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# STRONG CONVERGENCE OF HYBRID FIXED POINT ITERATIVE ALGORITHMS OF KIRK-NOOR TYPE WITH ERRORS IN AN ARBITRARY BANACH SPACE

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**Abstract:** The purpose of this paper is to introduce a new hybrid fixed point iterative algorithm of Kirk-Noor type with errors and to establish a general theorem to approximate the unique common fixed point of three operators satisfying a certain contractive condition in an arbitrary Banach space using the algorithms of Kirk-Noor type with errors. We use a more general contractive condition and prove a convergence theorem under weaker conditions on parameters than those of Rashwan et al. (2009). Our results, therefore, are improvements, generalization and extensions of the works of Rhoades (1976), Berinde (2004), Rashwan et al. (2009) and many others in the literature. An example to illustrate the validity of our results is also provided.

**AMS Subject Classification:** 47H06, 54H25
**Key Words:** Kirk type iterative schemes, quasi-contractive operators

## 1. Introduction

In the last decades, numerous papers were published on the iterative approximation of fixed points of contractive type self operators. When is it clear that consideration of errors terms in iterative schemes is an important part of the theory, in 1995, Liu [9] introduced iterative process with errors as follows :

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$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T y_n + u_n,$$
$$y_n = (1-\beta_n)x_n + \beta_n T x_n + v_n, \quad n \geq 0, \tag{1}$$

where $\{\alpha_n\}$, $\{\beta_n\}$ are sequences in $[0,1]$ and $u_n$, $v_n$ are summable sequences in $K$, $K$ being a closed, convex subset of a Banach space $X$. After that it was observed that the iterative process with errors introduced by Liu [9] was not satisfactory. The errors occur in a random way. The conditions imposed on the errors terms in (1.1) which say that they tend to zero as $n$ tends to infinity are, therefore, unreasonable. In the recent paper [22] Xu introduced Ishikawa iterative with errors with a more satisfactory error terms:

$$x_{n+1} = \alpha_n x_n + \alpha_n^1 T y_n + a_n u_n,$$
$$y_n = \beta_n x_n + \beta_n^1 T x_n + b_n v_n, \quad n \geq 0, \tag{2}$$

where $\{\alpha_n\}$, $\{\beta_n\}$, $\{a_n\}$, $\{b_n\}$, $\{\alpha_n^1\}$, $\{\beta_n^1\}$ are sequences in $[0,1]$ with $\alpha_n + \alpha_n^1 + a_n = \beta_n + \beta_n^1 + b_n = 1$ and $u_n$, $v_n$, are bounded sequences in $K$.

In 2002, Agarwal et al. [1] studied the following iterative process for a couple of quasi-contractive operators:

$$x_{n+1} = \alpha_n x_n + \alpha_n^1 S y_n + a_n u_n,$$
$$y_n = \beta_n x_n + \beta_n^1 T x_n + b_n v_n, \quad n \geq 0, \tag{3}$$

where $\{\alpha_n\}$, $\{\beta_n\}$, $\{a_n\}$, $\{b_n\}$, $\{\alpha_n^1\}$, $\{\beta_n^1\}$ are sequences in $[0,1]$ with $\alpha_n + \alpha_n^1 + a_n = \beta_n + \beta_n^1 + b_n = 1$ and $u_n$, $v_n$ are bounded sequences in $K$.

In 2009, Rashwan et al. [18] studied the convergence of the following three step iterative process with errors to approximate the common fixed point of three Z operators:

$$x_{n+1} = \alpha_n x_n + \alpha_n^1 T_1 y_n + a_n u_n$$
$$y_n = \beta_n x_n + \beta_n^1 T_2 x_n + b_n v_n$$
$$z_n = \gamma_n x_n + \gamma_n^1 T_3 x_n + c_n w_n, \quad n \geq 0, \tag{4}$$

where $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{a_n\}$, $\{b_n\}$, $\{\alpha_n^1\}$, $\{\beta_n^1\}$, $\{\gamma_n^1\}$ are sequences in $[0,1]$ with $\alpha_n + \alpha_n^1 + a_n = \beta_n + \beta_n^1 + b_n = \gamma_n + \gamma_n^1 + c_n = 1$ and $u_n$, $v_n$, $w_n$, are bounded sequences in $K$.

In 2009, Olatinwo [12] introduced the following iterative procedures:

(a) The sequence $\{x_n\}$ iteratively defined by

$$x_{n+1} = \alpha_{n,0} x_n + \sum_{i=1}^{k} \alpha_{n,i} T^i y_n, \qquad \sum_{i=0}^{k} \alpha_{n,i} = 1,$$

$$y_n = \beta_{n,0}x_n + \sum_{j=1}^{s} \beta_{n,j} T^j z_n, \qquad \sum_{j=0}^{s} \beta_{n,j} = 1, \quad n = 0, 1, 2, \dots \tag{5}$$

where $k$ and $s$ are fixed integers with $k \geq s \geq t$, $\alpha_{n,i} \geq 0$, $\alpha_{n,0} \neq 0$, $\beta_{n,j} \geq 0$, $\beta_{n,0} \neq 0$, $\alpha_{n,i}, \beta_{n,j} \in [0, 1]$, is known as Kirk-Ishikawa iterative scheme.

(b) The sequence $\{x_n\}$ iteratively defined by

$$x_{n+1} = \alpha_{n,0}x_n + \sum_{i=1}^{k} \alpha_{n,i} T^i x_n, \quad \sum_{i=0}^{k} \alpha_{n,i} = 1 \tag{6}$$

where $k$ is a fixed integer, $\alpha_{n,i} \geq 0$, $\alpha_{n,0} \neq 0$, $\alpha_{n,i} \in [0, 1]$, is known as Kirk-Mann iterative scheme.

## 2. Preliminaries

Now, we introduce the following Kirk-Noor iterative process with errors: Let $X$ be a Banach space, $K$ being a nonempty closed, convex subset of $X$ and $T_i : K \to K$, $i = 1, 2, 3$ are selfmaps of $K$ and $x_0 \in K$. Then, define the sequence $\{x_n\}_{n=0}^{\infty}$ by

$$\begin{aligned}
x_{n+1} &= \alpha_{n,0}x_n + \sum_{i=1}^{k} \alpha_{n,i} T_1^i y_n + a_n u_n, \qquad \sum_{i=0}^{k} \alpha_{n,i} + a_n = 1 \\
y_n &= \beta_{n,0}x_n + \sum_{j=1}^{s} \beta_{n,j} T_2^j z_n + b_n v_n, \qquad \sum_{j=0}^{s} \beta_{n,j} + b_n = 1 \\
z_n &= \sum_{l=0}^{t} \gamma_{n,l} T_3^l x_n + c_n w_n, \quad \sum_{l=0}^{t} \gamma_{n,l} + c_n = 1, \quad n = 0, 1, 2, \dots
\end{aligned} \tag{7}$$

where $k$, $s$ and $l$ are fixed integers with $k \geq s \geq t$, $\alpha_{n,i} \geq 0$, $\alpha_{n,0} \neq 0$, $\beta_{n,j} \geq 0$, $\beta_{n,0} \neq 0$, $\gamma_{n,l} \geq 0$, $\gamma_{n,0} \neq 0$, $\alpha_{n,i}$, $\beta_{n,j}$, $\gamma_{n,l}$, $a_n, b_n, c_n \in [0, 1]$ and $u_n$, $v_n$, $w_n$ are bounded sequences in $K$.

Putting $k = s = t = 1$ in (2.1) we obtain the iterative scheme with errors (1.4) used by Rashwan et al. [18].

Putting $t = 0$ in (2.1), we obtain the Kirk-Ishikawa iterative scheme with errors:

$$x_{n+1} = \alpha_{n,0}x_n + \sum_{i=1}^{k} \alpha_{n,i} T_1^i y_n + a_n u_n, \quad \sum_{i=0}^{k} \alpha_{n,i} + a_n = 1,$$

$$y_n = \beta_{n,0}x_n + \sum_{j=1}^{s}\beta_{n,j}T_2^j z_n + b_n v_n,\ \sum_{j=0}^{s}\beta_{n,j} + b_n = 1,\ n=0,1,2,\dots \tag{8}$$

Putting $t = s = 0$ in (2.1), we obtain the Kirk-Mann iterative scheme with errors:

$$x_{n+1} = \alpha_{n,0}x_n + \sum_{i=1}^{k}\alpha_{n,i}T_1^i x_n + a_n u_n, \quad \sum_{i=0}^{k}\alpha_{n,i} + a_n = 1 \tag{9}$$

Putting $s = k = t = 1$ and $T_1 = T_2 = T_3 = T$ in (2.1), we obtain the usual Noor iterative process with errors:

$$\begin{aligned} x_{n+1} &= (1-\alpha_{n,1})x_n + \alpha_{n,1}Ty_n + a_n u_n \\ y_n &= (1-\beta_{n,1})x_n + \beta_{n,1}Tx_n + b_n v_n \\ z_n &= (1-\gamma_{n,1})x_n + \gamma_{n,1}Tx_n + c_n w_n, \quad n \geq 0, \end{aligned} \tag{10}$$

where $\sum_{i=0}^{1}\alpha_{n,i} + a_n = \sum_{j=0}^{1}\beta_{n,j} + b_n = \sum_{l=0}^{1}\gamma_{n,l} + c_n = 1$, $\alpha_{n,1} = \alpha_n$, $\beta_{n,1} = \beta_n$, $\gamma_{n,1} = \gamma_n$.

Putting $t = 0$, $k = s = 1$ and $T_1 = T_2 = T$ in (2.1), we obtain the usual Ishikawa iterative process with errors:

$$\begin{aligned} x_{n+1} &= (1-\alpha_{n,1})x_n + \alpha_{n,1}Ty_n + a_n u_n \\ y_n &= (1-\beta_{n,1})x_n + \beta_{n,1}Tx_n + b_n v_n, \quad n \geq 0 \end{aligned} \tag{11}$$

where $\sum_{i=0}^{1}\alpha_{n,i} = \sum_{j=0}^{1}\beta_{n,j} = 1$, $\alpha_{n,1} = \alpha_n$, $\beta_{n,1} = \beta_n$.

Putting $s = t = 0$, $k = 1$ and $T_1 = T$, in (2.1), we obtain usual Mann iterative process with errors:

$$\begin{aligned} &x_{n+1} = (1-\alpha_{n,1})x_n + \alpha_{n,1}Tx_n + a_n u_n, \quad n \geq 0, \\ &\sum_{i=0}^{1}\alpha_{n,i} + a_n = 1, \quad \alpha_{n,1} = \alpha_n. \end{aligned} \tag{12}$$

Putting $s = 0$, $t = 0$ and $\alpha_{n,i} = \alpha_i$ in (2.1), we obtain the usual Kirk's iterative process with errors:

$$x_{n+1} = \sum_{i=0}^{k}\alpha_i T^i x_n + a_n u_n, \qquad \sum_{i=0}^{k}\alpha_i = 1, \quad n = 0,1,2,\dots \tag{13}$$

Putting $s = k = t = 1$ and $a_n = b_n = c_n = 0$ in (2.1), we obtain the usual Noor iterative process [11]:

$$\begin{aligned} x_{n+1} &= (1-\alpha_{n,1})x_n + \alpha_{n,1}Ty_n \\ y_n &= (1-\beta_{n,1})x_n + \beta_{n,1}Tz_n \\ z_n &= (1-\gamma_{n,1})x_n + \gamma_{n,1}Tx_n, \quad n \geq 0 \end{aligned} \tag{14}$$

**Remarks.** Usual Ishikawa [6] and usual Mann iterations [10] are special cases of usual Noor iteration (2.8).

We employ the following contractive definition: For a self map $T : K \to K$ there exist a real number $a \in [0, 1)$, and a monotone increasing function $\varphi : R^+ \to R^+$ with $\varphi(0) = 0$, such that $\forall\ x, y \in X$

$$\|Tx - Ty\| \leq \varphi(\|x - Tx\|) + a\|x - y\|. \tag{15}$$

However, we shall need the following Lemmas in the sequel.

**Lemma 2.1** ([2]). *If $\delta$ is a real number such that $0 = \delta < 1$ and $\{\epsilon_n\}_{n=0}^{\infty}$ is a sequence of positive numbers such that $\lim_{n\to\infty} \epsilon_n = 0$, then for any sequence of positive numbers $\{u_n\}_{n=0}^{\infty}$ satisfying*

$$u_{n+1} \leq \delta u_n + \epsilon_n, \quad n = 0, 1, 2, \ldots$$

*we have $\lim_{n\to\infty} u_n = 0$.*

**Lemma 2.2** ([13]). *Let $(X, \|\cdot\|)$ be a normed linear space and let $T : X \to X$ be a selfmap of $X$ satisfying (2.5). Let $\varphi : R^+ \to R^+$ be a subadditive, monotone increasing function such that $\varphi(0) = 0$, $\varphi(Lu) \leq L\varphi(u)$, $L \geq 0$, $u \in R^+$. Then, $\forall\ i \in N$ and $\forall\ x, y \in X$,*

$$\|T^i x - T^i y\| = \sum_{j=1}^{i} \binom{i}{j} a^{i-j} \varphi^j(\|x - Tx\|) + a^i\|x - y\| \tag{16}$$

**Remarks.** If $y = p$ is the fixed point of $T$, then (2.10) can be written as

$$\|T^i x - p\| = a^i\|x - p\| \quad \text{for all } \ i. \tag{17}$$

Inspired and motivated by the research work of Olatinwo [12] and Rashwan et al. [18], we study the convergence of iterative scheme (2.1) by employing contractive condition (2.9).

## 3. Main Results

**Theorem 3.1.** *Let $K$ be a non empty closed, convex subset of an arbitrary Banach space $X$ and $T_i$ $(i = 1, 2, 3) : K \to K$ be three operators satisfying the contractive condition (2.9) and $\varphi : R^+ \to R^+$ a subadditive monotone increasing function such that $\varphi(0) = 0$. If $F(T_1) \cap F(T_2) \cap F(T_3) \neq \phi$ and* $\lim a_n = \lim b_n = \lim c_n = 0$*, then the Kirk-Noor iterative process with errors $\{x_n\}_{n=0}^{\infty}$ defined by (2.1) converges strongly to a unique common fixed point of* $T_1$, $T_2$ and $T_3$.

*Proof.* First we prove uniqueness of fixed point. For this purpose, let $p, q \in F(T_1) \cap F(T_2) \cap F(T_3)$ with $p \neq q$. Then from (2.9), we have

$$\|Tp - Tq\| = \|p - q\| \leq \varphi(\|p - Tp\|) + a\|p - q\|.$$

Which implies $(1 - a)\|p - q\| \leq 0$ and hence $p = q$, since $a \in [0, 1)$.

Set $M = \max\big\{\sup_{n\geq 0} \|u_n - p\|, \sup_{n\geq 0} \|v_n - p\|, \sup_{n\geq 0} \|w_n - p\|\big\}$.

By using Lemma 2.2 and the triangle inequality, from (2.1) we have

$$\begin{aligned} \|x_{n+1} - p\| &\leq \alpha_{n,o}\|x_n - p\| + \sum_{i=1}^{k} \alpha_{n,i}\|T_1^i y_n - p\| + a_n\|u_n - p\| \\ &\leq \alpha_{n,o}\|x_n - p\| + \sum_{i=1}^{k} \alpha_{n,i}a^i\|y_n - p\| + a_n M \end{aligned} \tag{18}$$

Again using Lemma 2.2, (2.1) yields

$$\begin{aligned} \|y_n - p\| &\leq \beta_{n,o}\|x_n - p\| + \sum_{j=1}^{s} \beta_{n,j}\|T_2^j z_n - p\| + b_n\|v_n - p\| \\ &\leq \beta_{n,o}\|x_n - p\| + \sum_{j=1}^{s} \beta_{n,j}a^j\|z_n - p\| + b_n M \end{aligned} \tag{19}$$

Again, from (2.1), we have

$$\begin{aligned} \|z_n - p\| &\leq \sum_{l=0}^{t} \gamma_{n,l}\|T_3^l x_n - p\| + c_n\|w_n - p\| \\ &\leq \sum_{l=0}^{t} \gamma_{n,l}a^l\|x_n - p\| + c_n M \end{aligned} \tag{20}$$

Substituting (3.3) in (3.2), we get

$$\|y_n - p\| \le \beta_{n,o}\|x_n - p\| + \left(\sum_{j=1}^{s} \beta_{n,j} a^j\right)\left(\sum_{l=0}^{t} \gamma_{n,l} a^l\right)\|x_n - p\| + \left(\sum_{j=1}^{s} \beta_{n,j} a^j\right) c_n M + b_n M \tag{21}$$

Substituting (3.4) in (3.1) and rearranging the terms, we have

$$\begin{aligned}\|x_{n+1} - p\| &\le \left\{\alpha_{n,o} + \left(\sum_{i=1}^{k} \alpha_{n,i} a^i\right)\beta_{n,o} + \left(\sum_{i=1}^{k} \alpha_{n,i} a^i\right)\left(\sum_{j=1}^{s} \beta_{n,j} a^j\right)\left(\sum_{l=0}^{t} \gamma_{n,l} a^l\right)\right\} \\ &\times \|x_n - p\| + \left\{\left(\sum_{i=1}^{k} \alpha_{n,i} a^i\right)\left(\sum_{j=1}^{s} \beta_{n,j} a^j\right) c_n + \left(\sum_{i=1}^{k} \alpha_{n,i} a^i\right) b_n + a_n\right\} M. \end{aligned} \tag{22}$$

Now, we claim

$$\alpha_{n,o} + \left(\sum_{i=1}^{k} \alpha_{n,i} a^i\right)\beta_{n,o} + \left(\sum_{i=1}^{k} \alpha_{n,i} a^i\right)\left(\sum_{j=1}^{s} \beta_{n,j} a^j\right)\left(\sum_{l=0}^{t} \gamma_{n,l} a^l\right) = \delta(\text{say}) < 1$$

as follows:
Since

$$\sum_{i=0}^{k} \alpha_{n,i} + a_n = 1, a^i \in [0,1) \quad \text{and} \quad \alpha_{n,0} \ne 0,$$

so

$$\sum_{i=1}^{k} \alpha_{n,i} a^i \le \sum_{i=1}^{k} \alpha_{n,i} < 1 - \alpha_{n,0}\,.$$

Similarly

$$\sum_{j=1}^{s} \beta_{n,j} a^j \le \sum_{j=1}^{s} \beta_{n,j} \le 1 - \beta_{n,o}\,.$$

Also

$$\sum_{l=1}^{t} \gamma_{n,l} a^l < 1.$$

Hence

$$\alpha_{n,o} + \Big(\sum_{i=1}^{k} \alpha_{n,i} a^i\Big)\beta_{n,o} + \Big(\sum_{i=1}^{k} \alpha_{n,i} a^i\Big)\Big(\sum_{j=1}^{s} \beta_{n,j} a^j\Big)\Big(\sum_{l=0}^{t} \gamma_{n,l} a^l\Big)$$
$$< \alpha_{n,o} + (1-\alpha_{n,o})\beta_{n,o} + (1-\alpha_{n,o})(1-\beta_{n,o}) = 1.$$

So, from (3.5) we have

$$\|x_{n+1} - p\| \leq \delta\|x_n - p\| + \{a_n + b_n + c_n\}M \tag{23}$$

Now, $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = \lim_{n\to\infty} c_n = 0$. Hence using Lemma 2.1 and (3.6), we have $\lim_{n\to\infty} x_n = p$. □

**Corollary 3.2.** *Let $K$ be a non empty closed convex subset of an arbitrary Banach space $X$ and $T_i$ $(i = 1, 2) : K \to K$ be two operators satisfying the contractive condition (2.9) and $\varphi : R^+ \to R^+$ a subadditive monotone increasing function such that $\varphi(0) = 0$.*

*If $F(T_1) \cap F(T_2) \neq \phi$ and $\lim a_n = \lim b_n = 0$, then the Kirk-Ishikawa iterative process with errors $\{x_n\}_{n=0}^{\infty}$ defined by (2.2) converges strongly to a unique common fixed point of $T_1$ and $T_2$.*

**Corollary 3.3.** *Let $K$ be a non empty closed convex subset of an arbitrary Banach space $X$ and $T : K \to K$ be a operator satisfying the contractive condition (2.9) and $\varphi : R^+ \to R^+$ a subadditive monotone increasing function such that $\varphi(0) = 0$. If $F(T) \neq \phi$ and $\lim a_n = 0$, then the Kirk-Mann iterative process with errors $\{x_n\}_{n=0}^{\infty}$ defined by (2.3) converges strongly to a unique fixed point of $T$.*

**Remarks.**

1. We know that usual iterative procedures are special cases of iterative procedures with errors, so convergence theorems for Kirk-Noor, Kirk-Ishikawa, Kirk-Mann and usual Kirk iterations are obtained in Theorem 3.1, Corollary 3.2 and Corollary 3.3, respectively .

2. The $Z$ operators are included in the class of operators satisfying (2.9) and so their convergence theorems for Kirk-Noor iterative process with errors, Kirk-Ishikawa iterative process with errors, Kirk-Mann iterative process with errors and Kirk iterative process with error are obtained in Theorem 3.1, Corollary 3.2 and Corollary 3.3, respectively.

3. The condition on parameters i.e. $\lim a_n = \lim b_n = \lim c_n = 0$, is weaker than those of Rashwan et al. [18]

**Example.** Let $X$ be the real line with the usual norm and let $K = [0, 1]$. Define $T_i$ $(i = 1, 2, 3) : K \to K$ by

$$T_1 x = \begin{cases} 0, & x \in \left[0, \frac{1}{2}\right] \\ \frac{1}{4}, & x \in \left(\frac{1}{2}, 1\right] \end{cases}$$

$$T_2 x = \begin{cases} 0, & x \in \left[0, \frac{1}{2}\right] \\ \frac{1}{5}, & x \in \left(\frac{1}{2}, 1\right] \end{cases}$$

$$T_3 x = \begin{cases} 0, & x \in \left[0, \frac{1}{2}\right] \\ \frac{1}{6}, & x \in \left(\frac{1}{2}, 1\right] \end{cases}$$

Mappings $T_1, T_2, T_3$ and the set $K$ fulfill the requirements of the Theorem 3.1. If we take $k = s = t = 2$ and $\alpha_{n,0} = \frac{1}{n+5}$, $\alpha_{n,1} = \frac{n}{n+5}$, $\alpha_{n,2} = \frac{2}{n+5}$, $a_n = \frac{1}{n+5}$, $\beta_{n,0} = \frac{1}{n+4}$, $\beta_{n,1} = \frac{n}{n+4}$, $\beta_{n,2} = \frac{2}{n+4}$, $b_n = \frac{1}{n+4}$, $\gamma_{n,0} = \frac{1}{n+3}$, $\gamma_{n,1} = \frac{n-1}{n+3}$, $\gamma_{n,2} = \frac{2}{n+3}$, $c_n = \frac{1}{n+3}$, $u_n = v_n = w_n = \frac{1}{n+8}$, $n \geq 1$, then the sequence $\{x_n\}_{n=0}^{\infty}$ defined by (2.1) converges strongly to unique common fixed point 0 of $T_1$, $T_2$ and $T_3$.

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