International Journal of Pure and Applied Mathematics
Volume 80 No. 4 2012, 477-494
ISSN: 1311-8080 (printed version)
url: http://www.ijpam.eu

ijpam.eu

EXISTENCE AND STABILITY OF COLLINEAR EQUILIBRIUM POINTS IN ELLIPTICAL RESTRICTED THREE BODY PROBLEM UNDER THE EFFECTS OF PHOTOGRAVITATIONAL AND OBLATENESS PRIMARIES

C. Ramesh Kumar[1], A. Narayan[2 §]

[1]Department of Mathematics
R.C.E.T. Bhilai, 490024 (C.G.), INDIA

[2]Department of Mathematics
Bhilai Institute of Technology
Durg, 491001 (C.G.), INDIA

Abstract: The effects of the photogravitational and oblateness of the bigger primary and oblateness of the smaller primary to study the existence and stability of collinear equilibrium points in, the planar elliptical restricted three body problem have been discussed. We have analysed and investigated the stability of one of the collinear equilibrium points. The technique adopted in this research paper used Sahoo and Ishwar (see [1]) have been exploited to discuss the stability of one of the collinear equilibrium points. We have also adopted the simulation technique, using *MATLAB* 6.1 to analyze the stability of the system. We have also traced the different curves of zero velocity.

Key Words: collinear points, elliptical restricted three body problem, stability

1. Introduction

The present paper deals with the effects of the photogravitational and oblate-

Received: April 19, 2012

§Correspondence author

ness of the bigger primary and oblateness of the smaller primary on the existence and stability of collinear equilibrium points of the planar elliptical restricted three body problem. The elliptical restricted three body problem describes the dynamical system more accurately on account of realistic assumption of motion of the primaries are subjected to move along the elliptical orbit. Danby (see [2]) studied the elliptical restricted three body problem and used numerical integration to determine the linear stability of the elliptical Lagrange orbit. He obtained a stability diagram in the $\mu - e$ plane using the mass value μ and the eccentricity e. Ammer (see [3]) studied solar radiation pressure on the Lagrangian points in the elliptical restricted three body problem. Khasan (see [4]) studied libration solution to the photogravitational restricted three body problem by considering both of the primaries are radiating. He also investigated the stability of collinear and triangular points. Khasan (see [5]) also studies three dimensional periodic solutions to the photogravitational Hill problem. Sahoo and Ishwar (see [1]) studied the stability of collinear equilibrium points in the generalised photogravitational elliptical restricted three body problem. Selaru and Cucu-Dumitrescus (see [6]) performed an analytical investigation concerning the structure of asymptotic perturbative approximation for small amplitude motions, if the third point mass is in the neighboured of a Lagrangian equilateral libration position in the planer, elliptical restricted three bodies. Floria (see [7]), undertaken an approximate integration of the elliptical restricted three body problem by means, which leads to an approximate solution to the differential system of canonical equations of motion derived from the chosen Hamiltonian functions. In the present works, we have studied the existence and stability of collinear equilibrium points in elliptical restricted three body problem under the effects of the photogravitational and oblateness of the bigger primary and oblateness of the smaller primary. We have investigated the existence and stability of collinear equilibrium points, of the problem using the technique adopted in research paper, Sahoo and Ishwar (see [1]). The dimensionless variables are introduced by using the distance r between primaries given by:

$$r = \frac{a\left(1 - e^2\right)}{(1 + e\cos v)}$$

where a and e are the semi-major axis and the eccentricity of the elliptical orbit of the either primary around other and v is the true anomaly of one of the primary of mass m_1. A coordinate system which rotates with the variable angular velocity n is introduced. This angular velocity is given by

$$\frac{dn}{dt^*} = \frac{k\,(m_1 + m_2)^{\frac{1}{2}}\,(1 + e\cos v)^2}{a^{\frac{3}{2}}\,(1 - e^2)^{\frac{3}{2}}},$$

where t^* is dimensionless time.

The equation follows from the principal of the conversation of angular momentum in the problem of two bodies formed by the primaries of masses m_1 and m_2. This principle is expressed as follows:

$$nr^2 = \left[a\left(1-e^2\right)k^2\left(m_1+m_2\right)\right]^{\frac{1}{2}}$$

where $k = k_1 + k_2$where k_1 and k_2 are the product of the universal gravitational constants with the mass of primaries.

We have adopted simulation technique to analyze the stability of the system using *MATLAB* 6.1 version software. One of the collinear equilibrium points of elliptical restricted three body problem under the oblate and radiation primaries are found unstable. We have also traced the different curves of zero velocity.

2. Location of Collinear Equilibrium Points

The equation of motion for the photogravitational planar elliptical restricted three body problem with the effects of photogravitational and oblateness of the bigger primary and oblateness of the smaller primary in a dimensionless, barycentric, pulsating rotating, co-ordinate system are as follows,

$$x'' - 2y' = \frac{1}{(1+e\cos v)}\left(\Omega_x\right);$$

and

$$y'' + 2x' = \frac{1}{(1+e\cos v)}\left(\Omega_y\right), \tag{2.1}$$

where

$$\Omega = \frac{x^2+y^2}{2} + \frac{1}{1+3\left(\dfrac{A_1+A_2}{2}\right)}\left[\frac{(1-\mu)\,q}{r_1} + \frac{\mu}{r_2} + \frac{(1-\mu)\,qA_1}{2r_1^3} + \frac{\mu A_2}{2r_2^3}\right]$$

and

$$r_1^2 = (x+\mu)^2 + y^2, r_2^2 = (x-1+\mu)^2 + y^2, \tag{2.2}$$

where Ω_x denotes the partial differentiating $\Omega$with respect to x and, Ω_y denotes the partial differentiating Ω with respect to y ; q is the source of radiation of bigger primary. Now we averaged the potential function of the problem with respect to true anomaly. which is expressed as follows:

$$\Omega^{\star} = \frac{1}{2\pi}\int_0^{2\pi} G dv,$$

where

$$G = \frac{1}{(1 + e\cos v)}\left[\frac{x^2 + y^2}{2} + \frac{1}{1 + 3\left(\dfrac{A_1 + A_2}{2}\right)}\left\{\frac{(1-\mu)\,q}{r_1} + \frac{\mu}{r_2} + \frac{(1-\mu)\,qA_1}{2r_1^3} + \frac{\mu A_2}{2r_2^3}\right\}\right],$$

$$\Omega^{\star} = \frac{1}{(1-e^2)^{\frac{1}{2}}}\left[\frac{x^2 + y^2}{2} + \frac{1}{n^2}\left\{\frac{(1-\mu)\,q}{r_1} + \frac{\mu}{r_2} + \frac{(1-\mu)\,qA_1}{2r_1^3} + \frac{\mu A_2}{2r_2^3}\right\}\right] \tag{2.3}$$

is the modified potential function in the equation of motion in our problem. where e is the eccentricity of the orbit, μ is the mass parameter; v is the true anomaly of the system. By an analysis similar to Mc Cusky (see [8]) for the existence and position of collinear equilibrium points of planar elliptical restricted three body problem, which is given as follows (see Narayan, Ramesh, [10], [11]):

$$\frac{\partial\Omega^*}{\partial x} = \frac{\partial\Omega^*}{\partial y} = 0,$$

which gives the expression mentioned below:

$$x - \frac{1}{n^2}\left\{\frac{(1-\mu)\,(x+\mu)\,q}{r_1^3} + \mu\frac{(x-1+\mu)}{r_2^3} + \frac{3A_1\,(1-\mu)\,(x+\mu_1)\,q}{2r_1^5} + \frac{3A_2\mu\,(x-1+\mu)}{2r_2^5}\right\} = 0, \tag{2.4}$$

and

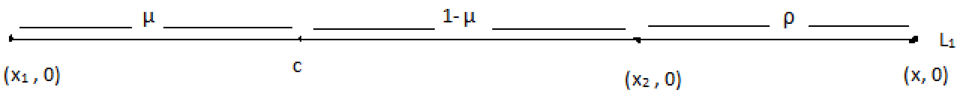
$$y\left[1 - \frac{1}{n^2}\left\{\frac{(1-\mu)\,q}{r_1^3} + \frac{\mu}{r_2^3} + \frac{3A_1\,(1-\mu)\,q}{2r_2^5} + \frac{3A_2\mu}{2r_2^5}\right\}\right] = 0. \tag{2.5}$$

In the elliptical restricted three body problem, however stationary equilibrium points in the same sense do not exist, for constant values of x and y, which cannot be found. Nevertheless, the libration points of the circular problem correspond, in the elliptical problem to the equilibrium five points that oscillate about average values. The three collinear points oscillate along the x-axis about locate roughly analogous to the their position in circular problem. The location of the collinear libration points relative to the rotating frame can be determined by assuming the existence of the collinear points in the elliptical problem such that$\dot{y} = y = o$ and that the ratio of x to r is constant.

Hence, the Lagrangian collinear points of the x-axis are given by setting $y = o$ in equation (2.4), we have:

$$2\left(x+\mu\right)^2\left(x+\mu-1\right)^2 \times\left\{xn^2\left(x+\mu\right)^2\left(x+\mu-1\right)^2-\left(1-\mu\right)q\left(x+\mu-1\right)^2-\left(x+\mu\right)^2\mu\right\} - 3\left(1-\mu\right)qA_1\left(x+\mu-1\right)^4-3\mu A_2\left(x+\mu\right)^4=0. \quad (2.6)$$

The equation (2.6) is the ninth degree equation in x, so we shall get nine roots of the equation and corresponding to nine values of x. Three equilibrium points lie on the x-axis. We have one of the values of roots, x is greater than x_2, another root lies between $x_1 and x_2$and other root is less than x_1.



Assuming $x > x_2$ we consider $x - x_2 = \rho; so that x - x_1 = 1 + \rho$, and we also have $x = 1 + \rho - \mu$; Substituting these values of the problem in equation (2.6), we get:

$$2\left(1+\rho\right)^2\rho^2\left\{n^2\left(1+\rho-\mu\right)\left(1+\rho\right)^2\rho^2-\left(1-\mu\right)q\rho^2-\mu\left(1+\rho\right)^2\right\} - 3\left(1-\mu\right)qA_1\rho^4-3\mu A_2\left(1+\rho\right)^4=0,$$

$$2n^2\rho^9+2n^2\left(5-\mu\right)\rho^8+2n^2\left(10-4\mu\right)\rho^7+\left\{2n^2\left(10-6\mu\right)-2q\left(1-\mu\right)-2\mu\right\}\rho^6 + \left\{2n^2\left(5-4\mu\right)4q\left(1-\mu\right)-8\mu\right\}\rho^5 + \left\{2n^2\left(1-\mu\right)-2q\left(1-\mu\right)-12\mu-3\left(1-\mu\right)qA_1-3\mu A_2\right\}\rho^4 - \left(8\mu+12\mu A_2\right)\rho^3-\left(2\mu+18\mu A_2\right)\rho^2-12\mu A_2\rho-3\mu A_2=0. \quad (2.7)$$

Assuming γ be the value of ρ in the classical restricted three body problem, when $e = 0$, $A_1 = A_2 = 0$ and $q = 1$.

For the presence of these terms, let the value of ρbe slightly changed and assuming the new value of ρ be defined by

$$\rho = \gamma + \delta \quad \text{and} \quad \delta \ll 1.$$

Substituting the value of ρ in the equation (2.7), setting the equation in the definite form, we get:

Let $q = 1 - \beta, \beta \ll 1$

$$\delta\left(P_1 + Q_1\beta + R_1A_1 + R_2A_2\right) = \left(L_1 + M_1\beta + N_1A_1 + N_2A_2\right), \tag{2.8}$$

where

$$P_1 = 18n^2\gamma^8 + 16n^2\left(5-\mu\right)\gamma^7 + 14n^2\left(10-4\mu\right)\gamma^6 + 6\left\{2n^2\left(10-6\mu\right)-2\right\}\gamma^5$$
$$+5\left\{2n^2\left(5-4\mu\right)-4\left(1+\mu\right)\right\}\gamma^4 + 4\left\{2n^2\left(1-\mu\right)-2\left(1+5\mu\right)\right\}\gamma^3 - 24\mu\gamma^2 - 4\mu\gamma,$$

$$\begin{aligned}
Q_1 =& 12\left(1-\mu\right)\gamma^5 + 20\left(1-\mu\right)\gamma^4 + 8\left(1-\mu\right)\gamma^3,\\
R_1 =& -12\left(1-\mu\right)\gamma^3,\\
R_2 =& -12\mu\gamma^3 - 36\mu\gamma^2 - 36\mu\gamma - 12\mu,\\
L_1 =& -\left[2n^2\gamma^9 + 2n^2\left(5-\mu\right)\gamma^8 + 2n^2\left(10-4\mu\right)\gamma^7 + \left\{2n^2\left(10-6\mu\right)-2\right\}\gamma^6\right.\\
&+\left\{2n^2\left(5-4\mu\right)-4(1+\mu)\right\}\gamma^5\\
&\left.+\left\{2n^2\left(1-\mu\right)-2\left(1+5\mu\right)\right\}\gamma^4 - 8\mu\gamma^3 - 2\mu\gamma^2\right],\\
M_1 =& -\left[2\left(1-\mu\right)\gamma^6 + 4\left(1-\mu\right)\gamma^5 + 2\left(1-\mu\right)\gamma^4\right],\\
N_1 =& 3\left(1-\mu\right)\gamma^4,\\
N_2 =& 3\mu\gamma^4 + 12\mu\gamma^3 + 18\mu\gamma^2 + 12\mu\gamma + 3\mu.
\end{aligned}$$

Now from the equation (2.8) we have

$$\begin{aligned}
\delta =& \frac{L_1 + M_1\beta + N_1A_1 + N_2A_2}{P_1 + Q_1\beta + R_1A_1 + R_2A_2},\\
\delta =& \frac{1}{P_1}\left[\left(L_1 + M_1\beta + N_1A_1 + N_2A_2\right)\left(1 - \frac{Q_1}{P_1}\beta + \frac{R_1}{P_1}A_1 + \frac{R_2}{P_1}A_2\right)\right], \qquad (2.9)\\
\delta =& \frac{1}{P_1}\left[L_1 + \left(M_1 - \frac{Q_1L_1}{P_1}\right)\beta + \left(N_1 - \frac{R_1L_1}{P_1}\right)A_1 + \left(N_2 - \frac{R_2L_1}{P_1}\right)A_2\right],
\end{aligned}$$

since

$$n^2 = \frac{1}{a}\left(1 + \frac{3e^2}{2} + \frac{3A_1}{2} + \frac{3A_2}{2}\right).$$

Neglecting the higher order terms, since $A_1, A_2 ande$ are very very small, we have:

$$P_1 = 18n^2\gamma^8 + 16n^2\left(5-\mu\right)\gamma^7 + 14n^2\left(10-4\mu\right)\gamma^6 + 6\left\{2n^2\left(10-6\mu\right)-2\right\}\gamma^5$$
$$+5\left\{2n^2\left(5-4\mu\right)-4\left(1+\mu\right)\right\}\gamma^4 + 4\left\{2n^2\left(1-\mu\right)-2\left(1+5\mu\right)\right\}\gamma^3 - 24\mu\gamma^2 - 4\mu\gamma.$$

Substituting the value of n^2 we get

$$P_1 = \frac{18}{a}\gamma^8 + \frac{16(5-\mu)}{a}\gamma^7 + \frac{28(5-2\mu)}{a}\gamma^6 + 12\left\{\frac{(10-6\mu)}{a} - 1\right\}\gamma^5$$
$$+ 5\left\{\frac{2(5-4\mu)}{a} - 4(1+\mu)\right\}\gamma^4 + 8\left\{\frac{(1-\mu)}{a} - (1+5\mu)\right\}\gamma^3$$
$$- 24\mu\gamma^2 - 4\mu\gamma + e^2\Big\{\frac{27}{a}\gamma^8 + \frac{24(5-\mu)}{a}\gamma^7 + \frac{42(5-2\mu)}{a}\gamma^6 + \frac{36(5-3\mu)}{a}\gamma^5$$
$$+ \frac{15(5-4\mu)}{a}\gamma^4 + \frac{12(1-\mu)}{a}\gamma^3\Big\}$$
$$+ \Big\{\frac{27}{a}\gamma^8 + \frac{24(5-\mu)}{a}\gamma^7 + \frac{42(5-2\mu)}{a}\gamma^6 + \frac{36(5-3\mu)}{a}\gamma^5 + \frac{15(5-4\mu)}{a}\gamma^4$$
$$+ \frac{12(1-\mu)}{a}\gamma^3\Big\}A_1$$
$$+ \Big\{\frac{27}{a}\gamma^8 + \frac{24(5-\mu)}{a}\gamma^7 + \frac{42(5-2\mu)}{a}\gamma^6 + \frac{36(5-3\mu)}{a}\gamma^5 + \frac{15(5-4\mu)}{a}\gamma^4$$
$$+ \frac{12(1-\mu)}{a}\gamma^3\Big\}A_2,$$

$$P_1^{-1} = X_1 + Y_1 e^2 + Y_1 A_1 + Y_1 A_2,$$

where

$$X_1 = \Big[\frac{18}{a}\gamma^8 + \frac{16(5-\mu)}{a}\gamma^7 + \frac{28(5-2\mu)}{a}\gamma^6 + 12\left\{\frac{(10-6\mu)}{a} - 1\right\}\gamma^5$$
$$+5\left\{\frac{2(5-4\mu)}{a} - 4(1+\mu)\right\}\gamma^4 + 8\left\{\frac{(1-\mu)}{a} - (1+5\mu)\right\}\gamma^3 - 24\mu\gamma^2 - 4\mu\gamma\Big]^{-1},$$

$$Y_1 = \Big(27\gamma^8 + 24(5-\mu)\gamma^7 + 42(5-2\mu)\gamma^6 + 36(5-3\mu)\gamma^5$$
$$+ 15(5-4\mu)\gamma^4 + 12(1-\mu)\gamma^3\Big)$$
$$\times \Big(a\Big[\frac{18}{a}\gamma^8 + \frac{16(5-\mu)}{a}\gamma^7 + \frac{28(5-2\mu)}{a}\gamma^6 + 12\left\{\frac{(10-6\mu)}{a} - 1\right\}\gamma^5$$
$$+5\left\{\frac{2(5-4\mu)}{a} - 4(1+\mu)\right\}\gamma^4 + 8\left\{\frac{(1-\mu)}{a} - (1+5\mu)\right\}\gamma^3 - 24\mu\gamma^2 - 4\mu\gamma\Big]\Big)^{-1},$$

similarly

$$L_1 = -\Big[2n^2\gamma^9 + 2n^2(5-\mu)\gamma^8 + 2n^2(10-4\mu)\gamma^7 + \left\{2n^2(10-6\mu) - 2\right\}\gamma^6$$

$$+\left\{2n^2\left(5-4\mu\right)-4(1+\mu)\right\}\gamma^5+\left\{2n^2\left(1-\mu\right)-2\left(1+5\mu\right)\right\}\gamma^4-8\mu\gamma^3-2\mu\gamma^2\Big].$$

Substituting the value of n^2, we get

$$\begin{aligned}L_1 =&-2\Big[\frac{1}{a}\gamma^9+\frac{(5-\mu)}{a}\gamma^8+\frac{2(5-2\mu)}{a}\gamma^7+\left\{\frac{6(10-6\mu)}{a}-1\right\}\gamma^6\\&+\left\{\frac{(5-4\mu)}{a}-2(1+\mu)\right\}\gamma^5\\&+\left\{\frac{(1-\mu)}{a}+(1+5\mu)\right\}\gamma^4-4\mu\gamma^3-\mu\gamma^2\Big]-\frac{3}{a}\Big\{\gamma^9+(5-\mu)\gamma^8\\&+2(5-2\mu)\gamma^7+(10-6\mu)\gamma^6+(5-4\mu)\gamma^5\\&+(1-\mu)\gamma^4\Big\}e^2+\left\{\gamma^9+(5-\mu)\gamma^8+2(5-2\mu)\gamma^7+(10-6\mu)\gamma^6\right.\\&\left.+(5-4\mu)\gamma^5+(1-\mu)\gamma^4\right\}A_1\\&+\left\{\gamma^9+(5-\mu)\gamma^8+2(5-2\mu)\gamma^7+(10-6\mu)\gamma^6+(5-4\mu)\gamma^5\right.\\&\left.+(1-\mu)\gamma^4\right\}A_2,\end{aligned}$$

$$L_1=U_1+V_1e^2+V_1A_1+V_1A_2,$$

where

$$\begin{aligned}U_1=&-2\Big[\frac{1}{a}\gamma^9+\frac{(5-\mu)}{a}\gamma^8+\frac{2(5-2\mu)}{a}\gamma^7+\left\{\frac{(10-6\mu)}{a}-1\right\}\gamma^6\\&+\left\{\frac{(5-4\mu)}{a}-2(1+\mu)\right\}\gamma^5+\left\{\frac{(1-\mu)}{a}+(1+5\mu)\right\}\gamma^4-4\mu\gamma^3-\mu\gamma^2\Big],\end{aligned}$$

$$\begin{aligned}V_1=-\frac{3}{a}\left\{\gamma^9+(5-\mu)\gamma^8+2(5-2\mu)\gamma^7+(10-6\mu)\gamma^6+(5-4\mu)\gamma^5\right.\\\left.+(1-\mu)\gamma^4\right\}.\end{aligned}$$

Hence substituting the value of $L_1, M_1, N_1, N_2, R_1, R_{2,}Q_1, P_1$ in the eqution (2.9), we obtain:

$$\begin{aligned}\delta =&U_1X_1+(U_1Y_1+V_1X_1)\left(e^2+A_1+A_2\right)+\left(X_1+Y_1e^2+Y_1A_1+Y_1A_2\right)\\&\Big[-2(1-\mu)\left\{\left(\gamma^6+2\gamma^5+\gamma^4\right)+\left(6\gamma^5+10\gamma^4+4\gamma^3\right)U_1X_1\right\}\Big]\beta\\&+\left(X_1+Y_1e^2+Y_1A_1+Y_1A_2\right)\\&\left[3\left(1-\mu\right)\left\{\gamma^4-4\gamma^3U_1X_1\right\}\right]A_1+\left(X_1+Y_1e^2+Y_1A_1+Y_1A_2\right)\end{aligned}$$

$$\left[3\mu\left\{\left(\gamma^4+4\gamma^3+6\gamma^2+4\gamma+1\right)+\left(4\gamma^3+12\gamma^2+12\gamma+4\right)U_1X_1\right\}\right]A_2.$$

We have $\rho=\gamma+\delta$, i.e.

$$\begin{aligned}\rho=&\gamma+U_1X_1+(U_1Y_1+V_1X_1)\left(e^2+A_1+A_2\right)+\left(X_1+Y_1e^2+Y_1A_1+Y_1A_2\right)\\&\Big[-2(1-\mu)\left\{\left(\gamma^6+2\gamma^5+\gamma^4\right)+\left(6\gamma^5+10\gamma^4+4\gamma^3\right)U_1X_1\right\}\Big]\beta\\&+\left(X_1+Y_1e^2+Y_1A_1+Y_1A_2\right)\\&\left[3\left(1-\mu\right)\left\{\gamma^4-4\gamma^3U_1X_1\right\}\right]A_1+\left(X_1+Y_1e^2+Y_1A_1+Y_1A_2\right)\\&\left[3\mu\left\{\left(\gamma^4+4\gamma^3+6\gamma^2+4\gamma+1\right)+\left(4\gamma^3+12\gamma^2+12\gamma+4\right)U_1X_1\right\}\right]A_2,\end{aligned}$$

where γ is the value of ρ in the classical case.

$$\begin{aligned}U_1=-2\Big[&\frac{1}{a}\gamma^9+\frac{(5-\mu)}{a}\gamma^8+\frac{2(5-2\mu)}{a}\gamma^7+\left\{\frac{(10-6\mu)}{a}-1\right\}\gamma^6\\&+\left\{\frac{(5-4\mu)}{a}-2(1+\mu)\right\}\gamma^5+\left\{\frac{(1-\mu)}{a}+(1+5\mu)\right\}\gamma^4-4\mu\gamma^3-\mu\gamma^2\Big],\end{aligned}$$

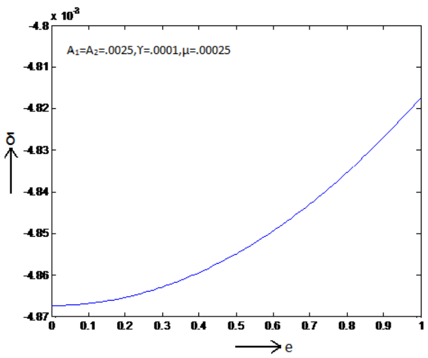
$$\begin{aligned}V_1=-\frac{3}{a}\big\{&\gamma^9+(5-\mu)\gamma^8+2(5-2\mu)\gamma^7+(10-6\mu)\gamma^6+(5-4\mu)\gamma^5\\&+(1-\mu)\gamma^4\big\},\end{aligned}$$

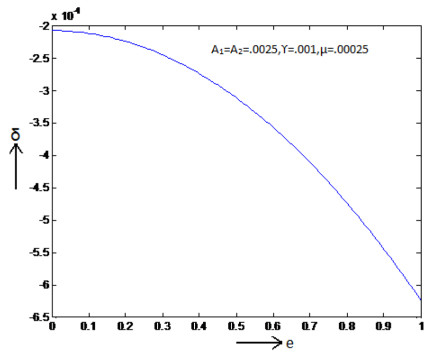
$$\begin{aligned}X_1=\Big[&\frac{18}{a}\gamma^8+\frac{16(5-\mu)}{a}\gamma^7+\frac{28(5-2\mu)}{a}\gamma^6+12\left\{\frac{(10-6\mu)}{a}-1\right\}\gamma^5\\&+5\left\{\frac{2(5-4\mu)}{a}-4(1+\mu)\right\}\gamma^4+8\left\{\frac{(1-\mu)}{a}-(1+5\mu)\right\}\gamma^3-24\mu\gamma^2-4\mu\gamma\Big]^{-1},\end{aligned}$$

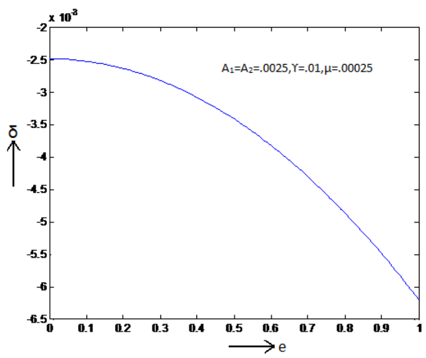
$$\begin{aligned}Y_1=&\left(27\gamma^8+24(5-\mu)\gamma^7+42(5-2\mu)\gamma^6+36(5-3\mu)\gamma^5\right.\\&\left.+15(5-4\mu)\gamma^4+12(1-\mu)\gamma^3\right)\\&\times\Bigg(a\Big[\frac{18}{a}\gamma^8+\frac{16(5-\mu)}{a}\gamma^7+\frac{28(5-2\mu)}{a}\gamma^6+12\left\{\frac{(10-6\mu)}{a}-1\right\}\gamma^5\\&+5\left\{\frac{2(5-4\mu)}{a}-4(1+\mu)\right\}\gamma^4\\&+8\left\{\frac{(1-\mu)}{a}-(1+5\mu)\right\}\gamma^3-24\mu\gamma^2-4\mu\gamma\Big]\Bigg)^{-1},\end{aligned}$$

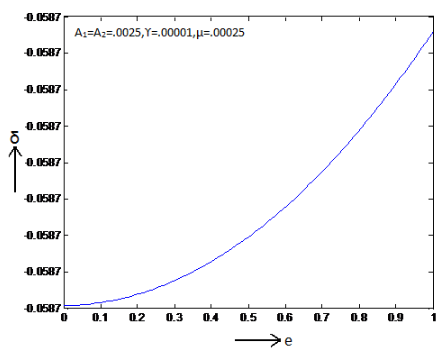
where μ is the mass parameter, $\beta = 1 - q$ where q is the radition parameter, $A_1 and A_2$ are oblateness parameter of $m_1 and m_2, e$ is the eccentricity and a is the semimajor axis of the orbit, where γ is the value of ρ, the distance between L_1 and the smaller primary.

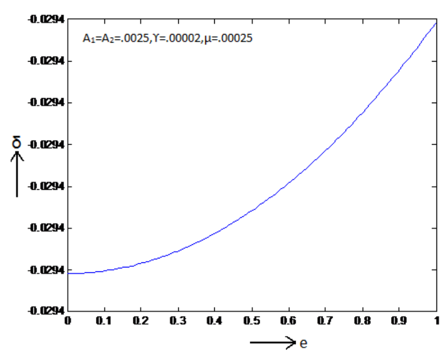
In order to investigate the effects of the oblateness of the primary on L_1, the simulation technique has been used by applying *MATLAB* 6.1, version software. As far as numerical calculation of δ is concern, we have used $a = .0001, \beta = .0001$ and plotted the curve between the deviation and eccentricity. Similarly, we have also plotted the curve between by taking into account of various values of oblateness parameters, which is indicated that the deviation δ is decreasing in one case, while this δ is increasing in some other cases. We have also investigate the effect of β on the position of L_1 but this effect is very insignificant, and the graphs are similar to the figure even if the values of A_1 and A_2 are changed.

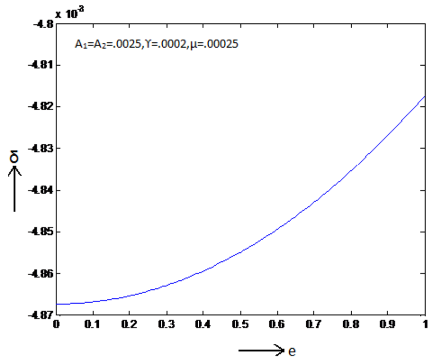


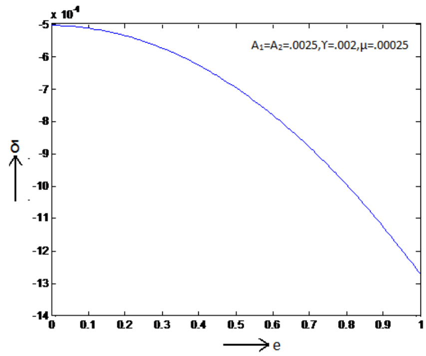


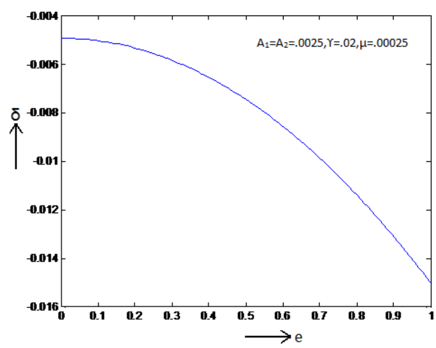


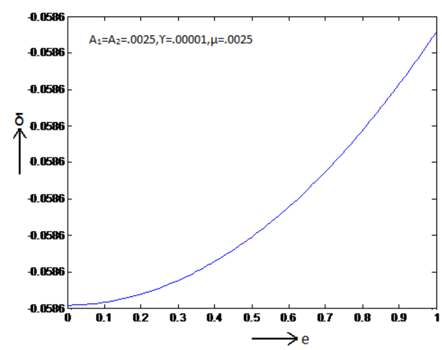


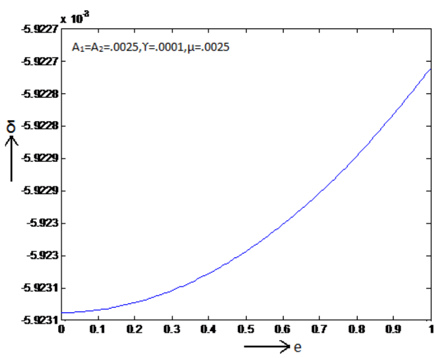

$A_1=A_2=.0025, \Upsilon=.00002, \mu=.00025$
δ
e

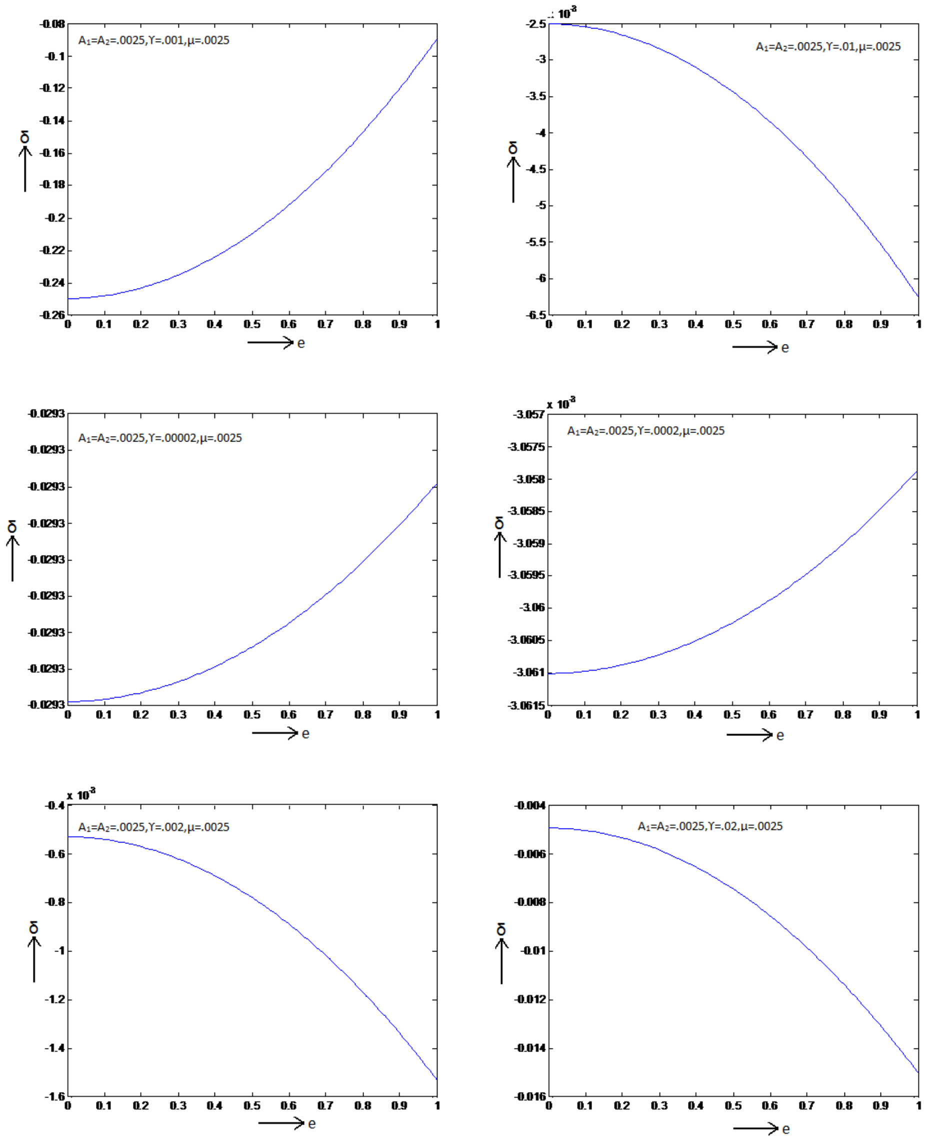

$A_1=A_2=.0025, \Upsilon=.0002, \mu=.00025$
δ
e


$A_1=A_2=.0025, \Upsilon=.002, \mu=.00025$
δ
e


$A_1=A_2=.0025, \Upsilon=.02, \mu=.00025$
δ
e


$A_1=A_2=.0025, \Upsilon=.00001, \mu=.0025$
δ
e


$A_1=A_2=.0025, \Upsilon=.0001, \mu=.0025$
δ
e



3. Stability of Collinear Equilibrium Points

Similarly, we have also plotted the curve between by taking into account of various values of oblateness parameters, which is indicated that the deviation

Assuming α, β denote small displacement of the infinitesimal particle from the collinear equilibrium points.

$$x = x_0 + \alpha, \quad y = y_0 + \beta. \tag{3.1}$$

Now

$$\Omega^*_x = \Omega^*_x (x, y) = \Omega^*_x (x_0 + \alpha, y_0 + \beta).$$

Expanding by Taylor's expansion and considering only first order terms, we have

$$\begin{aligned} \Omega^*_x =& \Omega^{*^0}_x + \alpha\Omega^{*^0}_{xx} + \beta\Omega^{*^0}_{xy}, \\ \Omega^*_y =& \Omega^{*^0}_y + \alpha\Omega^{*^0}_{yx} + \beta\Omega^{*^0}_{yy}, \end{aligned} \tag{3.2}$$

where $\Omega^{*^0}_x$ is the value of Ω^*_x at the point (x_0, y_0) and similarly the other values $\Omega^{*^0}_{xx}, \Omega^{*^0}_{xy}, \Omega^{*^0}_{yy}$ are the respective values at the points (x_0, y_0).

At the equilibrium points (x_0, y_0) we have

$$\Omega^0_x = \Omega^0_y = 0.$$

Hence the equation of motion of infinitesimal under the photogravitational and obletness of primary takes the form:

$$\begin{aligned} \alpha'' - 2\beta' =& \alpha\Omega^{*^0}_{xx} + \beta\Omega^{*^0}_{xy}, \\ \beta'' - 2\alpha' =& \alpha\Omega^{*^0}_{yx} + \beta\Omega^{*^0}_{yy}. \end{aligned} \tag{3.3}$$

In order to solve the equation (3.2) substitute $\alpha = Ae^{\lambda t}$ and $\beta = Be^{\lambda t}$ where A, B and λ are parameters to be found, substituting the values of α, β, α', β', α'', β'' in the equation (3.1), which takes the form:

$$\begin{aligned} A\left(\lambda^2 - \Omega^{*^0}_{xx}\right)e^{\lambda t} + B\left(-2\lambda - \Omega^{*^0}_{xy}\right)e^{\lambda t} =& 0, \\ A\left(2\lambda - \Omega^{*^0}_{xy}\right)e^{\lambda t} + B\left(\lambda^2 - \Omega^{*^0}_{yy}\right)e^{\lambda t} =& 0. \end{aligned} \tag{3.4}$$

The set of equation (3.4) has nontrivial solution if

$$\begin{vmatrix} \left(\lambda^2 - \Omega^{*^0}_{xx}\right) & \left(-2\lambda - \Omega^{*^0}_{xy}\right) \\ \left(2\lambda - \Omega^{*^0}_{xy}\right) & \left(\lambda^2 - \Omega^{*^0}_{yy}\right) \end{vmatrix} = 0, \tag{3.5}$$

$$\lambda^4 - \left(\Omega^{*^0}_{yy} + \Omega^{*^0}_{xx} - 4\right)\lambda^2 + \Omega^{*^0}_{yy}\Omega^{*^0}_{xx} - \left(\Omega^{*^0}_{xy}\right)^2 = 0. \tag{3.6}$$

In order to investigate the stability of collinear equilibrium points, we need to express the partial derivative $\Omega^{*^0}_{xx}, \Omega^{*^0}_{xy}, \Omega^{*^0}_{yy}$ are of the following forms:

$$\begin{aligned}\Omega^{*^0}_{xy} =& he^2 + h_1 A_1 + h_2 A_2 + h_3 \beta \qquad (\text{here } \beta = 1 - q),\\ \Omega^{*^0}_{xx} =& S_1^2 + S_2 e^2 + S_3 A_1 + S_4 A_2 + S_5 \beta,\\ \Omega^{*^0}_{yy} =& -T_1^2 + T_2 e^2 + T_3 A_1 + T_4 A_2 + T_5.\end{aligned} \tag{3.7}$$

We have discussed the stability of collinear equilibrium points, the following three possibilities may arise:

Case I:

$$\Omega^{*^0}_{xy} = 0, \quad \Omega^{*^0}_{xx} > 0, \quad \Omega^{*^0}_{yy} < 0.$$

The collinear equilibrium points will be unstable according to Szebehely (1967).

Case II:

$$\Omega^{*^0}_{xy} > 0, \quad \Omega^{*^0}_{xx} > 0, \quad \Omega^{*^0}_{yy} < 0.$$

The characteristic equation represented by (3.6) is given as follows:

$$\lambda^4 - \left(\Omega^{*^0}_{yy} + \Omega^{*^0}_{xx} - 4\right)\lambda^2 + \Omega^{*^0}_{yy}\Omega^{*^0}_{xx} - \left(\Omega^{*^0}_{xy}\right)^2 = 0$$

Applying the coordinate mentioned in, we get:

$$\Omega^{*^0}_{yy}\Omega^{*^0}_{xx} - \left(\Omega^{*^0}_{xy}\right)^2 = 0$$

Hence, applying the coordinate mentioned can be written in the following form

$$\Lambda^2 + 2\beta_2\Lambda - \beta_3^2 = 0, \tag{3.8}$$

where

$$\beta_2 = 2 - \frac{1}{2}\left(\Omega^{*^0}_{yy} + \Omega^{*^0}_{xx}\right)$$

$$\beta_3^2 = \left(\Omega^{*^0}_{xy}\right)^2 - \Omega^{*^0}_{yy}\Omega^{*^0}_{xx}$$

$$\lambda^2 = \Lambda so that \lambda = \pm\left(\Lambda\right)^{\frac{1}{2}}. \tag{3.9}$$

Now from the equation (3.8) we have

$$\Lambda = -\beta_2 \pm \left(\beta_2^2 + \beta_3^2\right)^{\frac{1}{2}}$$

$$Let \Lambda_1 = -\beta_2 + \left(\beta_2^2 + \beta_3^2\right)^{\frac{1}{2}}$$

$$\Lambda_2 = -\beta_2 - \left(\beta_2^2 + \beta_3^2\right)^{\frac{1}{2}}$$

For positive or negative value of β_2, Λ_1 is always positive and Λ_2 is always negative, i.e. they are of opposite sign. Again from the equation (3.9), we have

$$\lambda_{1,2} = \pm \left(\Lambda_1\right)^{\frac{1}{2}} = \pm \text{ real(since } \Lambda_1 \text{ is positive) and}$$

$$\lambda_{3,4} = \pm \left(\Lambda_2\right)^{\frac{1}{2}} = \pm \text{ imaginary(since } \Lambda_2 \text{ is negative).}$$

Hence, for only one real positive value of $\lambda = \lambda_1 (say) the solution \alpha = Ae^{\lambda t} and \beta = Be^{\lambda t}$ will be unbounded. Therefore the equilibrium poins is unstable.

Case III:

$$\Omega^{*^0}_{xy} < 0, \quad \Omega^{*^0}_{xx} > 0, \quad \Omega^{*^0}_{yy} < 0.$$

In this case also

$$\Omega^{*^0}_{yy} \Omega^{*^0}_{xx} - \left(\Omega^{*^0}_{xy}\right)^2 < 0.$$

Hence the equilibrium point in this case also is found unstable. Similarly, we can show that $L_2 and L_3$ are also unstable.

4. Different Curve of Zero Velocity

In order to discuss the different curves of zero velocity in elliptical restricted three body problem, when the bigger primary is oblate and radiating and smaller primary is oblate primary, multiplying the first equation of (2.1) by x' and the second equation by y' and adding we obtain:

$$x' x'' + y' y'' = \left(\frac{\partial \Omega}{\partial x}\right) x' + \left(\frac{\partial \Omega}{\partial y}\right) y'$$

$$\frac{1}{2} \frac{d}{dv} \left[x'^2 + y'^2\right] = \frac{d\Omega}{dv}. \tag{4.1}$$

Since Ω does not contain the time (true anomaly) explicitly. Therefore (4.1) can be integrated to given equation,

$$\frac{1}{2} \left[x'^2 + y'^2\right] = \int \frac{d\Omega}{1 + e \cos v} + c \tag{4.2}$$

Due to presence of $(1 + e\cos v)$ in the denominator of the integral (4.2), the equation is not possible to integrate to any definite form. Hence, in elliptical restricted three body problem, it does not adjust the Jecobi integral of classical circular problem at least in its usual sense.

The elliptical restricted three body problem, is different from the classical restricted problem in the sense that Jacobi integral does not exists, Floria (see [7]) and energy along any orbit is a time dependent quantity. As we know, no exact, complete and general solution to the elliptical restricted three body problem, can be obtained unlike in classical restricted three body problem, but this mathematical inconvenience is overcome along investigation of certain special cases of the problem based on simplifying the mathematical model under consideration Ammer (see [3]). Now if consider the potential function is

$$\overline{\Omega}\left(x,y\right) = \frac{\Omega\left(x,y\right)}{1+e\cos v} + c. \tag{4.3}$$

Here $\Omega\left(x,y\right)$ dependens not only on the position coordinate of the particle but also an independent variable. We select the initial point, $v = 0$ and we consider only a part of the trajectory $v = 0 and v = \delta$, where δ is sufficiently small positive quantity Ammer (see [3]). This restriction is subjected to sufficiently small time interval, during which the primaries describe sufficiently small ones, see Szebehely [9], with this restriction, we may define a Jacobi-constant in elliptical case as follows:

$$x'^2 + y'^2 - \frac{x^2+y^2}{1+e\cos v} - \frac{2}{(1+e\cos v)}\left[\frac{1}{1+3\left(\frac{A_1+A_2}{2}\right)}\left\{\frac{(1-\mu)\,q}{r_1} + \frac{\mu}{r_2} + \frac{(1-\mu)\,qA_1}{2r_1^3} + \frac{\mu A_2}{2r_2^3}\right\}\right] = c. \tag{4.4}$$

The equation (4.4) describes about different curves of zero velocity, at each given instant of time of elliptical restricted three body problem. The zero velocity curves are now pulsating with frequency of the nominal elliptic motion. Therefore, the plannar elliptical restricted three body problem, the zero velocity curves are obtains from the equation,

$$x^2 + y^2 - 2\left[\frac{1}{1+3\left(\frac{A_1+A_2}{2}\right)}\left\{\frac{(1-\mu)\,q}{r_1} + \frac{\mu}{r_2} + \frac{(1-\mu)\,qA_1}{2r_1^3} + \frac{\mu A_2}{2r_2^3}\right\}\right] + c^*$$

$$= 0,$$

where $c^* = c\,(1 + e\cos v)$

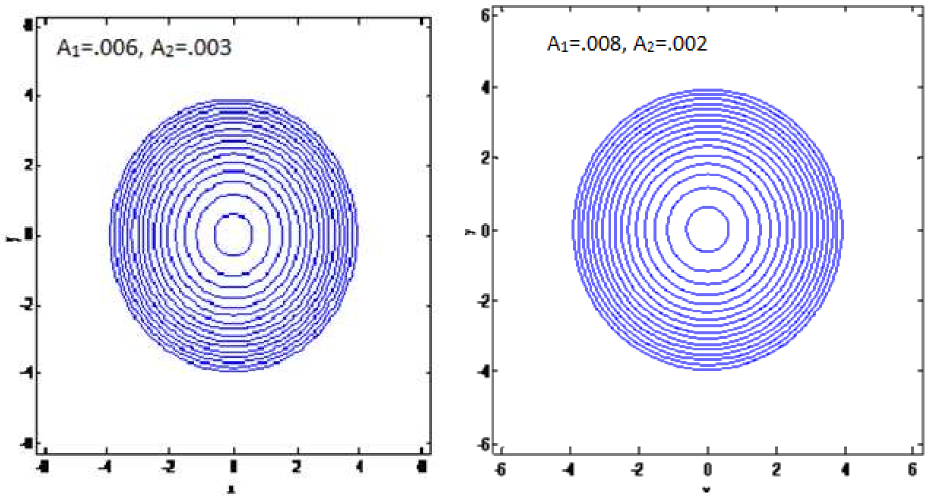


Figure 17. Difference curves of zero velocity

We arrived at the conclusion that at every time or any value of true anomaly, different set of zero velocity curves are to be constructed at every instant.

5. Discussion and Conclusion

The stability of one of the collinear equilibrium points of the planer elliptical restricted three body problem under the influence of oblateness and radiation of the bigger primary and oblateness of the smaller primary has been discussed. The problem is studied under the assumption that the eccentricity of the orbit of the gravitating bodies is small. The oblateness of the more massive primary does not affect the motion of the smaller primary due to its larger mass. whereas its effects the motion of the infinitesimal body. We have adopted the simulation technique using *MATLAB* software to investigate the stability of the infinitesimal oscillating arround L_1, we have also traced different curves of zero velocity.

We arrive at the conclusion that the infinitesimal oscillating around L_1 is found unstable. The same technique would be adopted to test the stability of infinitesimal around remaining of the collinear equilibrium points L_2 and L_3.

References

[1] S.K. Sahoo, B. Ishwar, Stability of the collinear equilibrium points in the generalized photogravitational elliptical restricted three body problem, *Bull Astr. Soc. India*, **28** (2000), 579-586.

[2] J.M.A. Danby, Stability of the triangular points in elliptic restricted problem of three bodies, *The Astronomical Journal*, 69 (1964), 165-172.

[3] M.K. Ammer, The effect of solar radiation pressure on the Lagrangian points in the elliptical restricted three body problem, *Astrophysics and Space Science*, **313** (2008), 393-408.

[4] S.N. Khasan, Liberation Solutions to the photogravitational restricted three body problem, *Cosmic Research*, **34**, No. 2 (1996), 146-151.

[5] S.N. Khasan, Three dimensional periodic solutions to the photogravitational Hill problem, *Cosmic Research*, **34**, No. 5 (1996), 504-507.

[6] D. Selaru, C. Cucu-Dumitrescu. An analytic asymptotical solution in the three body problem, *Rom Astron. J.*, **4**, No. 1 (1994), 59-67.

[7] L. Floria, On an analytical solution in the planar elliptic restricted three body problem,*Monografsem. Mat. Garacia de Galdeano*, **31** (2004), 135-144.

[8] S.W. Mc Cuskey, *Introduction to Celestial Mechanics*, Addison Wesley, New York (1963).

[9] V. Szebehely, *Theory of Orbits*, Academic, New York (1967).

[10] A. Narayan, C. Ramesh, Effects of photogravitational and oblateness on the triangular lagrangian points in the elliptical restricted three body problem, *Int. J. Pure and Appl. Math.*, **68** (2011), 201-224.

[11] A. Narayan, C. Ramesh, Stability of triangular equilibrium points in elliptical restricted three body problem under effects of oblateness primaries, *Int. J. Pure and Appl. Math.*, **70** (2011), 735-754.