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COMBINED EFFECTS OF THERMAL RADIATIONS AND HALL CURRENT ON MOVING VERTICAL POROUS PLATE IN A ROTATING SYSTEM WITH VARIABLE TEMPERATURE

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Abstract: The combined effects of Hall current and thermal radiations on unsteady flow of an electrically conducting fluid past an impulsively started infinite vertical porous plate with variable temperature is studied. A magnetic field of uniform strength is applied along an axis perpendicular to the plate and the entire system rotates about this axis. The plate temperature is raised linearly with time. An exact solution is obtained by Laplace Transform method. The dependence of the amplitude and phase difference of velocity and skin-friction on various parameters are discussed in detail.

AMS Subject Classification: 76Dxx, 76Sxx, 76Uxx
Key Words: thermal radiations, hall current, porous plate, rotation, variable temperature, magnetic field

1. Introduction

The problem of finding exact solution of the Navier-Stokes equations presents insurmountable mathematical difficulties. This is primarily due to the fact that NS equations are non-linear. But if the non-linear terms vanishes in natural way, one can get the exact solution. One of the exact solution of the Navier-

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Stokes equations of the flow due to an impulsively started infinite flat plate was first studied by Stokes[1]. Georgrantopoulous [2] has discussed the free convection effects of the oscillating flow in the Stokes problem past an infinite vertical porous plate with constant suction. Kafousias et al [3] have extended the above problem in the presence of a transverse magnetic field without taking into account the induced magnetic field. Very recently Garg et al [4] studied radiative free-convective and mass transfer flow past an accelerated plate in the presence of transverse magnetic field.

When the strength of magnetic field is strong enough then one cannot neglect the effects of Hall current. A comprehensive discussion of Hall effect is given by Cowling [5]. Hall effect is likely to be important in many astrophysical and geophysical situations as well as in flows of laboratory Plasmas. From this point of view, Mazumder et al [6] and Gupta [7] have studied hydromagnetic steady flow with Hall current. Very recently Singh and Garg [8] have analysed the Hall current effects on free convection flow past an accelerated porous plate in a rotating system with Heat source/sink by Laplace transformation technique. On the other hand current interests in the study of magnetohydynamic (MHD) of relating fluid has been motivated by several important problems such as maintenance and secular variations of earth's magnetic field, the internal rotation rate of sun, the structure of rotating magnetic stars, the planetary and solar dynamo problems and centrifugal machines etc. Hence the purpose of the present investigation is to study the effects of Hall current on unsteady free convective moving vertical porous plate in a rotating system in the presence of variable temperature and thermal radiations. The effects of different pertinent physical parameters on the velocity, temperature and skin-friction are presented graphically.

2. Mathematical Analysis

Consider an unsteady, free convective flow of an in compressible, electrically conducting, viscous fluid past an impulsively started infinite insulated vertical porous plate with variable temperature. A uniform magnetic field H_o is acting transverse to the plate in the presence of thermal radiations and Hall current. The fluid and plate rotate in unison with a constant angular velocity Ω' and $z' - axis$, taken normal to the plate. Initially, the plate and the fluid were at rest and temperatures of both are also same. At time $t' > 0$, plate is given impulsive motion in the vertical direction against gravitational field with constant velocity u'_0, the plate temperature is raised linearly with time. Since the plate occupying the plane $z' = 0$ is of infinite extent, all the physical quantities depend only on z' and t'. The equation of continuity $\nabla.\overrightarrow{V} = 0$ gives on integration $w' = -w_0$, where $\overrightarrow{V} \equiv (u', v', w')$. The constant $w_0 > 0$ representing the suction velocity at the plate. Using the relation $\nabla.\overrightarrow{H} = 0$ for the magnetic field $\overrightarrow{H} \equiv (H'_x, H'_y, H'_z)$, we obtain $H'_z = H_0$ everywhere in the fluid (H_0 is a constant), if $\overrightarrow{J} \equiv (J'_x, J'_y, J'_z)$ is the current density, from the relation $\nabla.\overrightarrow{J} = 0$, we have $J'_z =$ constant. Since the plate is non-conducing, $J'_z = 0$ at the plate and hence zero everywhere.

In addition, thermal radiation term is added in the energy equation. Taking Hall currents into account, the generalised Ohm's law, in absence of electric field (Cowling [5]) is of form

$$\overrightarrow{J} + \frac{\omega_e \tau_e}{H'_0} \overrightarrow{J} \times \overrightarrow{H} = \sigma(\mu_e \overrightarrow{V} \times \overrightarrow{H}) + \frac{1}{e\eta_e} \nabla.p_e. \tag{1}$$

Under the usual assumptions, the electron pressure (for a partially ionized gas), the thermoelectric pressure and ion slip are negligible (Meyer [9]). Hence equation (1) gives

$$J'_x + \omega_e \tau_e J'_y = \sigma \mu_e H'_0 \nu', \tag{2}$$

$$J'_y - \omega_e \tau_e J'_x = -\sigma \mu_e H'_0 u'. \tag{3}$$

Hence one can have

$$J'_x = \frac{\sigma \mu_e H'_0}{1 + m^2}(v' + mu'), \tag{4}$$

$$J'_y = \frac{\sigma \mu_e H'_0}{1 + m^2}(mv' - u'). \tag{5}$$

where $\sigma, \mu_e, \omega_e, \tau_e, e, \eta_e, p_e$ are respectively, the electric conductivity, the magnetic permeability, the cyclotron frequency, the electron collision time, the electric charge, the number density of the electron pressure and $m = \omega_e \tau_e$, the hall parameter.

The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesqs' approximations , the governing equations to the problem are obtained as:

$$u'_{t'} - w_0 u'_{z'} - 2\Omega' v' = g\beta(\theta' - \theta'_\infty) + \nu u'_{z'z'} + \frac{\sigma \mu_e^2 H_0^2 (mv' - u')}{\rho(1 + m^2)} \tag{6}$$

$$v'_{t'} - w_0 v'_{z'} + 2\Omega' u' = \nu v'_{z'z'} - \frac{\sigma \mu_e^2 H_0^2 (v' + mu')}{\rho(1 + m^2)} \tag{7}$$

$$\rho c_p(\theta'_{t'} - w_0\theta'_{z'}) = k\theta'_{z'z'} - q'_{z'} \tag{8}$$

The boundary conditions of the problem are

$$\left.\begin{array}{lr} u' = v' = 0, \quad \theta' = \theta'_\infty & for \quad all \quad z' \quad at \quad t' \leq 0, \\ u' = u'_0, \quad v' = 0, \quad \theta' = \theta'_\infty + (\theta'_{w'} - \theta'_\infty)ct' & at \quad z' = 0 \quad for \quad t' > 0 \\ u' = 0, \quad v' = 0, \quad \theta' = \theta'_\infty & as \quad z' \longrightarrow \infty \quad for \quad t' > 0. \end{array}\right\} \tag{9}$$

Where g, the acceleration due to gravity; β, the coefficient of volume expansion of the fluid; ν, the kinematic viscosity; ρ, the density; θ' the fluid temperature inside the thermal boundary layer; θ'_∞, fluid temperature away from the porous wall; k, the thermal conductivity, c_p the specific heat of the fluid under constant pressure and

$$c = \frac{(u'_0)^2}{v}.$$

The local radiant for the case of an optically thin gray gas is expressed by

$$q'_{z'} = -4a^*\sigma^*(\theta'^4_\infty - \theta'^4). \tag{10}$$

Here a^* is the absorption coefficient and σ^* is the Stefan-Boltzmann constant. We assume that the temperature differences within the flow are sufficiently small such that θ'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding θ'^4 in a Taylor series about θ'_∞ and neglecting higher order terms, thus

$$\theta'^4 \cong 4\theta'^3_\infty\theta' - 3\theta'^4_\infty. \tag{11}$$

By using equations (10) and (11), equation (8) reduces to

$$\rho c_p(\theta'_{t'} - w_0\theta'_{z'}) = k\theta'_{z'z'} + 16a^*\sigma^*\theta'^3_\infty(\theta'_\infty - \theta') \tag{12}$$

On introducing the following non-dimensional quantities

$$\left.\begin{array}{l} u = \dfrac{u'}{u'_0}, \quad t = \dfrac{t'u'^2_0}{v}, \quad v = \dfrac{v'}{u'_0}, \quad z = \dfrac{z'u'^2_0}{v}, \\ \theta = \dfrac{\theta' - \theta'_\infty}{\theta'_w - \theta'_\infty}, \quad w = \dfrac{w_0}{u'_0}, \quad \Omega = \dfrac{v\Omega'}{u'^2_0}, \quad Gr = \dfrac{vg\beta(\theta'_w - \theta'_\infty)}{u'^3_0}, \\ M = \dfrac{\sigma\mu_e^2 H_0^2 v}{u'^2_0\rho}, \quad R = \dfrac{16a^*v^2\sigma^*\theta'^3_\infty}{ku'^2_0}, \quad Pr = \dfrac{\rho v c_p}{k}, \end{array}\right\} \tag{13}$$

into (6), (7) and (12), we get

$$u_t - wu_z - 2\Omega v = Gr\theta + u_{zz} + M(mv - u), \tag{14}$$

$$v_t - wu_z + 2\Omega u = v_{zz} - M(mu + v), \tag{15}$$

$$Pr(\theta_t - w\theta_z) = \theta_{zz} - R\theta. \tag{16}$$

The transformed boundary conditions

$$\left.\begin{array}{llll} u = 0, \quad v = 0, \quad \theta = 0 & for \quad all \quad z, \quad at \quad t \leq 0, \\ u = 1, \quad v = 0, \quad \theta = t & at \quad z = 0, \quad for \quad t > 0, \\ u \longrightarrow 0 \quad v \longrightarrow 0, \quad \theta \longrightarrow 0 & as \quad z \longrightarrow \infty, \quad for \quad t > 0, \end{array}\right\} \tag{17}$$

The equations (14)-(16) subject to the boundary conditions (17) describe the hydromagnetic free-convection flow past the moving porous plate in a rotating system.

Introducing the complex velocity F=u+iv, we find that equations (14) and (15) can be combined into a single equation of the form.

$$F_t - wF_z = Gr\theta + F_{zz} - m_1 F, \tag{18}$$

where m_1 is given in the appendix.

The corresponding transformed boundary conditions in the complex notations are given as:

$$\left.\begin{array}{ll} F = 0, \quad \theta = 0 & for \quad all \quad z, \quad at \quad t \leq 0, \\ F = 1, \quad \theta = t & at \quad z = 0, \quad for \quad t > 0, \\ F \longrightarrow 0 \quad \theta \longrightarrow 0 & as \quad z \longrightarrow \infty, \quad for \quad t > 0. \end{array}\right\} \tag{19}$$

In order to find the solution of equation (18), we take Pr = 1. This is possible assumption since the Prandtl number is a measure of relative importance of viscosity and heat conductivity in the fluid. For most gases the Prandtl number is of unit order, so that the velocity and the thermal boundary layer will be of the same order of thickness (cf. Houghton and Boswell [10]). By using the Laplace transformation technique solutions of equations (16) and (18) subject to the boundary conditions (19) are derived as:

$$\begin{aligned} F = {} & A_1 A_3(\eta_1 expX_1 erfc\eta_1 - \eta_2 expX_2 erfc\eta_2) - A_2 A_3(\eta_3 expX_3 erfc\eta_3 \\ & - \eta_4 expX_4 erfc\eta_4) + \frac{1}{2}(expX_1 erfc\eta_1 + expX_2 erfc\eta_2), \end{aligned} \tag{20}$$

$$\theta = A_4\{\eta_5 expX_5 erfc\eta_5 - \eta_6 expX_6 erfc\eta_6\}. \tag{21}$$

Using equation (20), we get the following expression for the skin friction components τ_x and τ_y as:

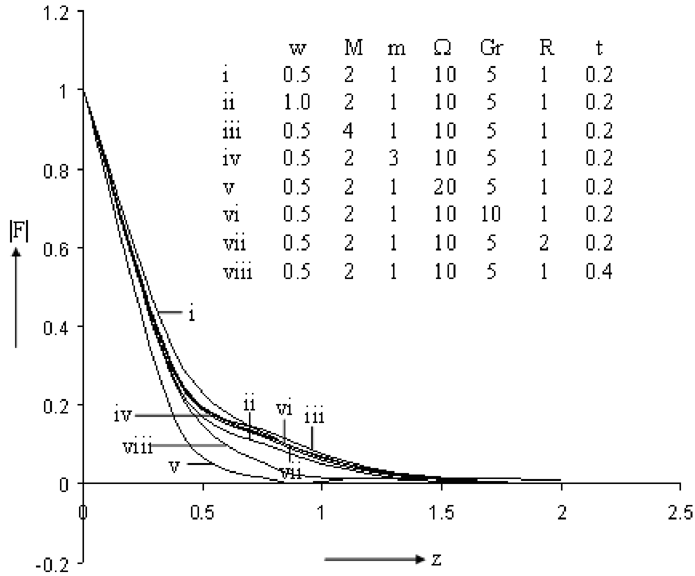

Figure 1: Amplitude of velocity

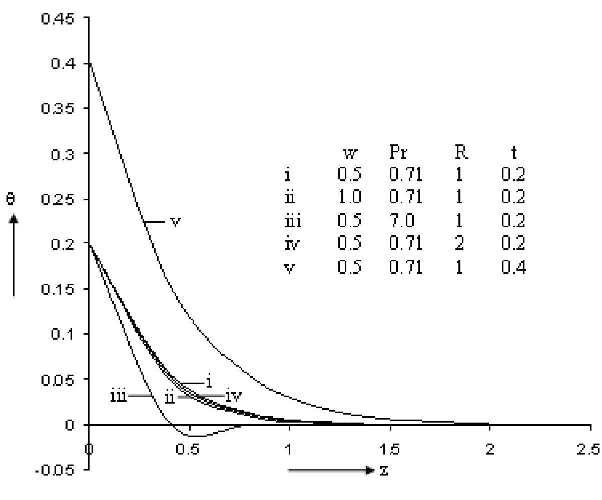

Figure 2: Temperature profiles

$$\tau_x + i\tau_y = -\left.\frac{dF}{dz}\right|_{z=0}$$

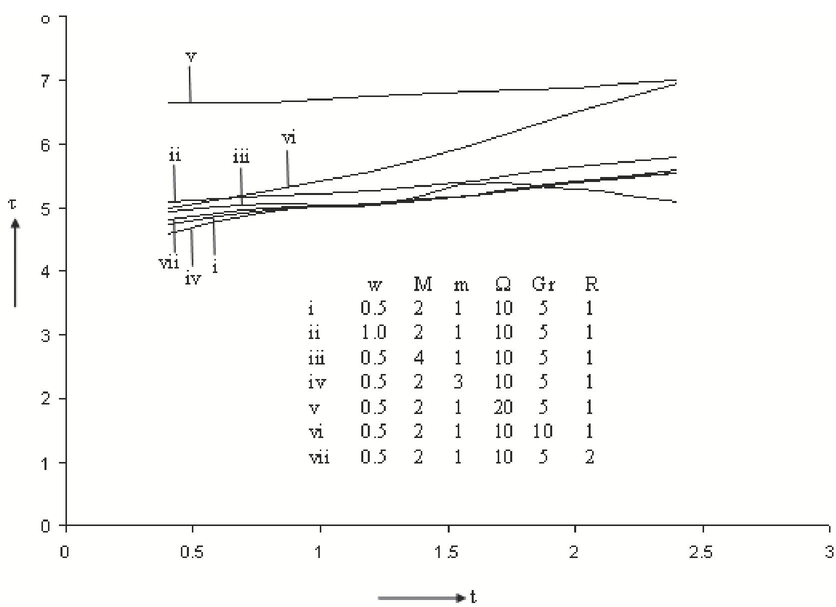

Figure 3: Amplitude of skin-friction profiles

$$= A_3\left[A_5 erf X_7 + \left(\frac{t}{\pi}\right)^{\frac{1}{2}}\left\{exp(-X_7^2) - exp(-X_8^2)\right\} - A_6 erf X_8\right]$$

$$+ b_1 erf X_7 + \frac{w}{2} + \frac{1}{\sqrt{\pi t}} exp(-X_7^2), \quad (22)$$

where all $A_i's$, $X_i's$ and $\eta_i's$ are given in the appendix.

In equations (20)-(22), the argument of the complementory error function and error function is complex. Hence in order to obtain the x- and y-components of velocity, temperature and skin-friction, it is necessary to introduce some properties of the error function with complex arguments due to Abramowitz and Stegun [11]: i.e.

$$erf(c + id) = erf \quad c + \frac{exp(-c^2)}{2\pi c}\{1 - \cos(2cd) + i\sin(2cd)\}$$

$$+ \frac{2exp(-c^2)}{\pi}\sum_{n=1}^{\infty}\frac{exp(-n^2/4)}{n^2 + 4c^2}\left\{f_n(c,d) + ig_n(c,d)\right\} + \in(c,d), \quad (23)$$

where

$$f_n(c,d) = 2c - 2c \quad \cosh(nd)\cos(2cd) + n\sinh(nd)\sin(2cd), \quad (24)$$

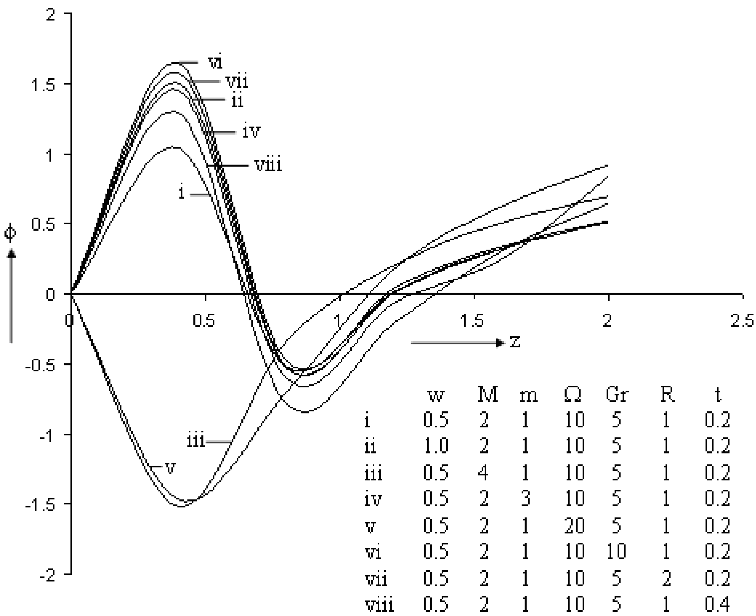

Figure 4: Phase angle of velocity profiles

$$g_n(c,d) = 2c\cosh(nd)\sin(2cd) + nsinh(nd)\cos(2cd), \tag{25}$$

and

$$|\in(c,d)| \cong 10^{-16} erf(c,id).$$

3. Result and Discussions

The expressions for resultant velocity $|F| = \sqrt{u^2+v^2}$ and the skin friction $|\tau| = \sqrt{\tau_x^2+\tau_y^2}$ are calculated numerically for different values of suction parameter ω, Hartmann number M, Hall parameter m, Rotation parameter Ω, Grashof number Gr, Radiation parameter R and time t. The resultant velocity profiles are shown graphically in Figure 1. It is observed that the amplitude $|F|$ of the velocity decreases with the increase of all the parameters except time t, where $|F|$ decreases near the moving plate and then increases away from the plate. The temperature profiles are presented in Figure 2. This figure clearly shows that the temperature decreases with the increase of suction parameter w, Prandtl number Pr and Radiation parameter R. But increases with the time

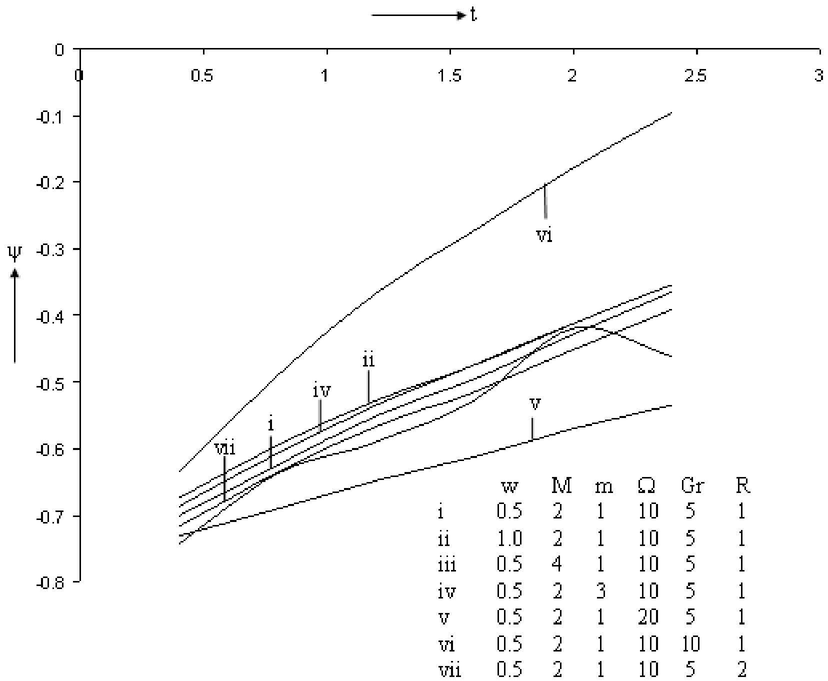

Figure 5: Phase angle of skin-friction profiles

t only. Fig. 3 shows the variation of the resultant skin friction with various values of different parameters. This figure clearly depicts that the resultant skin friction $|\tau|$ increases with the increase of suction parameter w, Rotation parameter Ω and Grashof number Gr. It is interesting to note that $|\tau|$ oscillates with the increase of Hartmann number M. There is slight but visible increase in $|\tau|$ with the increase of radiation parameter R. The increase of Hall current parameter m leads to decrease in $|\tau|$. The phase angle $\phi(= \tan^{-1}\ v/u)$ of the resultant velocity is plotted against z in Figure 4. This figure shows that near the plate there is always a phase lead and a phase lag thereafter and this phase lead increases with suction/injection parameter w, Hall parameter m, Grashof number Gr, Radiation parameter R and time t. However, the increase of Hartmann number M and Rotation parameter Ω affects the phase angle adversely i.e. there is a phase lag near the plate and phase lead thereafter.

The phase angle profiles of the skin friction $\psi(= tan^{-1}(\tau_y/\tau_x))$ are shown in Figure 5. It is clear from the figure that there is always a phase lag in the phase angle of skin-friction. This phase lag increases with the increase of suction/injection parameter w, Hall current parameter M and Grashof number Gr, however, it decreases with Hartmann number M. It is also noticed that phase angle starts oscillating with the increase of rotation parameter and radiation parameter R.

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Appendix

$$b_1 = \left(\frac{W^2}{4} + m_1\right)^{\frac{1}{2}}, \quad b_2 = \left(\frac{W^2}{4} + R\right)^{\frac{1}{2}}, \quad b_3 = \left(R + \frac{W^2 Pr^2}{4}\right)^{\frac{1}{2}}$$

$$\eta_1, \eta_2 = \frac{z \pm 2b_1 t}{2t^{\frac{1}{2}}}, \quad \eta_3, \eta_4 = \frac{z \pm 2b_2 t}{2t^{\frac{1}{2}}}, \quad \eta_5, \eta_6 = \frac{zPr \pm 2b_3 t}{2t^{\frac{1}{2}} Pr^{\frac{1}{2}}}$$

$$A_1 = \frac{t^{\frac{1}{2}}}{2b_1}, \quad A_2 = \frac{t^{\frac{1}{2}}}{2b_2}, \quad A_3 = \frac{Gr}{m_1 - R}, \quad A_4 = \frac{t^{\frac{1}{2}}}{2b_3}$$

$$A_5 = tb^{\frac{1}{2}} + \frac{1}{2b_1}, \quad A_6 = tb_2 + \frac{1}{2b_2}, \quad m_1 = -(M_0 + 2i\Omega),$$

$$M_0 = \frac{M}{1 - im}, \quad X_1 = \left(b_1 - \frac{w}{2}\right)z, \quad X_2 = -\left(b_1 + \frac{w}{2}\right)z,$$

$$X_3 = \left(b_2 - \frac{w}{2}\right)z, \quad X_4 = -\left(b_2 + \frac{w}{2}\right)z, \quad X_5 = \left(b_3 - \frac{wPr}{2}\right)z,$$

$$X_6 = -\left(b_3 + \frac{wPr}{2}\right)z, \quad X_7 = b_1 t^{\frac{1}{2}}, \quad X_8 = b_2 t^{\frac{1}{2}}.$$