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EXPLICIT FORMULAS OF AVERAGE RUN LENGTH FOR A MOVING AVERAGE CONTROL CHART FOR MONITORING THE NUMBER OF DEFECTIVE PRODUCTS

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Abstract: The objective of this paper is to study Statistical Process Control (SPC) with a Moving Average control chart (MA). The main characteristics of a control chart are the Average Run Length (ARL) (mean of false alarm times) and the Average Delay time (AD) (mean delay of true alarm times). The ARL should be sufficiently large while the process is still in-control and the AD should be small when the process goes out-of-control. Explicit MA-based formulas for ARL and AD are presented for observations from a binomial distribution. In particular, formulas are developed for the number of defective products in a production line. The new formulas are simple, and easy to implement and use by practitioners. The ARL and AD calculated from these new formulas are compared with numerical simulation results for the Shewhart np and EWMA charts. The results show that the new MA formulas perform better than the np and EWMA charts when the shifts in parameter values from in-control to out-of-control are moderate or large.

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1. Introduction

Statistical Process Control (SPC) charts are widely used for monitoring, measuring, controlling and improving quality of production in many areas of application, for example, in industry and manufacturing, finance and economics, epidemiology and health care, environmental sciences and other fields. Control charts are usually designed and evaluated under the assumption that the observations from a process are independent and identically distributed (i.i.d.) and from a normal distribution. In real applications, there are many processes which follow a non-normal distribution, for example, an Exponential, Weibull or Gamma distribution (see, e.g., Borror et al. [1]; Stoumbos and Reynolds [2]; Mititelu et al. [3]). Processes with data from a non-normal distribution need to be monitored by appropriate control charts.

In the past few decades, cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) charts have been proposed as good alternatives to the Shewhart chart for detecting small shifts. The CUSUM chart was initially presented by Page [4]. It has been shown that CUSUM charts are asymptotically optimal under minimax type criteria (see, e.g., Lorden [5]; Shiryayev [6]). The EWMA chart was initially introduced by Roberts [7]. It is a very flexible and effective chart for detecting small changes and has the advantage of showing robustness to non-normality (Borror et al. [1]; Stoumbos and Reynolds [2]). When the quality characteristic cannot be measured on a continuous scale, as, for example, in counting the number of defective products or the number of nonconformities in a production process, an attribute control chart must be used. Commonly-used attribute control charts are p, np, c, and u charts. EWMA and CUSUM methods for attribute data have also been applied to discrete processes (see, e.g., Montgomery [8]; Woodall and Adams [9]; Alwan [10]). Recently, the Moving Average control chart (MA) has been introduced for both continuous and discrete processes (see, e.g., Montgomery [8]; Alwan [10]). Michael and Khoo [11]) have studied the MA chart for monitoring the non-conforming or defective fraction in discrete processes.

The Average Run Length is an important characteristic for SPC charts. It is the expectation of the time before the control chart gives a false alarm that an in-control process has gone out-of-control. A second important characteristic for SPC charts is the Average Delay time. This is the expectation of the time between a process going out-of-control and the control chart giving the alarm that the process has gone out-of-control. The ARL of an acceptable chart should be large and the AD should be small. Many methods for evaluating the ARL and AD for control charts have been studied in the literature. A simple approach that is often used to test other methods is Monte Carlo (MC) simulation. Roberts [7] studied the ARL for EWMA charts by using simulations for processes following a normal distribution and derived nomograms that can be used to find the ARL for a variety of parameter values. Brook and Evans [12] obtained an approximate formula for the ARL of an EWMA chart by using a finite-state Markov Chain Approach (MCA). Crowder [13] used numerical quadrature methods to solve the exact Integral Equations (IE) for the ARL for the normal distribution. Knoth [14] and Borror et al. [1] have used the MCA to examine the ARL performance of the EWMA chart for both skewed and heavy-tailed symmetric non-normal distributions. Gan [15] studied the ARL for EWMA control charts for the exponential distribution by using differential equations. Recently, Areepong and Novikov [16] have derived explicit formulas for ARL and AD for EWMA charts for the exponential distribution. As discussed earlier, MC, MCA and IE are the most popular methods for evaluating the characteristics of control charts. These methods have the following features: MC is simple to program and is convenient for controlling and testing accuracy of analytical approximations. However, MC is usually based on a large number of sample trajectories so it is very time consuming. Moreover, it is difficult to obtain optimal designs using MC. MCA is considered as a popular technique. It is based on approximation of Markov Chains by using matrix inversions. Although there are at present no theoretical results on accuracy of this procedure, the results have been tested by direct comparison with MC simulations. IE is the most advanced method currently available. However, the results for ARL and AD usually cannot be obtained analytically and intensive programming or specialized software is required to obtain numerical results even for the case of the normal distribution. Martingale-based techniques have recently been introduced to compute ARL and AD for EWMA charts for a variety of light-tailed distributions (Sukparungsee and Novikov [17]). They are effective alternatives to traditional approaches as they are fast and easily to implement. However, they cannot be used on heavy-tailed distributions. In this paper, analytical formulas are derived for ARL and AD for MA charts when observations are from a binomial distribution. The performance of these analytical formulas is compared with numerical results obtained by simulation for the np and EWMA charts. The results show that the MA chart is superior to the other charts when the magnitudes of shift are moderate and large.

2. Control Charts and Their Properties

In this paper, SPC charts are considered under the assumption that sequential observations $X_1, X_2, ...$ of some process are identical, independently distributed random variables with a distribution function $F(x, n, p)$, where n is the total number of observations and p is a control parameter. It is assumed that while the process is in-control and $p = p_1 > p_0$ when the process goes out-of-control. It is assumed that there is a change-point time $\theta \leq \infty$ at which the parameter changes from $p = p_0$ to $p = p_1$. Note that $\theta = \infty$ means that the process always remains in the in-control state. All popular charts, such as Shewhart, Cumulative Sum (CUSUM) and EWMA charts (see e.g. [4, 7])are based on some function of parameter values that is used as a criterion for a process to go "out-of-control" if this function value goes above an upper control limit (UCL) or below a lower control limit (LCL). The minimum time required for a chart to signal out-of-control is defined as the stopping (alarm) time .

Let $E_\theta(.)$ denote the expectation that the change-point from $p = p_0$ to $p = p_1$ for a distribution function $F(x, n, p)$ occurs at time θ, where $\theta \leq \infty$. In the literature on quality control the quantity $E_\infty(\tau)$ is called the Average Run Length (ARL) of the chart for the given process. A typical condition imposed on an ARL is that:

$$ARL = E_\theta(\tau) = T, \tag{1}$$

(1) where T is given (usually large). For given distribution function and chart, this condition then determines choices for the UCL and LCL.

A typical definition of the AD is that

$$AD = E_1(\tau|\tau \geq 1), \tag{2}$$

i.e., that the change point occurs at $\theta = 1$. One could expect that a sequential control chart has a near optimal performance if its AD is close to a minimal value. There are many other criteria that could be used for optimality of an SPC chart. However, in practice, ARL and AD remain the most convenient and popular characteristics used for comparisons of different charts. For an np chart, let observations $X_1, X_2, ..., X_m$ be i.i.d random variables with binomial distribution, where X_i is the number of nonconforming items in sample i of m samples of size n. For these observations, the 3σ upper and lower control $n\bar{p}$ limits at after m samples are defined by

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}, \quad LCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})},$$

where $\bar{p}$ is estimated by $\bar{p} = \frac{\sum_{i=1}^{m} X_i}{nm}$.

The alarm time for the np chart is given by:

$$\tau = \inf\{i > 0 : X_i > UCL \quad \text{or} \quad X_i < LCL\}.$$

An EWMA chart is defined by a recursive formula

$$Y_i = (1-\lambda)Y_{i-1} + \lambda X_i, \quad i = 1, 2, ...$$

where $\lambda \in (0,1)$ is a smoothing parameter, X_i is a number of non-conforming items at i and Y_i is the weighted average between current and previous observations at i. The target mean is supposed to be steady and the initial value X_0 is usually chosen to be the process mean p_0. The upper control limit of the EWMA chart is the following:

$$UCL = h_U = p_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}},$$

and the lower control limit is:

$$LCL = h_L = p_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}},$$

where L is a constant to be chosen and σ is a standard deviation of a known underlying probability distribution. The process will be declared to be in an out-of-control state when $Y_i > UCL$ or $Y_i < LCL$. The alarm time for the EWMA is then given by

$$\tau = \inf\{i > 0 : Y_i > UCL \quad \text{or} \quad Y_i < LCL\}.$$

A Moving Average control chart is defined by the following statistics:

$$M_i = \frac{X_i + X_{i-1} + ... + X_{i-w+1}}{w} = \frac{\sum\limits_{j=i-w+1}^{i} X_j}{w}, i \geq w.$$

and

$$M_i = \frac{X_i + X_{i-1} + ... + X_{i-w+1}}{w} = \frac{\sum\limits_{j=1}^{i} X_j}{w}, 1 \leq i < w.$$

where w is the width of the moving average chart. For period $i \geq w$ the 3σ upper and lower control limits are given

$$UCL = np + 3\sqrt{\frac{np(1-p)}{w}}, \quad UCL = np - 3\sqrt{\frac{np(1-p)}{w}},$$

and for periods $i < w$, $\sqrt{\frac{np(1-p)}{w}}$ is replaced with $\sqrt{\frac{np(1-p)}{i}}$.

The alarm time for the MA procedure is given by

$$\tau = \inf\{i > 0 : M_i > UCL \quad \text{or} \quad M_i < LCL\}.$$

There are many other criteria for optimality of SPC (see e.g. [5, 6]); however, in practice, ARL and AD remain the most popular characteristics which are convenient to use for comparisons of different charts.

3. The Explicit Formula for Evaluating ARL for MA Chart

The ARL values of a Moving Average control chart can be derived as follows.

Let $ARL = n$, then

$$\begin{aligned}
\frac{1}{ARL} &= (\frac{1}{n})P(\text{o.o.c signal at time} \quad i < w) \\
&\quad + [\frac{n-(w-1)}{n}]P(\text{o.o.c signal at time} \quad i \geq w) \\
&= \frac{1}{n}\{\sum_{i=1}^{w-1}[P(\frac{\sum_{j=1}^{i} np_j}{i} > UCL_i) + P(\frac{\sum_{j=1}^{i} np_j}{i} < LCL_i)]\} \\
&\quad + [\frac{n-(w-1)}{n}][P(\frac{1}{w}\sum_{j=i-w+1}^{i} np_j > UCL_w) \\
&\quad + P(\frac{1}{w}\sum_{j=i-w+1}^{i} np_j < LCL_w)] \\
&= \frac{1}{n}\{\sum_{i=1}^{w}[P(\sum_{j=1}^{i} np_j > np_0 + 3\sqrt{\frac{np_0(1-p_0)}{i}})] \\
&\quad + [P(\sum_{j=1}^{i} np_j < np_0 - 3\sqrt{\frac{np_0(1-p_0)}{i}})]\} \\
&\quad + [\frac{n-(w-1)}{n}][P(\sum_{j=i-w+1}^{i} np_j > np_0 + 3\sqrt{\frac{np_0(1-p_0)}{w}}) \\
&\quad + P(\sum_{j=i-w+1}^{i} np_j < np_0 - 3\sqrt{\frac{np_0(1-p_0)}{w}})]
\end{aligned}$$

If we let

$$Z_1 = \frac{\sum_{j=1}^{i} np_j - np}{\sqrt{\frac{np(1-p)}{i}}} and \quad Z_2 = \frac{\sum_{j=i-w+1}^{i} np_j - np}{\sqrt{\frac{np(1-p)}{w}}},$$

then

$$\begin{aligned} \frac{1}{ARL} = \frac{1}{n}\{\sum_{i=1}^{w}[P(Z_1 > \frac{np_0 + 3\sqrt{\frac{np_0(1-p_0)}{i}} - np}{\sqrt{\frac{np(1-p)}{i}}}) \\ + P(Z_1 < \frac{np_0 - 3\sqrt{\frac{np_0(1-p_0)}{i}} - np}{\sqrt{\frac{np(1-p)}{i}}})]\} \\ + [\frac{n-(w-1)}{n}][P(Z_2 > \frac{np + 3\sqrt{\frac{np(1-p)}{w}} - np}{\sqrt{\frac{np(1-p)}{w}}}) \\ + P(Z_2 < \frac{np - 3\sqrt{\frac{np(1-p)}{w}} - np}{\sqrt{\frac{np(1-p)}{w}}})]. \end{aligned} \tag{3}$$

Let

$$U = \sum_{i=1}^{w-1}[P(Z_1 > \frac{np_0 + 3\sqrt{\frac{np_0(1-p_0)-np}{i}}}{\sqrt{\frac{np(1-p)}{i}}}) + P(Z_1 < \frac{np_0 - 3\sqrt{\frac{np_0(1-p_0)}{i}} - np}{\sqrt{\frac{np(1-p)}{i}}})],$$

$$V = \sum_{i=1}^{w-1}[P(Z_2 > \frac{np_0 + 3\sqrt{\frac{np_0(1-p_0)-np}{w}}}{\sqrt{\frac{np(1-p)}{w}}}) + P(Z_2 < \frac{np_0 - 3\sqrt{\frac{np_0(1-p_0)}{w}} - np}{\sqrt{\frac{np(1-p)}{w}}})].$$

Then, on substituting U and V into Equation 3, we obtain:

$$\begin{aligned} \frac{1}{n} &= \frac{1}{n}U + [\frac{n-(w-1)}{n}]V \\ n &= (1-U)V^{-1} + (w-1). \end{aligned}$$

As mentioned above, the value of the parameter p is equal to p_0 when the process is in-control. Therefore, substituting $p = p_0$ into Equation 3, we obtain the formula for the ARL as

$$\begin{aligned} ARL = (1 - \sum_{i=1}^{w-1}[P(Z_1 > 3) + P(Z_1 < -3)])[P(Z_2 > 3) \\ + P(Z_2 < -3)]^{-1} + (w-1). \end{aligned} \tag{4}$$

The formula for the AD can be obtained in a similar manner to the formulas for the ARL. When the process is out-of-control, the value of the parameter p in equation 3 will be $p = p_1$. The formula for AD can therefore be written as follows:

$$\begin{aligned} AD = (1 - \sum_{i=1}^{w-1}[P(Z_1 > \frac{np_0 + 3\sqrt{\frac{np_0(1-p_0)}{i}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{i}}}) \\ + P(Z_1 < \frac{np_0 - 3\sqrt{\frac{np_0(1-p_0)}{i}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{i}}})]) \\ \times [P(Z_2 > \frac{np_0 + 3\sqrt{\frac{np_0(1-p_0)}{w}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{w}}}) \\ P(Z_2 > \frac{np_0 - 3\sqrt{\frac{np_0(1-p_0)}{w}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{w}}})]^{-1} + (w-1). \end{aligned} \tag{5}$$

For general width of control limit (H), the formula for the desired ARL can be obtained by replacing 3 by H in equation 4. Then, the

$$\begin{aligned} ARL = (1 - \sum_{i=1}^{w-1}[P(Z_1 > H) + P(Z_1 < -H)][P(Z_2 > H) \\ + P(Z_2 < -H]^{-1} + (w-1). \end{aligned} \tag{6}$$

When the process is out-of-control, the value of the parameter p in equation 3 will be $p = p_1$. The formula for AD for a width of control limit H, can therefore be written as follows:

$$AD = (1 - \sum_{i=1}^{w-1}[P(Z_1 > \frac{np_0 + H\sqrt{\frac{np_0(1-p_0)}{i}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{i}}}) \tag{7}$$

$$+ P(Z_1 < \frac{np_0 - H\sqrt{\frac{np_0(1-p_0)}{i}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{i}}})])$$

$$\times [P(Z_2 > \frac{np_0 + H\sqrt{\frac{np_0(1-p_0)}{w}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{w}}})$$

$$P(Z_2 > \frac{np_0 - H\sqrt{\frac{np_0(1-p_0)}{w}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{w}}})]^{-1} + (w-1).$$

4. Comparison of Performance of Control Charts

In this section, the numerical results for ARL and AD for an MA chart were calculated from Equation 5 and Equation 7. Table 1 shows a comparison of these MA chart values with values obtained for np and EWMA charts by simulation. The parameter values for np, MA and EWMA charts were chosen by setting the desired ARL = 370, the value of the in-control parameter = 0.02 and the out-of-control parameter $p_1 \subseteq [0.025; 01]$. For the EWMA procedure, the parameter values $\lambda = 0.05$ and $h = 2.504$ and 3.610 were used for n=100 and 150 respectively. The MA chart shows a better performance than the other charts when the value of the parameter p had a moderate shift from its in-control value $p_0 = 0.02$ to its out-of-control valuep_1. For example, when $p_1 < 0.04$ the MA chart with $w = 5$ shows the best performance because it gives the minimum AD value. When $p_1 > 0.06$, the MA chart with $w = 2$ performs better than the np chart. However, the performance of the EWMA chart is superior to np and MA charts when $p_1 < 0.027$, i.e., when the shift is small. Note that, calculations with the explicit formulas in Equation 5 and 7 are simple and very fast with computational times of less than 1 second. Table 2 gives a comparison of results for ARL and AD for MA, np and EWMA charts presented in a similar manner to Table 1, but for a desired ARL=500. For the

EWMA procedure, the parameter values λ=0.05 and h=2.5404 and 3.655 were used for n=100 and 150 respectively. For MA, the boundary value is equal to 3.0905 for fixed ARL=500. The results for ARL=500 are in good agreement with the results for ARL=370. Consequently, use of the proposed formulas for ARL and AD for MA charts are easy to implement, can greatly reduce computation times, and are useful to practitioners.

n	p	ARL and AD					
		np	MA				EWMA
			w=2	w=3	w=4	w=5	λ=0.05,h=2.504
100	0.02	370.398	370.398	370.398	370.398	370.398	371.002
	0.025	98.0295	74.4752	59.6019	49.4974	42.2880	**33.9052**
	0.027	60.0202	41.1259	30.9959	24.8787	**20.9212**	22.1757
	0.03	31.7591	19.723	14.3492	11.5344	**9.9558**	14.3878
	0.04	7.6008	4.5863	3.7708	**3.5934**	3.6636	6.5875
	0.05	3.4313	2.3952	**2.2968**	2.4023	2.5475	4.3581
	0.07	1.6046	**1.4332**	1.4864	1.5303	1.5511	2.7309
	0.09	1.1960	**1.1678**	1.1948	1.1994	1.1901	2.0980
	0.1	1.1144	**1.1035**	1.1109	1.1120	1.1122	1.8975
n	p	np	w=2	w=3	w=4	w=5	λ=0.05,h=3.610
	0.02	370.398	370.398	370.398	370.398	370.398	369.3440
	0.025	84.6748	59.3018	45.0696	36.1697	30.2216	**25.762**6
	0.027	48.799	30.6212	22.0592	17.3371	**14.5133**	16.9460
	0.03	24.2102	13.9137	9.9242	8.0693	**7.1679**	11.1229
	0.04	5.3708	3.3010	**2.9045**	2.9240	3.0738	5.2437
	0.05	2.4701	**1.8802**	1.9035	2.0655	2.0912	3.5107
	0.07	1.2901	**1.2349**	1.2612	1.2703	1.2721	2.2631
	0.09	1.0675	**1.0634**	1.0659	1.0661	1.0661	1.7696
	0.1	1.032	**1.0310**	1.0317	1.0317	1.0316	1.5766

Table 1: Comparison of ARL and AD from proposed formula with np and EWMA charts for given ARL=370

5. Conclusion

Simple explicit formulas have been derived for the Average Run Length (ARL) and Average Delay (AD) for an MA control chart for observations from a bi-

n	p	ARL AND AD					
		np	MA				EWMA
			w=2	w=3	w=4	w=5	λ=0.05,H=2.5404
100	0.02	500.451	500.451	500.451	500.451	500.451	500.773
	0.025	123.190	92.455	73.2677	60.3173	51.1096	**38.0319**
	0.027	73.5919	49.6351	36.9327	29.2953	**24.3535**	24.5184
	0.03	37.7704	22.9943	16.4475	13.0111	**11.0620**	15.3631
	0.04	8.4625	4.9653	3.9987	**3.7595**	3.8030	7.0508
	0.05	3.6801	2.5026	**2.3719**	2.4716	2.6205	4.6583
	0.07	1.6551	**1.4608**	1.5152	1.5627	1.5859	2.9108
	0.09	1.2122	**1.1797**	1.1983	1.2034	1.2043	2.1594
	0.1	1.1241	**1.1115**	1.1241	1.1209	1.1211	1.9369
n	p	np	w=2	w=3	w=4	w=5	λ=0.05,H=3.655
	0.02	500.451	500.451	500.451	500.451	500.451	500.726
	0.025	105.737	72.9582	54.7943	43.5046	35.9768	**29.0941**
	0.027	59.3521	36.5482	25.9202	20.0727	**16.5631**	18.5430
	0.03	28.5144	16.002	11.1791	8.9178	**7.7901**	12.0859
	0.04	5.9062	3.5177	3.0370	**3.0290**	3.1724	5.6413
	0.05	2.6147	**1.9428**	1.9546	2.0596	2.1515	3.7744
	0.07	1.3168	**1.2519**	1.2803	1.2906	1.2928	2.3941
	0.09	1.0740	**1.0691**	1.0720	1.0722	1.0722	1.8622
	0.1	1.0353	**1.0341**	1.0353	1.0348	1.0348	1.6891

Table 2: Comparison of ARL and AD from proposed formula with np and EWMA charts for given ARL=500

nomial distribution. The new formulas are very easy to use and are easy to calculate and program. The performance of the new formulas for ARL and AD has been compared with the performance of np and EWMA control charts. For the binomial distribution, the results showed that for given ARL=370 and 500, the performance of the MA chart is superior to np and EWMA charts when the shifts in parameter values from an in-control to an out-of-control state are moderate or large.

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References

[1] C.M. Borror, D.C. Montgomery and G.C. Runger, Robustness of the EWMA Control Chart to Non-normality, *Journal of Quality Technology*, **31** (1999), 309-316.

[2] Z.G. Stoumbos and M.R. Reysnolds, Robustness to non-normality and autocorrelation of individuals control charts for monitoring the process mean and variance, *Journal of Statistical Computation and Simulation*, **66** (2000), 145-187.

[3] G. Mititelu, Y. Areepong, S. Sukpakrungsee and A. Novikov, Explicit Analytical Solutions for the Average Run Length of CUSUM and EWMA Charts, East-West Journal of Mathematics, **1** (2010), 253-265.

[4] E.S. Page, Continuous inspection schemes, *Biometrika*, **41** (1954), 100-114.

[5] G. Lorden, Procedures for reacting to a change in distribution, *Annals of Mathematical Statistics*, **42** (1971), 1897-1908.

[6] A.N. Shiryaev, *Optimal Stopping Rules*, Springer-Verlag, (1978).

[7] S.W. Roberts, Control chart tests based on geometric moving average, *Technometrics*, **1** (1959), 239-250.

[8] D.C. Montgomery, *Introduction to Statistical Quality Control*, Chichester: Wiley, New York, (2005).

[9] W.H. Woodall and B.M. Adams, The statistical design of CUSUM charts, *Quality Engineering*, **4** (1993), 559-570.

[10] L.C. Alwan, *Statistical Process Analysis*, Irwin, McGraw-Hill: New York (1980).

[11] M.B.C. Khoo, A Moving Average Control Chart for Monitoring the Fraction Non- conforming, *Journal of Quality and Reliability Engineering International*, **20** (2004), 617-635.

[12] D. Brook and D.A. Evans, An approach to the probability distribution of Cusum run length, *Biometrika*, **9** (1972), 539-548.

[13] S.V. Crowder, A simple method for studying run length distributions of exponentially weighted moving average charts, *Technometrics*, **29** (1978), 401-407.

[14] S. Knoth, Accurate ARL calculation for EWMA control charts monitoring normal mean and variance simultaneously, *Sequential Analysis*, **26** (2006), 251-263.

[15] F.F. Gan, Designs of one- and two-sided exponential EWMA char, *Journal of Quality Technology*, **30** (1998), 55-69.

[16] Y. Areepong and A.A. Novikov, Martingale approach to EWMA control chart for changes in Exponential distribution, *Journal of Quality Measurement and Analysis*, **4** (2008), 197-203.

[17] S. Sukparungsee and A.A. Novikov, Analytical approximations for detection of a change-point in case of light-tailed distributions, *Journal of Quality Measurement and Analysis*, **4** (2008), 373-380.