperature at earlier time step. Similarly $\theta^{k+1}(i,j)$is computed from equation (36) $C^{k+1}(i,j)$ and from equation (37). This procedure is repeated until $k = 320$ i.e.$t = 0.04$ for $j = 1$ i.e. $y = 0.05$. In our calculation the Plandtl number is taken as 0.71 which corresponds to air and the magnetic parameter $M^2 = 50.0$ which signifies strong magnetic field. Two Case are considered

1. When Grashof number, $Gr > 0(5.0)$ corresponding to cooling of the plate
2. When the Grashof number $Gr < 0(-0.5)$ corresponding to heating of the plate

1.2. Stability and Convergence

To ensure stability and convergence, which ensures that the finite difference scheme, the program is run using smaller values of$\triangle t = 0.0008, 0.0006, 0.001$. It is observed that there were no significant changes in the results, which 'ensure that the finite difference method used in the problem converge and is stable

1.3. Calculation of the Skin Friction and Rate of Mass Transfer

The skin friction is calculated from the velocity profiles using the equations

$$\tau_y = -\frac{\partial v}{\partial x} \mid_{x=0}, \tag{39}$$

and

$$\tau_z = -\frac{\partial w}{\partial x} \mid_{x=0}, \tag{40}$$

where

$$\tau = \frac{\tau^+}{\rho U^2}. \tag{41}$$

These are calculated by numerical differentiation using Newton's interpolation formula over the first five points,

$$\tau_y = \frac{5}{6}\left[25v(0,j) - 48v(1,j) + 36v(2,j) - 16v(2,j) + 3v(4,j)\right], \tag{42}$$

$$\tau_z = \frac{5}{6}\left[25w(0,j) - 48w(1,j) + 36w(2,j) - 16(2,j) + 3(4,j)\right], \tag{43}$$

$$sh = \frac{5}{6}\left[25C(0,j) - 48C(1,j) + 36C(2,j) - 16C(2,j) + 3C(4,j)\right]. \tag{44}$$

Results and Discussions The system of equations (33) – (37) with the boundary conditions 38 are solved Numerically by the finite difference method. A program was run for various values of velocity, temperature and concentration profiles for the finite difference equations. Results are obtained for various values of the parameters Hall parameter m, rotation parameter Er, heat parameter δ The velocities are classified as primary (v) and secondary (w) along the$y$ and z

axes respectively. Figures 2-9 represent the behaviours of the velocity, temperature and concentration profiles for different values of the parameters associated with the governing problem. The parameters have their initial values as $m = 1.0, Er = 1.0, Rm = 5000, Ec = 0.01, U0 = 0.5, Sc = 0.4, Gc = 1.5; \delta = 1.5$ and $Q = 0.005$.

Figures 2-5 are for case of $Gr = 5.0$ and Figures 6-9 for case of $Gr = -0.5$. Table 1 - 3 displays the skin friction and rate of mass transfer. The magnetic parameter $M = 50.0$, signify a strong magnetic field and the Prandtl number, $Pr = 0.71$ correspond to air. The Grashoff number $Gr > 0$ (where 5.0 corresponding to cooling of plate by free convection flow) and Grashoff number $Gr < 0$ (where – 0.5 corresponding to heating of plate by free convection flow) have been used.

For the cooling of the plate (Gr = 5.0)

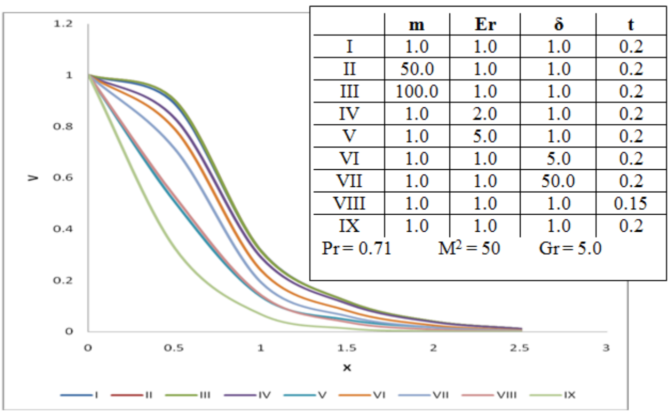


Figure 2: Primary Velocity Profiles

From Figure 2, we note that:

1. An increase in Hall current leads to an increase in primary velocity profiles. This is due to the fact that the effective conductivity decreases with the increase in Hall parameter which reduces the magnetic damping force.

2. An increase in the rotation parameter Er leads to a decrease in primary velocity profiles. The increase in frequency of oscillation decreases the thickness of the boundary layer thus increasing the velocity gradient and hence the velocity increases.

3. An increase in heat parameter leads to a decrease in primary velocity profile. The internal heat generation is increased and due to the cooling

of the plate, the rate of energy transfer is increased therefore the velocity of fluid will reduces.

4. Increase in time increases the primary velocity profiles. With time the flow gets to the free stream and therefore its velocity increases.

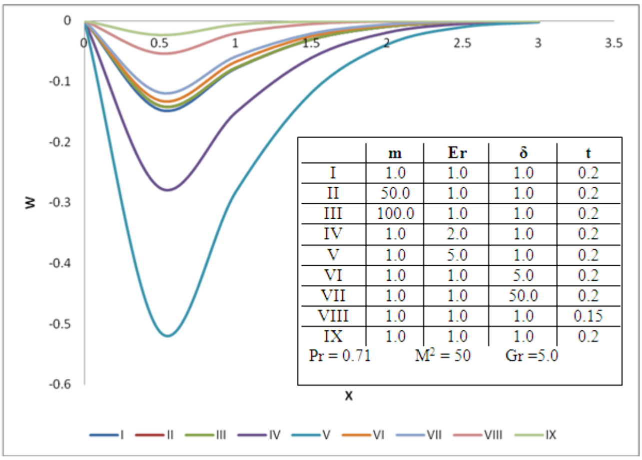


Figure 3: Secondary Velocity profile

From Figure 3, we note that:

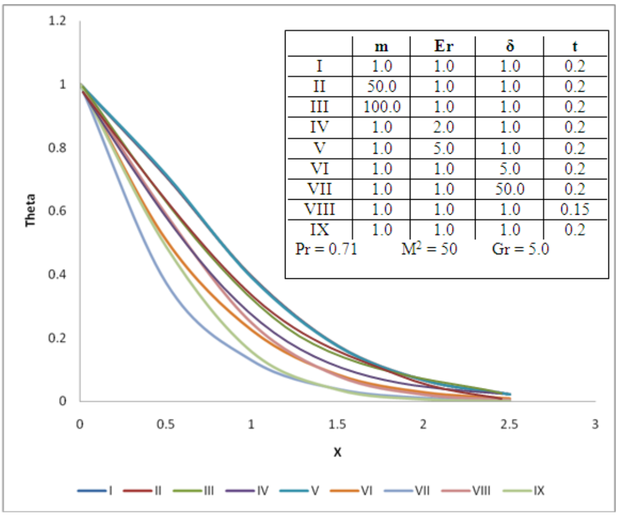
1. An increase in Hall current leads to an increase in secondary velocity profiles. This is due to decrease in the effective conductivity which decreases the magnetic damping force.

2. An increase in the rotation parameter Er leads to a decrease in secondary velocity profiles. The increase in frequency of oscillation increases the velocity gradient near the plate and hence the velocity decreases.

3. Increase in heat parameter leads a slight increase in the secondary velocity profiles. The internal heat generation is increased and due to cooling of the plate there will be high rate of energy transfer at the plate therefore the increase in secondary velocity.

4. Increase in time decreases the secondary velocity profiles. In the free stream the secondary velocity diminishes and therefore the decrease.

From Figure 4 we note that:



	m	Er	δ	t
I	1.0	1.0	1.0	0.2
II	50.0	1.0	1.0	0.2
III	100.0	1.0	1.0	0.2
IV	1.0	2.0	1.0	0.2
V	1.0	5.0	1.0	0.2
VI	1.0	1.0	5.0	0.2
VII	1.0	1.0	50.0	0.2
VIII	1.0	1.0	1.0	0.15
IX	1.0	1.0	1.0	0.2

Figure 4: Temperature Profile

1. An increase in Hall current leads to a slight increase in the temperature profiles. Increase in Hall current increases the thermal boundary layer therefore the temperature gradient decreases hence increasing the temperature of the fluid.

2. An increase in the rotation parameterEr increases the temperature profiles. The frequency of oscillation is increased thus increasing the temperature of the fliud.

3. An increase in the heat parameter δdecreases the temperature profiles. The internal heat generation is increased causing the temperature gradient to increase and hence the decrease in temperature.

4. Increase in time increases the temperature profiles. With time the flow gets to the free stream where the velocity is high the rate of energy transfer is increased and hence increases the temperature.

From Figure 5 we note that

1. An increase in Hall current leads to a slight increase in the magnetic field. Increase in Hall parameter decreases the effective conductivity which decreases the magnetic damping force and therefore the magnetic field is increased.

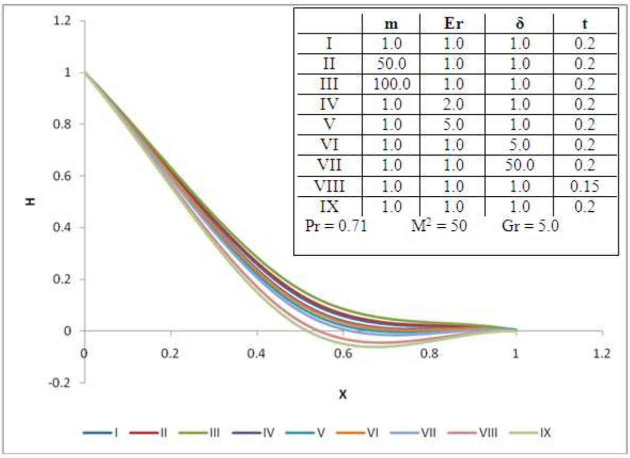


Figure 5: Magnetic field Profiles

2. An increase in the rotation parameter Er decreases the magnetic field. This is because increase in rotation parameter increases the frequency of oscillation which increases the damping force of the magnetic field.

3. An increase in the heat parameter decreases the magnetic field. The increase in heat generation reduces the magnetic field.

4. Increase in time increases the magnetic field profiles. In free stream the magnetic field is at its maximum.

From Figure 6 we note that

1. An increase in mass diffusion parameter Sc leads to an increase in the concentration profiles. This is because increase in Sc increases molecular diffusivity which results in a decrease of the concentration boundary layer hence the concentration in the concentration profile

2. Removal of injection leads to a decrease in the concentration profiles. This is due to the fact that this reduces the growth of the boundary layers and hence the decrease in the concentration profiles.

3. Increase in time increases the concentration profiles. With time the flow gets to the free stream and therefore its concentration increases.

For the heating of the plate

From figure 7, we note that:

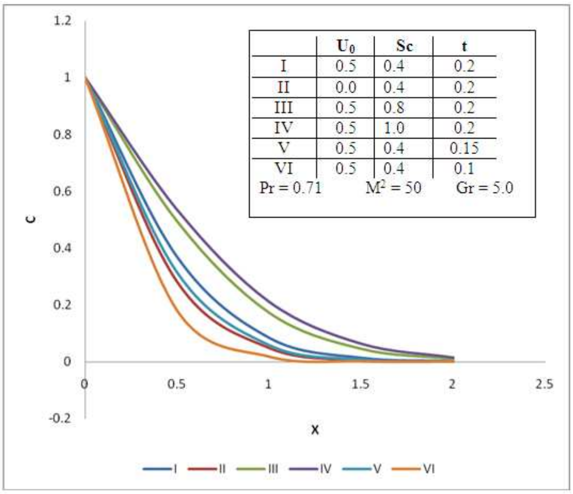


Figure 6: Concentration Profile

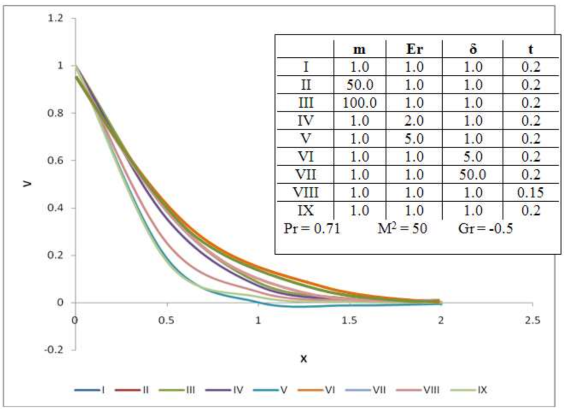


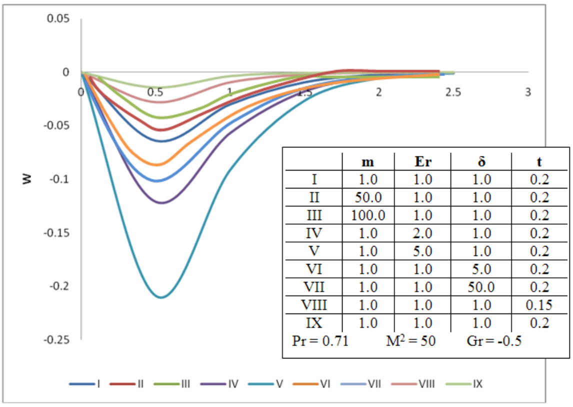
Figure 7: Primary Velocity Profile

1. An increase in Hall current leads to an increase in primary velocity profiles. This is due to the fact that the effective conductivity decreases with the increase in Hall parameter which reduces the magnetic damping force hence the increase in velocity.

2. An increase in the rotation parameter Er leads to a decrease in primary velocity profiles. The increase in frequency of oscillation decreases the thickness of the boundary layer thus increasing the velocity gradient and hence the velocity decreases.

3. An increase in heat parameter leads to an increase in primary velocity profile. The internal heat generation is increased and since the plate is being heated, the rate of energy transfer at the plate will be decreased therefore the increase in velocity.

4. Increase in time increases the primary velocity profiles. With time the flow gets to the free stream and therefore its velocity increases.



	m	Er	δ	t
I	1.0	1.0	1.0	0.2
II	50.0	1.0	1.0	0.2
III	100.0	1.0	1.0	0.2
IV	1.0	2.0	1.0	0.2
V	1.0	5.0	1.0	0.2
VI	1.0	1.0	5.0	0.2
VII	1.0	1.0	50.0	0.2
VIII	1.0	1.0	1.0	0.15
IX	1.0	1.0	1.0	0.2

Figure 8: Secondary Velocity profile

From figure 8, we note that:

1. An increase in Hall current leads to an increase in secondary velocity profiles. This is due to decrease in the effective conductivity which decreases the damping force.

2. An increase in the rotation parameter Er leads to a decrease in secondary velocity profiles. The increase in frequency of oscillation decreases the thickness of the boundary layer thus it increases the velocity gradient and hence the velocity decreases.

3. Increase in heat parameter leads a slight decrease in the secondary velocity profiles. The internal heat generation is increased and due to heating of the plate, the rate of energy transfer decreases hence the decrease in velocity.

4. Increase in time decreases the secondary velocity profiles. In the free stream the secondary velocity diminishes and therefore the decrease in its velocity.

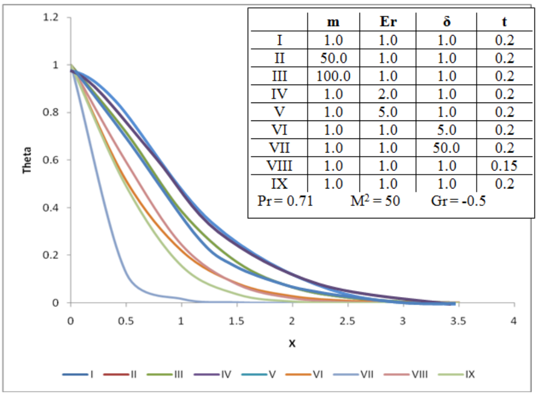


Figure 9: Temperature Profile

From Figure 9 we note that:

1. An increase in Hall current leads to a slight increase in the temperature profiles. Increase in Hall current increases the thermal boundary layer this decreases the temperature gradient and hence increase the temperature of the fluid.

2. An increase in the rotation parameter Er increases the temperature profiles. The frequency of oscillation is increased thus increase the temperature.

3. An increase in the heat parameter decreases the temperature profiles. The internal heat generation is increased which decreases the kinetic energy and hence the decrease in temperature.

4. Increase in time increases the temperature profiles. With time the flow gets to the free stream where the velocity is high the rate of energy transfer is increased and hence increases the temperature.

From Figure 10 we note that

1. An increase in Hall parameter leads to a slight increase in the magnetic field profiles. This may be attributed to the fact that increase in Hall parameter deceases the effect conductivity thus reducing the magnetic damping force hence the increase in the magnetic field.

2. An increase in the rotation parameter Er decreases the magnetic field profiles. This is due to increase in oscillations which increase the magnetic damping force.

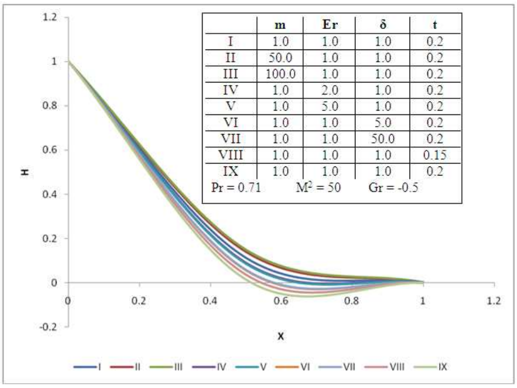


Figure 10: Magnetic field Profiles

3. An increase in the heat parameter decreases the magnetic field profiles.
4. Increase in time increases the magnetic field profiles.

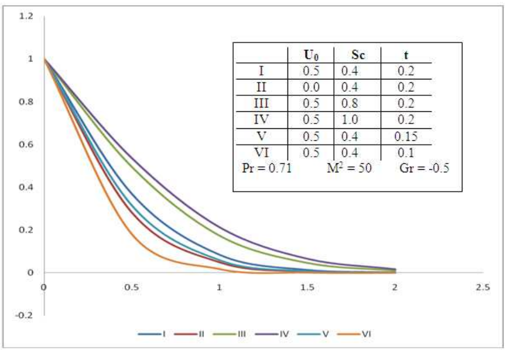


Figure 11: : Concentration Profile

From Figure 11

1. An increase in mass diffusion parameter Sc leads to an increase in the concentration profiles. This is because increase in Sc increases molecular diffusivity which results in an increase of the concentration boundary layer. Hence the concentration of the species is higher for large values of Sc.
2. Removal of injection leads to a decrease in the concentration profiles. This is due to the fact that this reduces the growth of the boundary layers and hence the decrease in the concentration profiles.

3. Increase in time increases the concentration profiles. With time the flow gets to the free stream and therefore its concentration increases.

M	Er	δ	t	τ_y	τ_Z
1.0	1.0	1.0	0.2	-6.75648	3.900639
50.0	1.0	1.0	0.2	-7.26806	3.660552
100.0	1.0	1.0	0.2	-7.26996	3.654071
1.0	2.0	1.0	0.2	-5.11567	7.250481
1.0	5.0	1.0	0.2	4.120271	13.53471
1.0	1.0	5.0	0.2	4.61625	3.552338
1.0	1.0	50.0	0.2	2.67565	3.255279
1.0	1.0	1.0	0.15	3.577195	1.588931
1.0	1.0	1.0	0.1	9.684261	0.758566

Table 2: Skin Friction $\tau_Y$andτ_Z. $Pr = 0.7$, $M2 = 5.0$ and $Gr = 5.0$

From Table 1 we observe that for Gr = 5.0

1. An increase in Hall current leads to a decrease in both $\tau_y$and τ_z, this is due to the fact that increase in Hall current increases the velocity and therefore shear resistance is decreased.

2. An increase in Er leads to an increase in both $\tau_y$andτ_z . Increase in Er decreases the velocity and therefore the shear resistance is increased.

3. An increase in heat parameterδ leads to an increase inτ_y but a slight decrease inτ_z. This is due to the effect of the heat parameter on the velocity which decreases the primary velocity but increases the secondary velocity

4. An increase in time leads a decrease in $\tau_y$but an increase in τ_z. in free stream the primary velocity is increased while the secondary velocity is decreased this leads to a decrease in the shear resistance is in the primary velocity while shear resistance is increased in the secondary velocity.

From Table 2 we observe that for $Gr = -0.5$

1. An increase in Hall current leads to a decrease in both and this is due to the fact that increase in Hall current increases the velocity and therefore shear resistance is decreased.

2. An increase in Er leads to an increase in both and. Increase in Er decreases the velocity and therefore the shear resistance is increased.

M	Er	δ	t	τ_y	τ_Z
1.0	1.0	1.0	0.2	7.751729	1.781171
50.0	1.0	1.0	0.2	7.696153	1.754483
100.0	1.0	1.0	0.2	7.695936	1.753763
1.0	2.0	1.0	0.2	8.638622	3.371353
1.0	5.0	1.0	0.2	13.54152	5.97752
1.0	1.0	5.0	0.2	7.488508	1.813468
1.0	1.0	50.0	0.2	6.724263	0.874377
1.0	1.0	1.0	0.15	12.03385	1.588931
1.0	1.0	1.0	0.1	14.5555	0.470796

Table 3: Skin Friction $\tau_Y$andτ_Z. $Pr = 0.7$, $M2 = 5.0$ and $Gr = -5.0$

3. An increase in heat parameter leads to a decrease in but a slight increase in. This is due to the effect of the heat parameter on the velocity which increases the primary velocity but decreases the secondary velocity

4. An increase in time leads a decrease in but an increase in. In free stream the primary velocity is increased which leads to a decrease in the shear stress while the secondary velocity is decreased which leads to an increase in the shear stress.

U_0	Sc	T	Sh
0.5	0.4	0.2	8.421213
0.0	0.4	0.2	10.95473
0.5	0.8	0.2	5.65355
0.5	1.0	0.2	4.947739
0.5	0.4	0.15	9.913319
0.5	0.4	0.1	14.12577

Table 4: Rate of mass transfer, Sh

From table 3 we observe that

1. Removal of injection leads to an increase in rate of mass transfer. Velocity of fluid increases with removal of injection since the flow towards the plate will be deceased and therefore the rate of mass transfer increases.

2. Increase in mass diffusion parameter Sc, leads an increase in rate of mass transfer. This is because the diffusion is increased.

3. Increase in time leads to a decrease in rate of mass transfer. With time the flow gets to the free stream where the velocity of the fluid is high and hence the rate of mass transfer.

Conclusion

From the results above we have seen that the parameters in the governing equations affect the velocity, temperature or concentration profiles. Consequently their effects alter the skin friction and the rate of mass transfer

1. An increase in the Hall current increases the primary velocity profiles, secondary velocity profiles and the temperature profiles.

2. An increase in the rotation parameters Er decreases the primary velocity profiles and secondary velocity profiles but increase in the temperature profiles.

3. While an increase in heat parameter δleads to a decrease in primary velocity profiles and an increase in secondary velocity profiles for the case of $Gr = 5.0$ it leads to an increase in primary velocity profiles and a decrease in secondary velocity profiles for $Gr = -0.5$. The temperature profiles decreases.

4. Increase in time increases the primary profiles, temperature and concentration profiles but decreases the secondary profiles. It is noticed that while the primary velocity profiles for $Gr = 5.0$ are higher compared to those of $Gr = -0.5$, the secondary velocity profiles are lower.

5. The temperature profiles for the case of $Gr = 5.0$ are lower than those for $Gr = -0.5$. This shows that in presence of Hall current cooling of the plate by free convection current decreases the thermal boundary layer.

6. The value of the skin frictionτ_y due to primary velocity for $Gr = 5.0$ are lower than the corresponding values for $Gr = -0.5$. However the values of the skin friction $\tau_z$due to secondary velocity for $Gr = 5.0$ are higher than the corresponding values for $Gr = -0.5$. This implies that whereas decrease in Gr leads to increase inτ_y it causes a decrease in τ_z.

7. Removal of injection, increase in the mass transfer parameter Sc or increase in times leads to an increase in rate of mass transfer.

When the variable magnetic field is removed and it is considered a constant in a laminar flow other than turbulent, the results agreed with those of [5] who investigated the MHD Stoke problem for a vertical infinite plate in a dissipative rotating fluid with Hall current.

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