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LYAPUNOV OPTIMIZING SLIDING MODE CONTROL FOR ATTITUDE TRACKING OF SPACECRAFT

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Abstract: This research studies the robust optimal control for attitude tracking of a rigid spacecraft. The objective is to zero the states of the attitude control system with a bounded control, while considering some state dependent cost. Lyapunov optimizing control and the concept of the descent function are employed to derive a control algorithm that offers the asymptotic stability of the origin. The proposed control includes a singular control regime that produces a switching surface, whose existence and placement is forced by function minimization in addition to stability of reduced order dynamics. The second method of Lyapunov is used to show that trajectories of the control system reach the switching surface and the asymptotic stability of the origin can be guaranteed. An example of multiaxial attitude manoeuvres is presented and simulation results are given to verify the usefulness of the proposed controller.

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Key Words: attitude tracking, Lyapunov optimizing sliding mode control, quickest descent control, descent function

1. Introduction

The optimal attitude control problem for a rigid spacecraft has been studied by using various nonlinear controller design techniques. In [1] and [2] the SDRE method was successfully applied to spacecraft attitude control. Tewari [3]

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devised a nonlinear optimal control design using the Hamilton-Jacobi formulation for spacecraft attitude manoeuvres. Krstic and Tsiotras [4] developed the inverse optimal control law was designed to solve the attitude stabilization problem of a rigid spacecraft. In [5] and [6] nonlinear H_∞ suboptimal controllers presented were applied to achieve attitude tracking. The main limitation of optimal or suboptimal control is the sensitivity to small system uncertainties or disturbances. It may fail to work properly or even cause instability when applied to a system with uncertainties or disturbances. Thus, the robust control of attitude tracking motion under these conditions has been investigated by Di Gennaro [7], and Park and Tahk [8].

As extensions of the above studies, optimal control and robust control have been merged to obtain robust optimal attitude control laws (see, e.g., [9] and [10]). An effective method to design a robust optimal controller is to use an optimal sliding mode controller design scheme. Sliding mode control (SMC) has been shown to be a potential approach when applied to a system with disturbances which satisfy the matched uncertainty condition ([11], [12]). However, the optimal sliding mode of nonlinear systems has rarely been studied. Optimal SMC was proposed in [13] to deal with the attitude tracking control problem. In Li *et al.* [14] an equivalent control and optimal sliding manifold were studied for attitude control systems. However, these researches have not considered the attitude control problem with bounded inputs. Based on Lyapunov-based control and sliding mode control techniques a robust optimal control algorithm can be developed for the attitude tracking. It is well-known that Lyapunov's second method has been extended to develop controllers for linear and nonlinear systems. One such controller is Lyapunov optimizing control (LOC) that offers feedback controls by selecting a candidate Lyapunov function and choosing the control to minimize this Lyapunov function as much as possible along system trajectories ([15] and [16]). Supposing that the target is the origin, if the candidate Lyapunov function is decreased everywhere outside the origin, a sufficient condition for asymptotic stability is satisfied [16]. In addition, the LOC approach provides an algorithm to design feedback controllers where cost accumulation is considered explicitly. An approach to merge LOC and SMC has been presented in McDonald [17] and [18] and is called Lyapunov optimizing sliding mode control (LOSMC). This resulting algorithm was applied to control linear systems with bounded disturbances. However, this technique has not been investigated for nonlinear models of real-life problems.

In this research the LOC approach and the singular (sliding mode) control concept are merged to develop a novel robust optimal controller. The resulting controller can be applied to nonlinear attitude control systems. A variant of

LOC known as quickest descent control (QDC) is used to derive a suboptimal attitude controller design for the finite-time nonlinear optimal control problem. This control algorithm offers the asymptotic stability of the origin and minimizes the given cost functional.

This paper is organized as follows. In Section 2 the dynamic equation of error rate and the kinematics of attitude error ([19], [20]) are described. Section 3 presents the LOSMC approach to develop the optimal controller. The QDC approach that provides bang-bang and singular controls is proposed. The stability proofs of the proposed control law are given in Section 4. In Section 5 an example of spacecraft attitude tracking manoeuvres is presented to verify the usefulness of the proposed controller. In Section 6 we present conclusions.

2. Mathematical Model of Spacecraft Attitude Tracking Control

2.1. Dynamic Equations of the Error Rate

In [19], the dynamic equation for a rigid spacecraft rotating under the influence of body-fixed devices is given as

$$J\dot{\omega} = -\omega^{\times} J\omega + \tau + d, \tag{1}$$

where $\omega = [\omega_1 \quad \omega_2 \quad \omega_3]^T$ is the angular rate of the spacecraft, $\tau = [\tau_1 \quad \tau_2 \quad \tau_3]^T$ represents the control vector, $d = [d_1 \quad d_2 \quad d_3]^T$ are bounded disturbances, J is the inertia matrix, and the skew-symmetric matrix $\omega^{\times}$ is defined by

$$\omega^{\times} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}. \tag{2}$$

We denote $\omega_r = [\omega_{1r} \quad \omega_{2r} \quad \omega_{3r}]^T$ as the desired reference rate and $\omega_e = \omega - \omega_r$ as the error rate. Then substitution into (1) gives the dynamic equation of the error rate in the form [10]

$$\begin{aligned} J\dot{\omega}_e \quad = \quad & -\left[(\omega_e^{\times} + \omega_r^{\times})J\omega_e + \omega_e^{\times} J\omega_r\right] + \tau + d \\ & -J\dot{\omega}_r - \omega_r^{\times} J\omega_r \end{aligned} \tag{3}$$

For the sake of simplicity, we now define the extended disturbance as [10]

$$\tilde{d} = D - \omega_r^{\times} J\omega_e - \omega_e^{\times} J\omega_r, \tag{4}$$

where $D = d - J\dot{\omega}_r - \omega_r^\times J\omega_r$ is the sum of the signal $\dot{\omega}_r$ and ω_r and the external disturbance $d(t)$. Substituting $\tilde{d}$ into (3), we obtain the dynamic equation of the error rate as

$$J\dot{\omega}_e = -\omega_e^\times J\omega_e + \tau + \tilde{d}. \tag{5}$$

The tracking problem formulation given in (5) has been discussed in [10]. It has a suitable form for the application and development of optimal feedback controllers for attitude tracking.

2.2. Kinematics of the Attitude Error

We now briefly explain the use of quaternions for description of the attitude error. We define the quaternion $Q = [q^T \quad q_4]^T \in \mathcal{R}^3 \times \mathcal{R}$ with $q = [q_1 \quad q_2 \quad q_3]^T \in \mathcal{R}^3$ and

$$Q_r = [q_r^T \quad q_{4r}]^T,$$

where $q_r = [q_{1r} \quad q_{2r} \quad q_{3r}]^T \in \mathcal{R}^3$ is the desired reference attitude. The quaternion for the attitude error is $Q_e = [q_e^T \quad q_{4e}]^T \in \mathcal{R}^3 \times \mathcal{R}$ with $q_e = [q_{1e} \quad q_{2e} \quad q_{3e}]^T \in \mathcal{R}^3$. Using the multiplication law for quaternions, we then obtain

$$Q_e = \begin{bmatrix} q_{4r}q - q_4 q_r - q_r^\times q \\ q_4 q_{4r} + q^T q_r \end{bmatrix} \tag{6}$$

subject to the constraint

$$Q_e^T Q_e = (q^T q + q_4^2)(q_r^T q_r + q_{4r}^2) = 1 \tag{7}$$

The kinematic equation for the attitude error can then be expressed as (see, e.g., [19], [20])

$$\dot{q}_e = \frac{1}{2} T(Q_e)\omega_e \tag{8a}$$

$$\dot{q}_{4e} = -\frac{1}{2} q_e^T \omega_e \tag{8b}$$

where $T(Q_e) = q_e^\times + q_{4e} I_{3\times 3}$ with $I_{3\times 3}$ the 3×3 identity matrix.

To avoid the singularity of $T(Q_e)$ that will occur at $q_{4e} = 0$, we let the attitude of the spacecraft be restricted to the workspace W defined by [18]

$$\begin{aligned} W &= \{Q_e | Q_e = [q_e^T \quad q_{4e}]^T, \|q_e\| \le \beta < 1, \\ &\qquad q_{4e} \ge \sqrt{1 - \beta^2} > 0\}, \end{aligned} \tag{9}$$

where β is a positive constant.

After taking the time derivative of (8a) and premultiplying both sides of the result by $T^{-1}(Q_e)JT(Q_e)$, one obtains [20]

$$J_e\ddot{q}_e = \frac{1}{2}J_e\dot{T}(Q_e)\omega_e + \frac{1}{2}P^T J\dot{\omega}_e, \tag{10}$$

where $J_e(Q_e) \in \mathcal{R}^{3\times3}$ is an auxiliary matrix defined as $J_e = P^T JP$ and $P(Q_e) \in \mathcal{R}^{3\times3}$ is defined as $P = T^{-1}(Q_e)$. Then, after substituting (5) into (10), one obtains

$$J_e\ddot{q}_e = -C^*(q_e, q_{4e}, \dot{q}_e)\dot{q}_e + u + \xi, \tag{11}$$

where the new control input $u \in \mathcal{R}^3$ and the new disturbance vector are defined as

$$u = \frac{1}{2}P^T\tau \quad \text{and} \quad \xi = \frac{1}{2}P^T\tilde{d} \tag{12}$$

and $C^*(q_e, q_{4e}, \dot{q}_e) \in \mathcal{R}^{3\times3}$ is defined as follows:

$$C^*(q_e, q_{4e}, \dot{q}_e) = -J_e\dot{P}^{-1}P - 2P^T(JP\dot{q}_e)^\times P. \tag{13}$$

Let $x = [x_1^T \quad x_2^T]^T = [q_e^T \quad \dot{q}_e^T]^T \in \mathcal{R}^6$ and ignore the external disturbance, then transform system (12) into the new coordinate as

$$\dot{x} = F(x) + B(x)u + B(x)\xi \tag{14}$$

where

$$F(x) = \begin{bmatrix} x_2 \\ -J_e^{-1}C^*(q_e, q_{4e}, \dot{q}_e)x_2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0_{3\times3} \\ J_e^{-1} \end{bmatrix}.$$

In (14) we suppose that the i^{th} component of disturbance vector ξ is bounded by $\|\xi_i\| \leq d_{max}, \quad i = 1, 2, 3$. According to the results of [21], the following properties of dynamic mode in (13) will be used in the subsequent stability analysis.

Proposition 1. *The inertia and centripetal-Coriolis matrices satisfy the following skew symmetric relationship*

$$\vartheta^T(\frac{1}{2}\dot{J}_e - C^*)\vartheta = 0, \quad \forall\vartheta \in \mathcal{R}^3. \tag{15}$$

Proposition 2. *The norm of the centripetal-coriolis matrix can be upper bounded as*

$$\|C^*(q_e, q_{4e}, \dot{q}_e)\| \leq \zeta\|\dot{q}_e\|, \tag{16}$$

where ζ is a positive constant.

Proposition 3. *The inertia matrix J_e is symmetric and positive definite, and satisfies the following inequalities*

$$m_1\|\vartheta\|^2 \le \vartheta^T J_e \vartheta \le m_2\|\vartheta\|^2, \quad \forall \vartheta \in \mathcal{R}^3 \tag{17}$$

where m_1 and m_1 are positive constants.

3. Lyapunov Optimizing Sliding Mode Control

In this section we discuss an optimal control law minimizing the performance index

$$I = \int_0^{t_f} x^T \Psi x dt \tag{18a}$$

$$\text{where} \quad \Psi = \frac{1}{2} x^T \frac{\partial^2 \Psi}{\partial x^2} x \tag{18b}$$

which is associated with driving the state $x \in \mathcal{R}^n$ form some initial point to the origin. The matrix $\frac{\partial^2 \bar{\Psi}}{\partial x^2} \in \mathcal{R}^{n\times n}$ is constant, symmetric and positive definite matirx. We consider the attitude control system

$$\dot{x} = F(x) + B(x)u, \tag{19}$$

which ignores the disturbance term. Here, the i^{th} component of the control variable u is restricted by

$$-u_{max} \le u_i \le u_{max}, \quad i = 1, 2, \dots m \tag{20}$$

where u_{max} is a positive constant.

Quickest descent control (QDC) offers control algorithms that consider cost accumulation, control bounds and stability of the origin. QDC feedback controls are designed by first choosing a descent function W. According to the work by [16] a function $W(x)$ is a descent function if the following properties hold. 1) $W(0) = 0$, 2) $W(x) > 0$ $\forall x \ne 0$ and 3) $\frac{\partial W}{\partial x} \ne 0 \quad \forall x \ne 0$. Next, the control u for the optimal control problem (18)-(20) is chosen to decrease W as quickly as possible along trajectories $x(t)$. For that reason, u is selected by

$$\min_u \frac{dW}{dt}, \tag{21}$$

where the cost accumulation rate Ψ is chosen as the descent function. Thus, letting $W(x) = \Psi(x)$ the control is chosen as

$$\min_u \dot{\Psi}, \tag{22}$$

where $\dot{\Psi} = \dfrac{d\Psi}{dt} = \dfrac{dW}{dt}$. Next, we employ the QDC algorithm to design a supoptimal controller. Using the condition in [17], for the tracking system (14) the function $W(x)$ can be defined as

$$W(x) = \frac{1}{2}ax_1^T x_1 + bx_1^T J_e x_2 + cx_2^T J_e x_2. \tag{23}$$

Clearly, the condition $acJ_e > b^2 J_e$ is required to ensure that $W(x)$ is positive definite.

Letting $f(x) = -J_e^{-1}C^*(q_e, q_{4e}, \dot{q}_e)x_2$ and taking the time derivative of $W(x)$, we obtain

$$\begin{aligned}\dot{W}(x) &= \left[\frac{\partial W}{\partial x}\right]^T \dot{x} \\ &= \left[\left[\frac{\partial W}{\partial x_1}\right]^T \quad bx_1^T J_e + cx_2^T J_e\right]\begin{bmatrix} x_2 \\ f(x) + J_e^{-1}u\end{bmatrix} \\ &= \left[\frac{\partial W}{\partial x_1}\right]^T x_2 + (bx_1^T J_e + cx_2^T J_e)(f(x) + J_e^{-1}u). \end{aligned} \tag{24}$$

According to the basic concept in [16], the switching function $\sigma(x) \in \mathcal{R}^3$ is defined as

$$\begin{aligned}\sigma(x) &= \left[\frac{\partial W}{\partial u}\right]^T \\ &= bx_1 + cx_2. \end{aligned} \tag{25}$$

Applying the basic concept in [16], we obtain the QDC law

$$u_i(x) = \begin{cases} u_{max} & \text{for} \quad \sigma_i(x) < 0 \\ \bar{u}_i & \text{for} \quad \sigma_i(x) = 0 \\ -u_{max} & \text{for} \quad \sigma_i(x) > 0 \end{cases} \tag{26}$$

where $\bar{u}_i$ denotes the i^{th} component of the singular control $\bar{u}$ $(i = 1, 2, 3)$ and the switching surface is defined by $S \equiv \{x : \sigma(x) = 0\}$. Using the equivalent control method the singular control $\bar{u}$ requires $\sigma = \dot{\sigma} = 0$, which from (25), is

$$\begin{aligned}\dot{\sigma}(x) &= b\dot{x}_1 + c\dot{x}_2 \\ 0 &= bx_2 + c\left(f(x) + J_e^{-1}\bar{u}\right). \end{aligned} \tag{27}$$

Solving for $\bar{u}$, one obtains

$$\bar{u} = J_e\left(-\frac{b}{c}x_2 - f(x)\right). \tag{28}$$

The control law (26) switches discontinuously across the switching surface $\sigma(x) = 0$. A control $u(x) = \bar{u}(x)$ results in $\sigma(x) = 0$ for a nonzero time interval. The singular control over a nonzero time interval requires $\sigma = \dot{\sigma} = 0$.

4. Stability Results

The proposed control law is useful only if it can be shown that the trajectories of closed-loop system under the control bound (20) reach the switching surface S and then asymptotically approach the origin.

We first prove that there exist initial states whose trajectories under the QDC law reach the switching surface S.

Lemma 4. *With the descent function (23), for nonlinear system (19) subject to the control bound (20) the state may be upper bounded by* $\|x_1(0)\|$ *and the maximum value of* $\|\sigma\|$ *attained in that time interval.*

Proof. Using the first component of $\dot{x}$ in (14), one has

$$\dot{x}_1 = x_2 = -\frac{b}{c}x_1 + \frac{\sigma}{c}. \tag{29}$$

We can express the solution to (29) to obtain

$$x_1(t) \leq \exp(-\frac{b}{c}t)x_1(0) + \frac{1}{c}\int_0^t \exp(-\frac{b}{c}(t-s))\sigma(s)ds. \tag{30}$$

Taking the norm of both sides we have

$$\begin{aligned} \|x_1(t)\| &\leq \|\exp(-\frac{b}{c}t)\|\|x_1(0)\| + \frac{1}{c}\int_0^t \|\exp(-\frac{b}{c}(t-s))\|\|\sigma(s)\|ds \\ &\leq \|x_1(0)\| + \frac{1}{c}\max_{0\leq s\leq t}\|\sigma(s)\|\int_0^\infty \|\exp(-\frac{b}{c}(t-s))\|ds \end{aligned} \tag{31}$$

and

$$\begin{aligned} \|x_2(t)\| &\leq \|-\frac{b}{c}x_1 + \frac{1}{c}\sigma\| \\ &\leq \|\frac{b}{c}x_1\| + \|\frac{\sigma}{c}\| \\ &\leq \frac{b}{c}\|x_1\| + \frac{1}{c}\max_{0\leq s\leq t}\|\sigma\| \\ &\leq \frac{b}{c}\left[\|x_1(0)\| + \frac{1}{c}\max_{0\leq s\leq t}\|\sigma(s)\|\int_0^\infty \|\exp(-\frac{b}{c}(t-s))\|ds\right] \end{aligned}$$

$$+ \max_{0 \le s \le t} \frac{\|\sigma\|}{c} \tag{32}$$

Using $\|x\| \le \|x_1\| + \|x_2\|$, the state $x(t)$ is bounded by

$$\begin{aligned} \|x(t)\| \;\; & \le \;\; \left(\frac{b}{c}+1\right)\left[\|x_1(0)\| + \max_{0\le s\le t}\frac{\|\sigma(s)\|}{c}\int_0^\infty \|\exp(-\frac{b}{c}(t-s))\|ds\right] \\ & + \max_{0\le s\le t}\frac{\|\sigma\|}{c}, \end{aligned} \tag{33}$$

which completes the proof. □

The following theorem provides the condition for values of u_{max} to ensure that sliding mode is attained.

Theorem 5. *All trajectories $x_1(0)$ and u_{max} that satisfy*

$$\begin{aligned} u_{max} \;\; \ge \;\; & \frac{b}{c}\lambda_{max}(J_e)\left[\frac{b}{c}\left(\|x_1(0)\| + \frac{\sigma_{max}}{c}\int_0^\infty \|h(s)\|ds\right) + \frac{\sigma_{max}}{c}\right] \\ & + b\zeta\left[\frac{b}{c}\left(\|x_1(0)\| + \frac{\sigma_{max}}{c}\int_0^\infty \|h(s)\|ds\right) + \frac{\sigma_{max}}{c}\right] \\ & \times \left[\|x_1(0)\| + \frac{\sigma_{max}}{c}\int_0^\infty \|h(s)\|ds\right] + d_{max} \end{aligned} \tag{34}$$

reach the surface S. Here, $\lambda_{max}(J_e)$ denotes maximum singular value of the matrix J_e, $h(s) = exp(-\frac{b}{c}(t-s))$ and $\sigma_{max} = \max_{0\le s\le t}\|\sigma(s)\|$.

Proof. We now choose a Lyapunov function as

$$V = \frac{1}{2}\sigma^T J_e \sigma. \tag{35}$$

The time derivative of V can be expressed as

$$\dot{V} = \sigma^T J_e \dot{\sigma} + \frac{1}{2}\sigma^T \dot{J}_e \sigma. \tag{36}$$

Substituting (25) and (27) into (36), $\dot{V}$ can be rearranged as

$$\dot{V} = \sigma^T\left[bJ_e x_2 + u + d\right] - c\sigma^T C^* x_2 + \frac{1}{2}\sigma^T \dot{J}_e \sigma. \tag{37}$$

Substituting $x_2 = \dfrac{\sigma}{c} - \dfrac{b}{c}x_1$ into (37), now (37) becomes

$$\dot{V} \;\; = \;\; \sigma^T\left[bJ_e x_2 + u + d\right] - c\sigma^T C^*(\frac{\sigma}{c} - \frac{b}{c}x_1) + \frac{1}{2}\sigma^T \dot{J}_e \sigma$$

$$= \sigma^T \left[bJ_e x_2 + u + d \right] - \sigma^T C^* \sigma + \frac{1}{2}\sigma^T \dot{J}_e \sigma + b\sigma^T C^* x_1. \quad (38)$$

Using the property 1 implies $\sigma^T(\frac{1}{2}\dot{J}_e - C^*)\sigma = 0$, we obtain

$$\dot{V} = \sigma^T \left(J_e x_2 + bC^* x_1 + u + d \right). \quad (39)$$

For the region $\sigma < 0$, we can obtain $\dot{\sigma} > 0$ by choosing u_{max} as

$$u_{max} \geq b\lambda_{max}(J_e)\|x_2\| + b\|C^*\|\|x_1\| + d_{max}. \quad (40)$$

Using the property 2 one obtains $\|C^*(q_e, q_{4e}, \dot{q}_e)\| \leq \zeta\|x_2\|$ and u_{max} can be lower bounded as

$$u_{max} \geq b\lambda_{max}(J_e)\|x_2\| + b\zeta\|x_1\|\|x_2\| + d_{max}. \quad (41)$$

On the other hand, for the region $\sigma > 0$, we can obtain $\dot{\sigma} < 0$ by choosing $-u_{max}$ as

$$-u_{max} \leq b\lambda_{max}(J_e)\|x_2\| + b\zeta\|x_1\|\|x_2\| + d_{max}, \quad (42)$$

which can be further written as

$$u_{max} \geq b\lambda_{max}(J_e)\|x_2\| + b\zeta\|x_1\|\|x_2\| + d_{max}. \quad (43)$$

Obviously, for both cases we can use the same condition of u_{max} to obtain $\dot{V} < 0$. Using upper bounds of $\|x_1\|$ and $\|x_2\|$, one has

$$\begin{aligned} u_{max} \geq\ & \frac{b}{c}\lambda_{max}(J_e)\left[\frac{b}{c}\left(\|x_1(0)\| + \frac{\sigma_{max}}{c}\int_0^\infty \|h(s)\|ds\right) + \frac{\sigma_{max}}{c}\right] \\ +\ & b\zeta\left[\frac{b}{c}\left(\|x_1(0)\| + \frac{\sigma_{max}}{c}\int_0^\infty \|h(s)\|ds\right) + \frac{\sigma_{max}}{c}\right] \\ & \times\left[\|x_1(0)\| + \frac{\sigma_{max}}{c}\int_0^\infty \|h(s)\|ds\right] + d_{max} \end{aligned} \quad (44)$$

Therefore, the trajectories and u_{max} satisfying (34) reach the switching surface S. □

The following theorem shows asymptotic stability results of the proposed control law.

Theorem 6. *Trajectories that reach S with $\bar{u}$ satisfying (20) asymptotically approach the origin.*

Proof. Substituting (28) into (19), we obtain

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -\frac{b}{c}x_2.\end{aligned} \tag{45}$$

Solving (45) we have $x_2(t) = \exp(-\frac{b}{c}t)x_2(0)$. Clearly, $x_2(t) \to 0$ as $t \to \infty$. Also $x_1(t) \to 0$ and the proof is completed. □

Hence, the proofs of reaching the switching surface S and stability phases of the control proposed law have been conducted.

5. Simulation Results

An example of a rigid-body satellite [22] is presented with numerical simulations to verify the performance of the developed controller. The spacecraft is assumed to have the inertia matrix

$$J = \begin{bmatrix} 21 & 0.6 & 1.1\\ 0.6 & 22 & 0.3\\ 1.1 & 0.3 & 20 \end{bmatrix} \quad \text{kg}\cdot\text{m}^2.$$

Suppose that the workspace W is defined by $\beta^2 = 0.75$. For the descent function (23) the positive scalars are chosen as $a = 5.0$, $b = 1.5$ and $c = 5.0$. The initial conditions for the state are $Q_e(0) = [0.3 \quad -0.2 \quad -0.3 \quad 0.8832]^T$, $\omega_e(0) = [0.06 \quad -0.04 \quad 0.05]^T$ rad/s.

$$\omega_d(t) = \begin{bmatrix} 0.1\cos(0.025t)\\ -0.1\sin(0.02t)\\ -0.1\cos(0.0167t) \end{bmatrix} \quad \text{N-m}. \tag{46}$$

The attitude control problem is considered in the presence of external disturbance $d(t)$. The disturbance model [22] is

$$d(t) = 0.01 \times \begin{bmatrix} 3\cos(0.1t)+1\\ 1.5\sin(0.1t)+3\cos(0.1t)\\ 3\sin(0.1t)+1 \end{bmatrix} \quad \text{N-m} \tag{47}$$

As shown Figs. 1 and 2 responses of the quaternion tracking and angular velocity errors reach zero after 30 seconds. Obviously, the effect of external disturbances on both responses is totally removed. As shown in Fig. 3 the

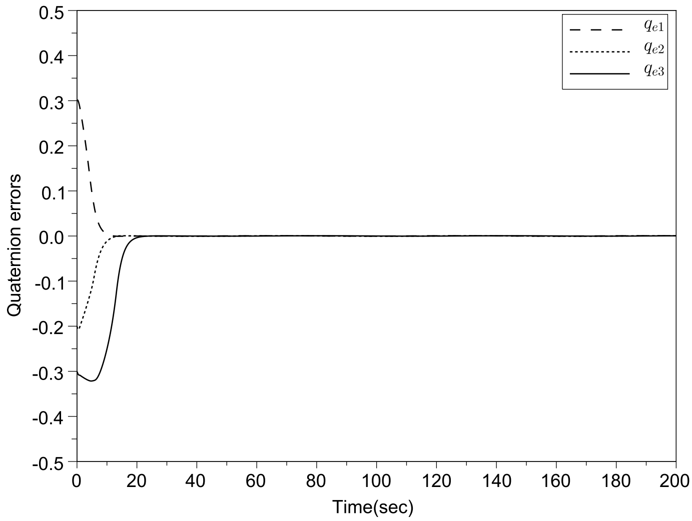

Figure 1: Quaternion tracking errors.

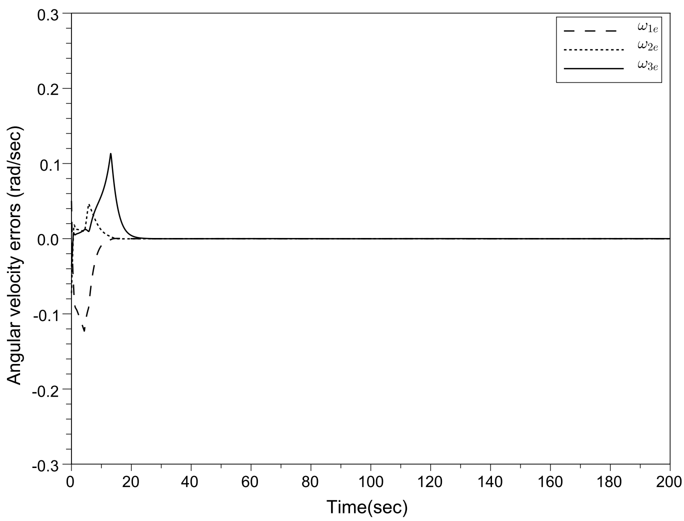

Figure 2: Angular velocity tracking errors.

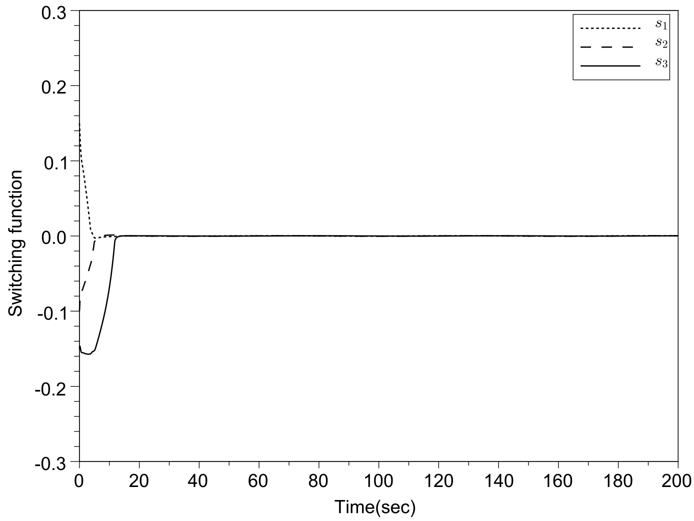

Figure 3: Switching function.

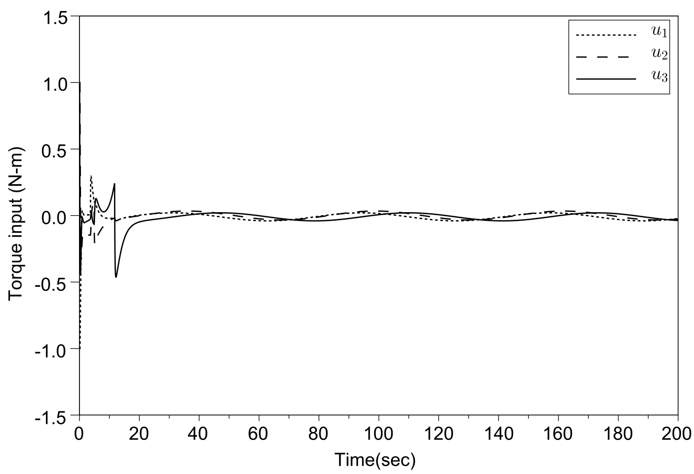

Figure 4: Control torques.

sliding vector remains on the sliding manifold in about 20 seconds. From Fig. 4 it can be seen that the proposed controller approximates the harmonic curves which are limited to a steady state level.

6. Conclusion

A novel robust optimal feedback controller design based on the LOC method has been successfully applied to the attitude tracking control problem. To obtain this controller design, the QDC approach and the SMC concept have been employed to solve the finite time optimal control problem. The proposed control includes a singular control regime that produces a switching surface, whose existence and placement is forced by function minimization in addition to stability of reduced order dynamics. The second method of Lyapunov is used to show that trajectories of the control system reach the switching surface and the asymptotic stability of the origin is achieved. An example of multiaxial attitude manoeuvres is presented and simulation results are included to verify the usefulness of the proposed controller.

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