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THE MODIFIED MCSP-C CONTINUOUS SAMPLING PLAN

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Abstract: This paper presents the MCSP-2-C for the concept of a two-level continuous sampling plan that has been developed from the single-level continuous sampling plan MCSP-C. The parameters used in MCSP-2-C are i (the number of consecutive conforming units that must be produced during a 100% inspection), c (the acceptance number), m (the number of conforming units to be found before allowing c non-conforming units in the sampling inspection), f_1 and f_2 (the specified sampling frequency at level 1 and 2, respectively). Three performance measure formulas were developed, namely average fraction inspected (AFI), average outgoing quality (AOQ) and average outgoing quality limit ($AOQL$). The validity of these formulas have been tested by extensive simulations for all sets of parameters and the probability of non-conforming units (p). The AFI and AOQ values from the two plans were compared and it was found that MCSP-2-C does not give an appreciable difference with results at $p = 0.005$, 0.008, 0.01 and 0.02 when r is large for all sets of i and c. However, it was found that MCSP-2-C gives a lower number of units inspected than MCSP-C when i is large, $p = 0.03$ and 0.05 for all sets of r and c.

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1. Introduction

A continuous sampling plan (CSP) is a sampling inspection plan for inspecting each product unit and the result of the inspection is either accept or reject for a continuous process. CSP alternates between two phases of inspection, i.e. 100% inspection (phase 1) and sampling inspection (phase 2). Dodge [2] presented a sampling of the first type, namely CSP-1 which was the single-level continuous sampling plan. This plan is well known and was used to develop plans such as CSP-2 and CSP-3 by Dodge and Torrey [3], CSP-M by Lieberman and Solomon [8] and CSP-C by Govindaraju and Kandnsamy [4]. Reviews of these continuous sampling plans can be seen in many statistical quality control textbooks such as Grant [5], Stephens [11] and [12] and Montgomery [9].

MCSP-C has been developed as a continuous sampling plan based on CSP-C by Balamurali and Subramani [1] and it is a single-level continuous sampling plan. The operating procedure of the MCSP-C plan starts at 100% inspection (phase 1) when units are inspected one by one consecutively in the order of production. When i successive units are found to conform, phase 1 is discontinued and sampling inspection (phase 2) is started which inspects only a fraction f of the units selected at random. If c non-conforming units are found after the first m sampled units have been found to conform then inspection continues with a sampling rate f. If a non-conforming unit is found within m sampled or when c+1 non-conforming units are found after m sampled conforming units then reverts immediately to phase 1. This plan allows c non-conforming units during the sampling inspection and leads to the acceptance of low quality products. In the MCSP-C plan, sampling inspection is continued if the first m sampled units have been found to conform or until the event of c+1 non-conforming units. So MCSP-C imports an additional requirement that, after invoking sampling inspection, the event of a non-conforming unit before finding m sampled units.

In this paper, the MCSP-C plan was modified and extended from a single level to two levels during the sampling inspection, namely the MCSP-2-C plan. The MCSP-2-C plan added two sampling inspection rates at the sampling inspection phase, f_1 and f_2. The difference between the two plans is if a non-conforming unit is found within m sampled conforming units then MCSP-C reverts to 100% inspection but MCSP-2-C starts sampling inspection at level 2, inspects only a fraction f_2 until a total of c+1 non-conforming sampled units have been found then reverts to a 100% inspection.

The main objectives of this paper are as follows:

1. The design of a two-level continuous sampling plan which is called MCSP-2-C and was developed from the one-level continuous sampling plan MCSP-C.

2. To give a detailed derivation of the theory of the MCSP-2-C sampling plan and the formulas for performance measures in MCSP-2-C, such as the average fraction inspected (AFI), the average outgoing quality (AOQ) and the average outgoing quality limit ($AOQL$).

3. To summarize the results of tests of the validity of the formulas for performance measures by comparison of the values computed from the formulas with values obtained through extensive simulations.

4. To give a comparison of performance measures of the MCSP-C plan with MCSP-2-C.

2. Design and Theory of MCSP-2-C Plan

2.1. The Operating Procedure of the MCSP-2-C

For completeness, a summary is given of the procedure in this section. The MCSP-2-C uses five parameters (i, c, m, f_1 and f_2) for inspection of the units being produced on the production line, which are defined by:

i = the number of consecutive conforming units that must be produced during a 100% inspection of the line,

c = the acceptance number,

m = the number of conforming units to be found before allowing c non-conforming units in the sampling inspection,

f_1 = the specified sampling frequency at level 1 on the line or $f_1 = 1/r$,

f_2 = the specified sampling frequency at level 2 on the line or $f_2 = 2/f_1$.

The procedure for inspection of the MCSP-2-C plan represented schematically as in Figure 1.

2.2. The Performance Measures of MCSP-2-C

The performance measures that were defined in MCSP-2-C are generalizations of the performance measures used in MCSP-C. MCSP-2-C computes 3 performance measures, average fraction inspected (AFI), average outgoing quality (AOQ) and average outgoing quality limit ($AOQL$). A derivation of these performance measures assumed that the production process is under statistical control and based on Markov Chain formulation.

Let p be the probability of a unit produced by the process being non-conforming and q =1-p, the following formulas for performance measures may be obtained:

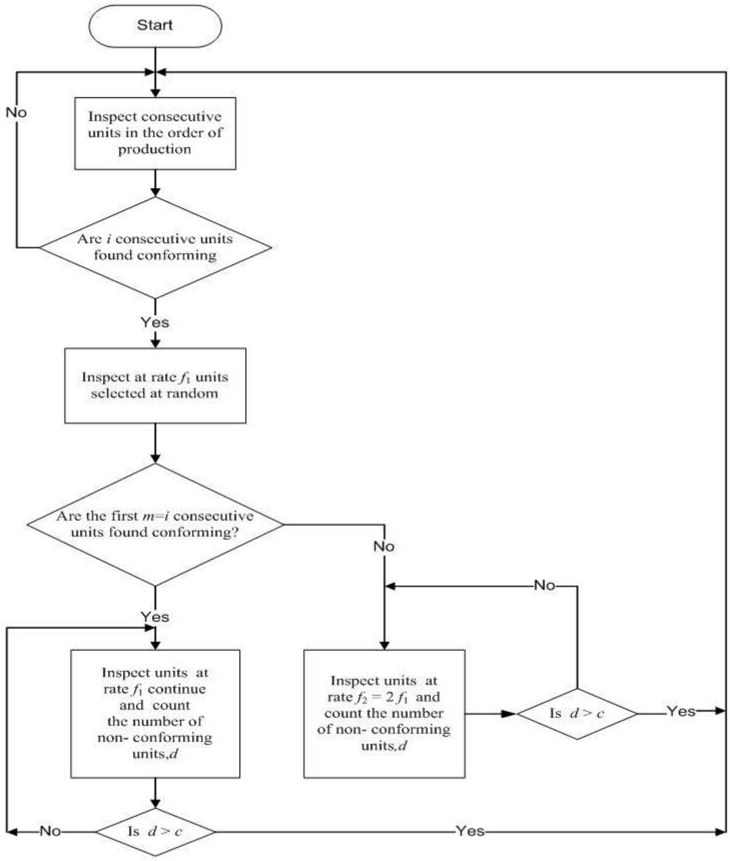

Figure 1: Flow diagram of the procedure for inspection using MCSP-2-C plan.

The average number of units inspected in a 100% screening sequence following the finding of a non-conforming unit, u:

$$u = \frac{1 - q^i}{pq^i} \tag{1}$$

The average number of units passed under the sampling inspection, v:

$$v = \frac{f_2(1 + cq^m) + (c + 1)f_1(1 - q^m)}{pf_1f_2} \tag{2}$$

The average cycle length, which is the average number of units, passed on each cycle, ACL:

$$ACL = \frac{f_1 f_2(1-q^i) + q^i f_2(1+cq^m) + (c+1)q^i f_1(1-q^m)}{pq^i f_1 f_2} \tag{3}$$

The average fraction inspected, AFI:

$$AFI = \frac{f_1 f_2\{1+(c+1)q^i - q^{i+m}\}}{f_1 f_2(1-q^i) + q^i f_2(1+cq^m) + q^i f_1(c+1)(1-q^m)} \tag{4}$$

The average outgoing quality, AOQ:

$$AOQ = \frac{pq^i\{(1-q^m)(1-f_1)f_2 + (c+1)[q^m(1-f_1)f_2 + f_1(1-q^m)(1-f_2)]\}}{f_1 f_2(1-q^i) + q^i f_2(1+cq^m) + q^i f_1(c+1)(1-q^m)} \tag{5}$$

The average outgoing quality limit, $AOQL$:

$$AOQL = \max_p(AOQ) \tag{6}$$

Full details of the derivation of these performance measures can be found in Guayjarernpanishk and Mayureesawan [6].

2.3. Test of the Validity of Performance Measures

The validity of the performance measures were tested by comparing the results from our formulas with the values for the performance measures obtained from extensive simulations. A summary of our tests will be given in this section and results in Section 3.

Four different levels were examined for the incoming probability p of non-conforming units produced on the line. The four levels were: low level ($p = 0.005$, 0.008), medium level ($p = 0.01$, 0.02), high level ($p = 0.03$) and very high level ($p = 0.05$). For each set of values for p, values for i, m, r and c were selected. The values of i were selected from 10, 15, 20, 30, 40, 50, the values of $m = i$, the values of r were selected from 4, 10, and the values of c were selected from 2, 3.

For each set of values of the parameters p, i, m, r and c a simulation was carried out to compute the values of AFI and AOQ. The simulation was repeated for 200 different product lines and the values of AFI and AOQ were calculated and then compared with the values of AFI and AOQ computed from the formulas given in equations (4) and (5).

An AFI formula is accepted as a valid formula if the percentage difference between the AFI values from the formula and the AFI values from the simulations were less than or equal to 2. In testing the validity of an AOQ formula, the formula was accepted as a valid formula if the percentage difference between the AOQ values from the formula and the AOQ values from the simulations were less than or equal to 2. The validity of the formulas was then compared for each set of p, i, m, r and c values.

2.4. Comparisons of the Performance Measures of MCSP-C Plan with MCSP-2-C Plan

In this section, the values of AFI and AOQ for MCSP-C were compared with the values of AFI and AOQ respectively obtained for MCSP-2-C. Extensive simulations for two inspections were carried out using the same parameter values that were used for testing the validity of performance measures in MCSP-2-C.

The $DIFF(AFI)$ and $DIFF($AOQ) values for comparing the AFI and AOQ values respectively of MCSP-C plan with MCSP-2-C plan were defined by:

$$DIFF(AFI) = AFI(\text{MCSP-C}) - AFI(\text{MCSP-2-C}) \tag{7}$$

and

$$DIFF(AOQ) = AOQ(\text{MCSP-C}) - AOQ(\text{MCSP-2-C}) \tag{8}$$

Where

AFI(MCSP-C) = the AFI values of MCSP-C plan,
AFI(MCSP-2-C) = the AFI values of MCSP-2-C plan,
AOQ(MCSP-C) = the AOQ values of MCSP-C plan,
AOQ(MCSP-2-C) = the AOQ values of MCSP-2-C plan.

The results for the comparisons are presented in the next section.

3. Results

3.1. The Validity Performance Measures for MCSP-2-C

In Table 1, the percentage differences for each set of p, i, m, r and c values of the AFI and AOQ values from the formula and the AFI and AOQ values from the simulations are shown. It was found that the percentage differences were less than 2 for all sets of p, i, m, r and c values. So the simulations signified that the AFI and AOQ formulas are valid.

3.2. The Comparisons of Performance Measures

In this section, the results of the comparison of the average fraction of units inspected using our MCSP-C plan with the average fraction inspected using the MCSP-2-C plan for $c = 2$, 3, and $r = 4$ and 10 are shown in Figure 2. It can be seen that for a low or medium level of probability of a unit produced by the process being non-conforming ($p = 0.005$, 0.008, 0.01 and 0.02) for all sets of i, c and r values, the *DIFF(AFI)* values are lower than 0 which means the AFI values for MCSP-2-C are higher than MCSP-C but when r is large ($r = 10$) the AFI values for MCSP-2-C are close to the values from MCSP-C.

At a higher level of p ($p = 0.03$) for all sets of c, the *DIFF(AFI)* values are lower than 0 when i is less than or equal to 40 for $r = 4$ and when i is less than or equal to 30 for $r = 10$, which means that for these cases, the AFIvalues for MCSP-2-C are higher than MCSP-C but for $i = 50$ the AFI values for MCSP-2-C are lower than MCSP-C for all sets of c and r. However, for the higher level of r ($r = 10$) the differences between the AFI values are relatively small.

For a very high level of p ($p = 0.05$) the AFI values for MCSP-2-C are greater than MCSP-C for all sets of c, r, when i is less than or equal to 20 but for the higher level of i ($i = 30$, 40 and 50) the AFI values for MCSP-2-C are less than MCSP-C and the differences between the AFI values from the two plans become large as the value of i is increased. From the effect of r on the AFI values from the two plans when the level of i is very high ($i = 30$, 40, 50) it was found that the AFI values of MCSP-2-C are smaller than MCSP-C when r is large ($r = 10$)

A comparison of the average outgoing quality using the MCSP-2-C plan with MCSP-C plan for $c = 2$, 3, and $r = 4$ and 10 are shown in Figure 3. It was found that the results of the comparisons of the AOQ values are opposite to the results of the comparisons of the AFI values. It was found that at a low or medium level of p ($p = 0.005$, 0.008, 0.01 and 0.02) for all sets of i, c and r values, the *DIFF(AOQ)* values were higher than 0 which means the AOQ values for MCSP-2-C are lower than MCSP-C. But for all sets of i and c when r is large ($r = 10$) the AOQ values from the two plans are very close.

In the case of a higher level of p ($p = 0.03$) for all sets of c, when i is less than or equal to 40, $r = 4$ and when i is less than or equal to 30 for $r = 10$, the AOQ values for MCSP-2-C are lower than MCSP-C but for $i = 50$ the AOQ values for MCSP-2-C are higher than MCSP-C for all sets of c and r. However, the differences between AOQ values are quite small when r is small ($r = 4$).

At a very high level of p ($p = 0.05$) for all sets of c and r, the AOQ values

			p=0.005		p=0.008		p=0.01		p=0.02		p=0.03		p=0.05	
i	r	c	AFI	AOQ	AFI	AOQ	AFI	AOQ	AFI	AOQ	AFI	AOQ	AFI	AOQ
10	4	2	1.95	0.37	1.18	1.19	0.53	0.13	0.92	0.95	0.70	0.28	1.70	1.51
10	4	3	0.22	0.97	0.80	1.44	0.95	1.62	1.35	1.26	0.15	0.04	0.56	0.28
10	10	2	1.37	1.44	1.77	1.70	1.00	0.61	1.27	1.15	1.29	0.32	1.11	1.07
10	10	3	0.81	1.35	1.89	0.32	0.79	0.38	1.81	0.72	1.36	0.14	1.14	0.09
15	4	2	0.55	1.43	0.78	0.42	0.40	0.02	0.49	0.43	1.02	1.43	0.36	0.25
15	4	3	0.24	0.58	0.48	0.63	1.34	0.01	0.31	0.97	0.28	1.33	0.77	1.36
15	10	2	0.97	1.47	0.32	1.60	0.84	0.98	1.73	0.06	1.34	0.24	1.83	0.30
15	10	3	1.63	1.44	0.59	0.65	1.53	1.02	0.88	0.40	0.42	0.07	0.97	0.43
20	4	2	0.95	1.40	0.08	1.00	0.31	1.33	1.57	1.57	1.42	1.95	0.62	1.68
20	4	3	0.90	1.57	0.68	1.77	1.38	0.04	0.23	1.23	0.83	0.42	1.43	0.36
20	10	2	1.39	1.46	1.17	1.27	1.29	1.53	0.77	1.92	1.57	0.21	1.19	0.29
20	10	3	1.13	1.74	1.15	0.33	1.24	0.16	0.30	0.90	1.69	0.18	0.85	0.13
30	4	2	0.57	1.43	1.25	1.26	0.65	0.33	0.08	0.65	0.36	1.21	0.38	0.78
30	4	3	0.41	0.22	0.74	1.27	1.54	0.15	1.82	0.20	0.91	0.28	0.86	1.48
30	10	2	1.43	0.76	0.46	1.93	1.72	1.02	1.90	0.15	0.57	0.34	1.10	1.33
30	10	3	1.27	1.02	1.16	0.47	1.41	0.01	0.18	0.22	0.30	0.08	0.86	0.93
40	4	2	0.89	1.55	0.06	1.76	1.49	1.31	1.09	1.45	0.29	0.36	0.37	0.94
40	4	3	0.52	1.07	1.21	1.43	0.31	0.72	0.89	1.67	0.76	1.46	0.34	1.18
40	10	2	1.92	1.88	1.40	1.60	1.63	1.45	0.49	1.04	0.13	0.82	0.37	0.88
40	10	3	1.68	0.97	0.60	0.29	0.14	0.31	0.96	1.28	0.80	0.07	0.86	0.28
50	4	2	1.76	1.52	0.85	0.78	1.65	1.41	1.98	1.11	0.52	0.17	0.82	1.30
50	4	3	0.73	0.39	0.41	0.78	0.20	1.37	0.00	1.23	0.15	1.26	0.31	0.90
50	10	2	1.45	1.85	1.71	1.11	1.90	0.69	0.64	0.91	0.62	1.03	0.36	0.64
50	10	3	1.31	1.55	0.22	0.06	0.99	0.54	0.81	1.16	1.71	0.08	0.88	1.99

Table 1: The percentage differences between the AFI and AOQ values from the formula and the AFI and AOQ values from the simulations for MCSP-2-C.

for MCSP-2-C are greater than MCSP-C when i is more than or equal to 30 and it was also found that the differences are relatively large when the value of i is increased.

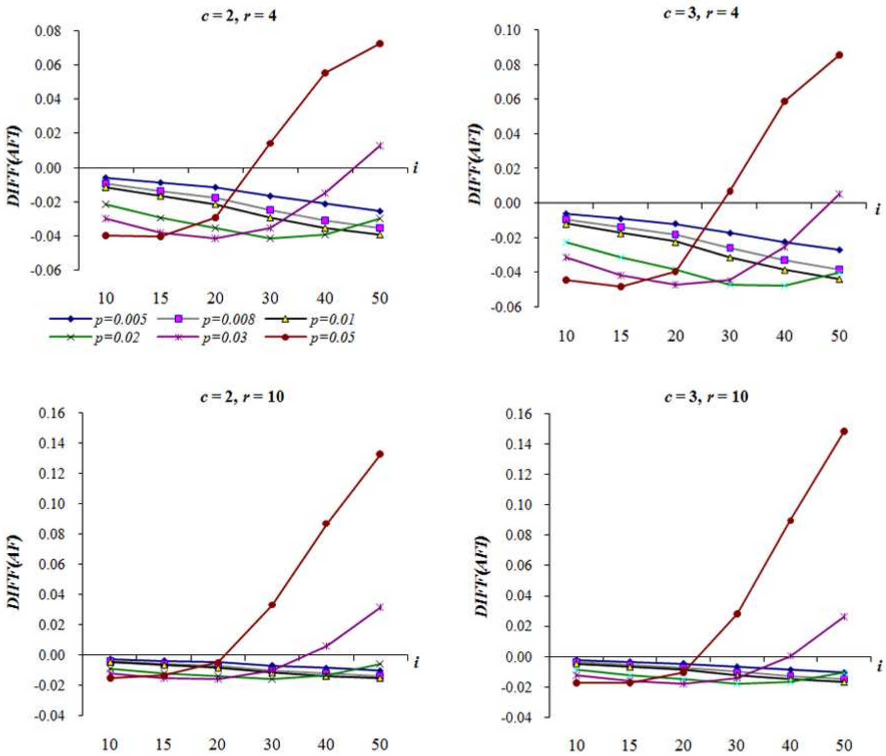

Figure 2: The different AFI values ($DIFF(AFI)$) between MCSP-C and MCSP-2-C for each set of c, r and for all sets of i and p values.

4. Discussions and Conclusions

The MCSP-2-C plan has been proposed to reduce inspection or extended restart 100% inspection in the MCSP-C plan process and formulas have been derived for performance measures such as the average fraction inspected (AFI), the average outgoing quality (AOQ) and the average outgoing quality limit ($AOQL$) of the MCSP-2-C plan.

The validity of AFI and AOQ for MCSP-2-C has been tested by extensive simulations over wide and representative ranges of values of the four parameters (p, i, c and r), where p is the probability of a unit produced by the process being non-conforming, i is the number of consecutive conforming units that must be produced during a 100% inspection of the line, c is the acceptance number and $f_1 = 1/r$, $f_2 = 2/f_1$ are the specified sampling frequency at level

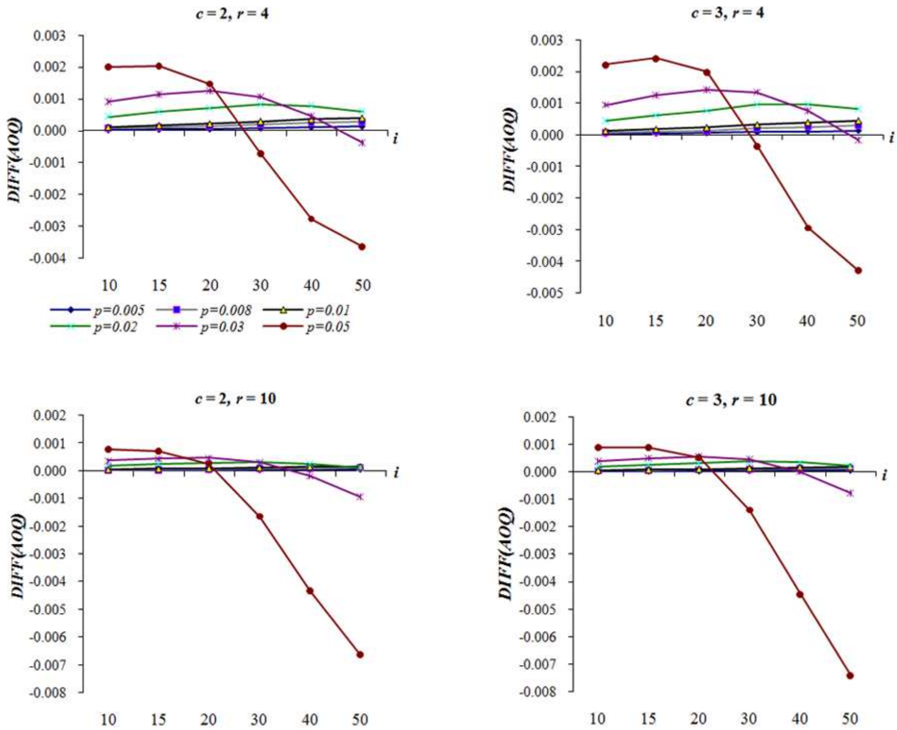

Figure 3: The different AOQ values ($DIFF(AOQ)$) between MCSP-C and MCSP-2-C for each set of c, r and for all sets of i and p values.

1 and 2, respectively. The percentage difference between the AFI and AOQ values from the formula and the AFI and AOQ values from the simulations were found to agree within 2% in all simulations.

Extensive simulations have been carried out to compare the AFI and AOQ values obtained from the MCSP-C plan with AFI and AOQ values from the MCSP-2-C plan. In table 2, the conclusion of the comparison of the AFI values between MCSP-2-C and MCSP-C are shown, for a low and medium level of p (p = 0.005, 0.008, 0.01 and 0.02) for all sets of i, c values when $r = 4$, theAFI values of MCSP-2-C are higher than MCSP-C and when $r = 10$, the AFI values of MCSP-2-C are close to the AFI values of MCSP-C. At a higher level of p (p = 0.03) for all sets of c, i is less than or equal to 40 when $r = 4$, the AFI values of MCSP-2-C are greater than MCSP-C and when $r = 10$, the AFI values of the two plans are nearly the same but for all sets of c, r when i is more than

p	i	c	r	Results of the comparison of AFI values (MCSP-2-C versus MCSP-C)
0.005, 0.008	all	all	4	$>$
0.01, 0.02	all	all	10	similar
0.03	≤ 40	all	4	$>$
	≤ 40	all	10	similar
	> 40	all	all	$<$
0.05	≤ 20	all	4	$>$
	≤ 20	all	10	similar
	> 20	all	all	$<$

Table 2: The comparison of the AFI values for MCSP-2-C plan and MCSP-C plan for all sets of p, i, c and r.

p	i	c	r	Results of the comparison of AOQ values (MCSP-2-C versus MCSP-C)
0.005, 0.008	all	all	4	$<$
0.01, 0.02	all	all	10	similar
0.03	≤ 40	all	4	$<$
	≤ 40	all	10	similar
	> 40	all	all	$>$
0.05	≤ 20	all	4	$<$
	≤ 20	all	10	similar
	> 20	all	all	$>$

Table 3: The comparison of the AOQ values for MCSP-2-C plan and MCSP-C plan for all sets of p, i, c and r.

40, the AFI values of MCSP-2-C are less than MCSP-C. For a very high level of p ($p = 0.05$) for all sets of c, when i is less than or equal to 20 and when $r = 4$, the AFI values of MCSP-2-C are higher than MCSP-C but when $r = 10$, the AFI values of MCSP-2-C and MCSP-C do not give different results but for all sets of c, r when i is more than 20, the MCSP-2-C plan gives AFI values which are less than MCSP-C.

A comparison of the AOQ values from the two plans is shown in Table 3, and this is contrary to the conclusion of the comparisons of the AFI values in Table 2.

When considering the conclusions from Table 2 and 3, in cases where the two plans give similar results of the AFI and AOQ comparisons, the operator may choose to use MCSP-C for inspection of production lines because this plan gives values of performance measures close to MCSP-2-C and is also an easier operating process of inspection than MCSP-2-C. But MCSP-2-C grants better performance measures for some levels of parameters p, i, c and r and meets the needs of the operator. Although the MCSP-2-C plan is a more complicated inspection procedure than MCSP-C, it gives a lower number of units inspected when higher and very high level of p ($p = 0.03$ and 0.05) for all sets of r, c when i is more than 40 for $p = 0.03$ and i is more than 20 for $p = 0.05$.

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