INVESTIGATIONS OF AN AXISYMMETRIC COMPOUND FLOW BEHAVIOR OF SINTERED PREFORM:
AN UPPER BOUND APPROACH

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Abstract: The paper reports on an investigation into the various aspects of an axisymmetric sintered compound flow behaviours of powder preforms which have been compacted and sintered from atomized metal powder. Many forging processes involve flow of preform in more than one direction. For the purpose of die and process design it is often necessary to determine the flow behavior and the forging force. Analysis of a general axisymmetric problem is discussed and presented graphically. In addition the fundamental interest that it emphasizes the power of upper-bound when applied to the prediction of compound sintered metal flow.

AMS Subject Classification: 97M10, 74A05
Key Words: compound flow, sintering preform, interfacial friction

1. Introduction

The forging of powder preforms have been considerably developed during the

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last few years in industry, as they are being used successfully in a wide range of applications. Both the mechanical and the metallurgical properties of the metal powder components compare favourably with those of wrought materials. Bulk processing of metal powder preforms is a convenient method of reducing or eliminating the porosity from conventional powder metallurgy products. Process is attractive because it avoids large number of operations, high scrap losses and high-energy consumption associated with the conventional manufacturing processes such as casting, machining etc. Sintered porous powder preforms are used as starting materials in metal forming processes. Metal powder products manufactured by this new technology are comparable and in some cases even superior to those of cast and wrought products. The deformation pattern in forging of metal powder preform is quite different.

- A change in density occurs during plastic deformation.
- The yielding of porous metals is not completely insensitive to the hydrostatic stress imposed during processing.

Analysis of an axisymmetric sintered compound flow problem (Figure 1) is made. As the preform is compressed, preform while flowing radially outwards may or may not flow through the annular hole of the upper flat die. The theoretical analysis suggests that depending upon the dimensions of the preform and the annular hole in the die friction, one of the following three situations may occur:

1. The thickness of preform within annular hole may increase.
2. This thickness may not change.
3. This thickness may reduce.

Assuming the preform to flow through the die hole with a positive or negative velocity, separate velocity fields are postulated for the deforming preform. The expressions for the rate of energy dissipations of the process for the different sets of velocity fields are obtained. Minimizing the expressions for the power according to the upper bound theorem, the geometric dimensions and friction conditions valid for any one of the above-mentioned three situations are computed. The material of the preform is assumed as Mises’ material and the friction is represented by constant friction factor. Many forging processes involve flow of preform in more than one direction. For the purpose of die and process design it is often necessary to determine the flow behavior and forging force. A simple axisymmetric problem i.e. formation of a central boss on a small disc by compression between a platen by Jain and Bramley [1], and
Newnham and Rowe [2]. The analysis of axisymmetric problem is presented here. In addition it emphasizes the power of upper-bound when applied to the prediction of metal flow.

2. Plastic Deformation of Metal Powder Preform

In an investigation of the plastic deformation of sintered deformation of sintered metal powder preforms it is clear that change in volume occurs due to porosity. A preform with a high relative density yields at a relative high stress whereas a low relative density preform yields at a relatively low stress. Even hydrostatic stress can cause the sintered powder preforms to yield, as the yield surface is closed on the hydrostatic stress axis. The density distribution does not seem to be uniform, being high in the central region and low at the edges. The density distribution will be more uniform for a smaller coefficient of friction and for a higher initial-density preform. Tabata and Masaki [3] proposed the following
yield criterion for porous metal powder preforms:
\[\rho^k = \sqrt{3J'_2} \pm 3\eta\sigma_m.\]  
(1)

The negative sign is taken for \(\sigma_m \leq 0\) and the positive sign is taken for \(\sigma_m \leq 0\). The values of \(\eta\) and \(k\) were determined experimentally from simple compression and tension test of sintered copper-powder preforms as

\[\eta \begin{cases} 0.54(1 - \rho)^{1.2} & \text{for } \sigma_m \leq 0; \\ 0.55(1 - \rho)^{0.83} & \text{for } \sigma_m < 0 \text{ and } k = 2. \end{cases}\]

For the axisymmetric condition the yield criterion reduces to
\[\sigma_1 = \frac{\rho^k\sigma_0}{(1 - 2\eta)} + \frac{(1 + \eta)}{(1 - 2\eta)}\sigma_2.\]  
(2)

3. Interfacial Friction Condition

Many solutions to problem in forging are obtained with constant shear factor, Coulomb friction and viscous friction assumed between die and preform. These theories are associated with the forging practice. The basic shortcomings of these theories are the absence of a sufficient justification for the value of constant shear forces adopted. The correct determination of friction at the interface is of great importance. In the plastic deformation, the surface of the preform is distorted and takes on an impression of the tool surface. Therefore, the actual contact area, as far as the specific cohesion of the contact surface is concerned, is not negligible. Hence, friction in plastic deformation is essentially different from sliding friction. Direct metal-to-metal contact between the die and the preform is usually undesirable, since this contact results in welding and shearing of surface and gives rise to high friction values and deterioration of the product. Contact is prevented either by the presence of a surface oxide film or by the introduction of Lubricant [8]. High relative velocities between the preform or die surface combined with high interface pressure and/or deformation modes will cause breakdown of the surface film and will allow the new surface to come into contact with the die surface and causes adhesion. We also can not deny completely the existence of slip between the preform and dies, therefore [6]
\[\tau = \mu[p_{av} + \rho_0\phi_0].\]  
(3)
4. Velocity Fields and Strain Rates

Simple velocity fields are postulated by dividing the deforming preform into three cylindrical zones (Figure 1). In each zone the velocity components should be continuous, satisfying mass constancy and boundary conditions. bg, cf, and bc are the surfaces of velocity discontinuity. The normal components of velocities at these surfaces in any two adjoining zones should be equal to satisfy the mass constancy. In each zone parallel velocity field is assumed.

**Case I.** The metal of Zone 1 partly flows into the die hole.

At \( r_n \), radial velocity is zero. For \( r > r_n \), \( U_r \) is (+)ve and \( r < r_n \), \( U_r \) is (−)ve

\[
\text{Zone 1} \quad U_z = -\frac{Z}{h}; \quad \varepsilon_z^* = \frac{\partial U_z^*}{\partial z} = -\frac{1}{h}; \quad U_r = \frac{(1 - 2\eta)}{2(1 + \eta)rh} (r^2 - r_n^2); \\
\varepsilon_r^* = \frac{\partial U_r^*}{\partial r} = \frac{(1 - 2\eta)}{2(1 + \eta)h} \left[ 1 + \frac{r_n^2}{r^2} \right] 
\]

\[
\text{Zone 2} \quad U_z = \frac{Z}{h} V_e; \quad \varepsilon_z^* = \frac{V_e}{h}; \\
U_r = \frac{(1 - 2\eta)r_0^2}{2(1 + \eta)rh} - \frac{(1 - 2\eta)V_e(r^2 - r_0^2)}{2(1 + \eta)rh}; \\
\varepsilon_r^* = \frac{(1 - 2\eta)}{2(1 + \eta)h} \left[ -\frac{r_0^2}{r^2} - V_e \left( 1 + \frac{r_0^2}{r^2} \right) \right] 
\]

\[
\text{Zone 3} \quad U_z = -\frac{Z}{h}; \quad \varepsilon_z^* = -\frac{1}{h}; \quad U_r = \frac{(1 - 2\eta)r}{2(1 + \eta)h}; \\
\varepsilon_r^* = \frac{(1 - 2\eta)}{2(1 + \eta)h}. 
\]

**Case II.** No material from Zone 1 flows into the die hole. The material coming out of Zone 3 flows partly into the die hole and partly into Zone 1. \( V_e \) is (+)ve

\[
\text{Zone 1} \quad U_z = -\frac{Z}{h}; \quad \varepsilon_z^* = -\frac{1}{h}; \\
U_r = \frac{(1 - 2\eta)}{2(1 + \eta)hr} [r_0^2(1 + V_e) - r_1^2(1 + V_e) + r^2]; 
\]
\[ \varepsilon_r = \frac{(1 - 2\eta)}{2(1 + \eta)h} \left[ (1 + V_e) \left( \frac{r_1^2 - r_0^2}{r^2} \right) + 1 \right] \]  \hspace{1cm} (7)

**Zone 2**  
\[ U_z = -\frac{z}{h} V_e \varepsilon_z = \frac{V_e}{h}; \]
\[ U_r = \frac{(1 - 2\eta)}{2(1 + \eta)hr} [r_0^2(1 + V_e) - V_e r^2] \]
\[ \varepsilon_r = \frac{(1 - 2\eta)}{2(1 + \eta)h} \left[ -\frac{r_0^2}{r^2} (1 + v_e) - V_e \right] \]  \hspace{1cm} (8)

**Zone 3**  
\[ U_z = -\frac{z}{h} V_e \varepsilon_z = -\frac{1}{h}; \]
\[ U_r = \frac{(1 - 2\eta)r}{2(1 + \eta)h}; \]
\[ \varepsilon_r = \frac{(1 - 2\eta)}{2(1 + \eta)h} \]  \hspace{1cm} (9)

**Case III.** Material from Zone 3 and Zone 2 together flows to Zone 1. \( V_e \) is (+)ve.

**Zone 1**  
\[ U_z = -\frac{z}{h} V_e; \varepsilon_z = -\frac{1}{h}; \]
\[ U_r = \frac{(1 - 2\eta)}{2(1 + \eta)hr} [r_0^2r(1 - V_e) - r_1^2r(1 - V_e) + r^2] \]
\[ \varepsilon_r = \frac{(1 - 2\eta)}{2(1 + \eta)h} \left[ (1 - V_e) \left( \frac{r_1^2 - r_0^2}{r^2} \right) + 1 \right] \]  \hspace{1cm} (10)

**Zone 2**  
\[ U_z = -\frac{z}{h} V_e; \varepsilon_z = -\frac{1}{h}; \]
\[ U_r = \frac{(1 - 2\eta)}{2(1 + \eta)hr} [r_0^2(1 - v_e) - V_e r^2] \]
\[ \varepsilon_r = \frac{(1 - 2\eta)}{2(1 + \eta)h} \left[ -\frac{r_0^2}{r^2} (1 - V_e) + V_e \right] \]  \hspace{1cm} (11)

**Zone 3**  
\[ U_z = -\frac{z}{h}; \varepsilon_z = -\frac{1}{h}; \]
\[ U_r = \frac{(1 - 2\eta)r}{2(1 + \eta)h} \varepsilon_r = \frac{(1 - 2\eta)}{2(1 + \eta)h} \]  \hspace{1cm} (12)

5. Rate of Energy Dissipation

Corresponding to the three sets of velocity fields the expressions for components of strain rates and velocity discontinuities are obtained. Using this strain rate
components and velocity discontinuities

\[ J^* = \frac{2}{\sqrt{3}} \sigma_0 \int_v \sqrt{\frac{1}{2} e_{ij} \cdot e_{ij}} dv + \int_s \tau |\nabla V| ds \]

\[ + \int_v p a_i U_i dv - \int_{st} T_i V_i ds . \] (13)

We are considering internal energy dissipation and frictional force.

### 6. Internal Energy Dissipation

\[ dW_i = \frac{\sqrt{2}}{3} \rho^k \sigma_0 \sqrt{\left( \varepsilon_{r}^* - \varepsilon_{\theta}^* \right)^2 + \left( \varepsilon_{\theta}^* - \varepsilon_{z}^* \right)^2 + \left( \varepsilon_{z}^* - \varepsilon_{r}^* \right)^2} \] (14)

For axisymmetric condition, \( \varepsilon_{r}^* = \varepsilon_{\theta}^* \), equation reduces \( dW_i = \frac{2}{3} \rho^k \rho_0 (\varepsilon_{r}^* - \varepsilon_{z}^*) \).

**Case I.**

\[ W_i = \frac{2}{3} \pi \sigma_0 \rho^k \left[ \frac{(1 - 2n)}{2(1 + \eta)} \left\{ \left( r_2^2 - r_1^2 \right) - 2r_0^2 \frac{\ln r_2}{r_1} \right\} + \left( r_2^2 - r_1^2 \right) \right], \]

\[ W_i = \frac{2}{3} \rho^k \sigma_0^\pi \left[ \frac{(1 - 2n)}{2(1 + \eta)} \right] \left\{ - 2r_0^2 r \ln \frac{r_1}{r_0} \ight. \]

\[ - V_e \left( r_1^2 r_0^2 + 2r_0^2 \ln \frac{r_1}{r_0} \right) \left\} - V_e (r_1^2 - r_0^2) \right\}, \]

\[ W_i = \frac{2}{3} \rho^k \sigma_0^\pi \left( \frac{3r_0^2}{2(1 + \eta)} \right) \] (Zone 3) (17)

**Case II.**

\[ W_i = B \left[ A \left\{ 2(1 + V_e)(r_1^2 - r_0^2) \ln \frac{r_2}{r_1} + \left( r_2^2 - r_1^2 \right) \right\} + \left( r_2^2 - r_1^2 \right) \right] \] (Zone 1) (18)

\[ W_i = B \left[ A2(1 + V_e)(r_1^2 - r_0^2) \ln \frac{r_2}{r_1} + \left( r_2^2 - r_1^2 \right) - 2r_0^2 (1 + V_e) \right. \]

\[ \cdot \ln \frac{r_1}{r_0} - V_e - V(r_1^2 - r_0^2) + (r_2^2 - r_1^2) \]

\[ - V_e (r_1^2 - r_0^2) + \frac{3r_0^2}{2(1 + \eta)} \] (Zone 2) (19)
\[ W_i = B \left[ \frac{3r_0^2}{2(1 + \eta)} \right] \quad (\text{Zone 3}) \quad (20) \]

**Case III.**

\[ W_i = B \left[ A \left\{ 2(1 - V_e)(r_1^2 - r_0^2) \ln \frac{r_2}{r_1} + (r_2^2 - r_1^2) \right\} \right] \quad (\text{Zone 1}) \quad (21) \]

\[ W_i = B \left[ A \left\{ -2r_0^2(1 - V_e) \ln \frac{r_1}{r_0} + V_e(r_1^2 + r_0^2) \right\} + V_e(r_1^2 - r_0^2) \right] \quad (\text{Zone 1}) \quad (22) \]

\[ W_i = \frac{2}{3} \rho^k\sigma_0 \pi \left[ \frac{3r_0^2}{2(1 + \eta)} \right] \quad (\text{Zone 3}) \quad (23) \]

Total internal work done for all three zones:

**Case I.**

\[ W_i = B \left[ A \left\{ (r_2^2 - r_1^2) + 2r_0^2 \ln \frac{r_2}{r_1} - 2r_0^2 \ln \frac{r_1}{r_0} \\
- V_e \left( r_1^2 - r_0^2 + 2r_0^2 \ln \frac{r_1}{r_0} \right) \right\} + (r_2^2 - r_1^2) \\
- V_e(r_1^2 - r_0^2) + \frac{3r_0^2}{2(1 + \eta)} \right] \quad (24) \]

**Case II.**

\[ W_i = B \left[ A \left\{ 2(1 + V_e)(r_1^2 - r_0^2) \ln \frac{r_2}{r_1} + (r_2^2 - r_1^2) \\
- 2r_0^2(1 + V_e) \ln \frac{r_1}{r_0} - V_e(r_1^2 - r_0^2) \right\} \\
+ (r_2^2 - r_1^2) - V_e(r_1^2 - r_0^2) + \frac{3r_0^2}{2(1 + \eta)} \right] \quad (25) \]

**Case III.**

\[ W_i = B \left[ A \left\{ 2(1 - V_e)(r_1^2 - r_0^2) \ln \frac{r_2}{r_1} + (r_2^2 - r_1^2) \\
- 2r_0^2(1 - V_e) \ln \frac{r_1}{r_0} + V_e(R_1^2 - r_0^2) \right\} \\
+ \left\{ (r_2^2 - r_1^2) + V_e(r_1^2 - r_0^2) + \frac{3r_0^2}{2(1 + \eta)} \right\} \right] \quad (26) \]
where

\[ B = \frac{2}{3} \rho^k \sigma_0 \pi \quad \text{and} \quad A = \frac{(1 - 2\eta)}{2(1 + \eta)}. \]  

(27)

### 7. Frictional Energy Dissipation

The frictional energy dissipation is

\[ W_f = \int s \tau |\Delta v| ds \] where

\((\Delta v)\) is the velocity discontinuities along surfaces \( ds = 4\pi rdr \);

\[ \tau = \mu[p + \rho_0 \varphi_0]. \]  

(28)

**Case I. Zone 1**

\[ \Delta v = \frac{(1 - 2\eta)(r^2 - r_n^2)}{2(1 + \eta)h}; \]

\[ W_f = \frac{4\pi \mu}{2(1 + \eta)} \int_s \frac{(r^2 - r_n^2)}{rh} r dr \]

\[ \Rightarrow W_f = \frac{2\pi \mu}{3(1 + \eta)h} \left[ (r_2^3 - r_1^3) - 3r_n^2 (r_2 - r_1) \right] \]  

(29)

**Zone 2**

\[ \Delta v = \frac{(1 - 2\eta)}{2(1 + \eta)hr} \left[ r_0^2 - r_e V (r^2 - r_0^2) \right]; \]

\[ W_f = \frac{4\pi \mu}{2(1 + \eta)h} \int_{r_0}^{r_1} \frac{[r_0^2 - V_e (r^2 - r_0^2)]}{r} r dr; \]

\[ W_f = \frac{2\pi \mu(p + \rho_0 \varphi_0) (1 - 2\eta)}{3(1 + \eta)h} \left[ 3r_0^2 (r_1 - r_0) - V_e (r_1^3 + 2r_0^3 - 3r_0^2 r_1) \right] \]  

(30)

**Zone 3**

\[ \Delta v = \frac{(1 - 2\eta)r}{2(1 + \eta)h}; \]

\[ W_f = \frac{4\pi \mu(p + \rho_0 \varphi_0) (1 - 2\eta)}{2(1 + \eta)h} \int_0^{r_0} r \cdot r dr; \]

\[ W_f = \frac{2\pi \mu(p + \rho_0 \varphi_0) (1 - 2\eta)}{3(1 + \eta)h} r_0^3 \]  

(31)
Total frictional work done for all three zones:

\[ W_f = \frac{2\pi \mu (p + \rho_0 \phi_0)(1 - 2\eta)}{3(1 + \eta)h} \left[ (r_2^3 - r_1^3) - 3r_0^2(r_2 - r_1) \\
+ 3r_0^2(r_1 - r_0) - V_e(r_1^3 + 2r_0^3 - 3r_0^2r_1r) + r_0^3r \right] \] (32)

**Case II. Zone 1**

\[ \Delta v = \frac{(1 - 2\eta)}{2(1 + \eta)r_h} \left[ r_0^2(1 + V_e) - r_1^2(1 + V_e) + r^2 \right]; \]

\[ W_f = \frac{4\pi \mu (p + \rho_0 \phi_0)(1 - 2\eta)}{2(1 + \eta)h} \int_{r_1}^{r_2} \left[ r_0^2(1 + V_e) - r_1^2(1 + V_e) + \frac{r^2}{r} \right] r dr; \]

\[ W_f = \frac{2\pi \mu (p + \rho_0 \phi_0)(1 - 2\eta)}{3(1 + \eta)h} \left[ 3r_0^2(1 + V_e)(r_2 - r_1) - 3r_1^2(1 + V_e)(r_2 - r_1) + (r_2^3 - r_1^3) \right] \] (33)

**Zone 2**

\[ \Delta v = \frac{(1 - 2\eta)}{2(1 + \eta)hr} \left[ r_0^2(1 + V_e) - V_e r^2 \right]; \]

\[ W_f = \frac{4\pi \mu (p + \rho_0 \phi_0)(1 - 2\eta)}{2(1 + \eta)h} \int_{r_0}^{r_1} \left[ r_0^2(1 + V_e) - V_e \cdot r^2 \right] r dr; \]

\[ W_f = \frac{2\pi \mu (p + \rho_0 \phi_0)(1 - 2\eta)}{3(1 + \eta)h} \left[ 3r_0^2(1 + V_e)(r_1 - r_0) - V_e(r_1^3 - r_0^3) \right] \] (34)

**Zone 3**

\[ \Delta v = \frac{(1 - 2\eta)r}{2(1 + \eta)h}; \] (35)

\[ W_f = \frac{4\pi \mu (p + \rho_0 \phi_0)(1 - 2\eta)}{2(1 + \eta)h} \int_{0}^{r_0} r dr; \]

\[ \Rightarrow W_f = \frac{2\pi \mu (p + \rho_0 \phi_0)(1 - 2\eta)}{3(1 + \eta)h} r_0^3 \] (36)

Total frictional work done for all three zones:

\[ W_f = \frac{2\pi \mu (p + \rho_0 \phi_0)(1 - 2\eta)}{3(1 + \eta)h} \left[ 3r_0^2(1 + V_e)(r_2 - r_1) - 3r_1^2(1 + V_e)(r_2 - r_1) + (r_2^3 - r_1^3) \right] \]
\[+ 3r_0^2(1 + V_e)(r_1 - r_0) - V_e(r_1^3 - r_0^3) + r_0^3\]  \hspace{1cm} (37)

**Case III. Zone 1**
\[
\Delta v = \frac{(1 - 2\eta)}{2(1 + \eta)hr} [r_0^2(1 - v_e) - r_1^2(1 - v_e) + r_2^2];
\]
\[
W_f = \frac{4\pi \mu (p + \rho_0 \phi_0)(1 - 2\eta)}{2(1 + \eta)h} \int_{r_0}^{r_1} [r_0^2(1 - V_e) - r_1^2(1 - V_e) + r_2^2] dr;
\]
\[
\Rightarrow \quad W_f = \frac{2\pi \mu (p + \rho_0 \phi_0)(1 - 2\eta)}{3(1 + \eta)h} \times [3r_0^2(1 - V_e)(r_2 - r_1) - 3r_1^2(1 - V_e)(r_2 - r_1) + (r_2^3 - r_1^3)]  \hspace{1cm} (38)
\]

**Zone 2**
\[
\Delta v = \frac{(1 - 2\eta)}{2(1 + \eta)hr} [r_0^2(1 - V_e) - V_e r_2^2];
\]
\[
W_f = \frac{4\pi \mu (p + \rho_0 \phi_0)(1 - 2\eta)}{2(1 + \eta)h} \int_{r_0}^{r_1} [r_0^2(1 - V_e) + V_e r_2^2] dr;
\]
\[
\Rightarrow \quad W_f = \frac{2\pi \mu (p + \rho_0 \phi_0)(1 - 2\eta)}{3(1 + \eta)h} \times [3r_0^2(1 - V_e)(r_2 - r_1) + V_e(r_2^3 - r_1^3)]  \hspace{1cm} (39)
\]

**Zone 3**
\[
\Delta v = \frac{(1 - 2\eta)r}{2(1 + \eta)h};
\]
\[
W_f = \frac{4\pi \mu (p + \rho_0 \phi_0)(1 - 2\eta)}{2(1 + \eta)h} \int_{0}^{r_0} r \cdot r dr;
\]
\[
W_f = \frac{2\pi \mu (p + \rho_0 \phi_0)(1 - 2\eta)}{3(1 + \eta)h} r_0^3  \hspace{1cm} (40)
\]

**Total frictional work done for all three zones:**
\[
\Rightarrow \quad W_f = \frac{2\pi \mu (p + \rho_0 \phi_0)(1 - 2\eta)}{3(1 + \eta)h} [3r_0^2(1 - V_e)(r_2 - r_1) - 3r_1^2(1 - V_e)(r_2 - r_1) + (r_2^3 - r_1^3) + 3r_0^2(1 - V_e)(r_1 - r_0) + V_e(r_1^3 - r_0^3) + r_0^3]  \hspace{1cm} (41)
\]
8. Relative Average Pressure

\[ \frac{P_{\text{av.}}}{\sigma_0} = \frac{2\rho k}{3r_2^2} \left[ A \left\{ (r_2^2 - r_1^2) + 2r_2^2 \ln \frac{r_2}{r_1} - 2r_0^2 \ln \frac{r_1}{r_0} \right. \right. \\
- V_e \left( r_2^2 - r_0^2 + r_0^2 \ln \frac{r_1}{r_0} \right) \left. \left. \right\} \left( r_2^2 - r_1^2 \right) - V_e (r_1^2 - r_0^2) + \frac{3r_0^2}{2(1 + \eta)} \right] \\
\times \left[ 1 - \frac{2\mu(p + \rho_0 \phi_0)(1 - 2\eta)}{3(1 + \eta) hr_2^2} \right] \left\{ (r_2^3 - r_1^3) - 3r_0^2(r_2 - r_1) \\
+ 3r_0^2(r_1 - r_0) - V_e (r_1^3 + 2r_0^3 - 3r_0^2r_1) + r_0^3 \right\}^{-1} \\
\frac{P_{\text{av.}}}{\sigma_0} = \frac{2\rho k}{3r_2^2} \left[ A \left\{ 2(1 + V_e)(r_1^2 - r_0^2) \ln \frac{r_1}{r_0} + (r_2^2 - r_1^2) \right. \right. \\
- 2r_0^2(1 + V_e) \ln \frac{r_1}{r_0} - V_e (r_1^2 - r_0^2) \right\} + (r_2^2 - r_1^2) \\
- V_e (r_1^2 - r_0^2) + \frac{3r_0^2}{2(1 + \eta)} \right] \\
\times \left[ 1 - \frac{2\mu(p + \rho_0 \phi_0)(1 - 2\eta)}{3(1 + \eta) hr_2^2} \right] \left\{ 3r_0^2(1 + V_e)(r_2 - r_1) \\
- 3r_1^2(1 + V_e)(r_2 - r_1) + (r_2^3 - r_1^3) + 3r_0^2(1 + V_e)(r_1 - r_0) \\
- V_e (r_1^3 - r_0^3) + r_0^3 \right\}^{-1} \right) \right) \right) \right) \right) \right) \right)

(42)

9. Experimental Analysis

9.1. Metal Powder Used

Electrolytic Copper powder of greater than 99% purity was used
Physical characteristics of copper powder used.
Apparent Density: 2.60 g/cc Tap Density: 8.96 g/cc

9.2. Specimens and Density Measurement

In the preparation of metal powder compacts the following steps are necessary:
**Maximum Limits of Impurities**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>99.80%</td>
</tr>
<tr>
<td>Phosphorous</td>
<td>&lt; 0.001%</td>
</tr>
<tr>
<td>Iron</td>
<td>&lt; 0.006%</td>
</tr>
<tr>
<td>Silicon</td>
<td>&lt; 0.002%</td>
</tr>
</tbody>
</table>

Table 1: Chemical analysis of sintered copper powder (weight percentage)

(I) **Compaction:** Copper powder was compacted in a closed circular die using a 40 Ton hydraulic press (Figure 2) at various pressures. Compaction is shown in Figure 3.

(II) **Sintering:** Sintering of copper compacts was carried at 600, 625 and 650°C for three hours in appropriate atmosphere. All sintering operations were carried out in a muffle type silicon carbide furnace (Figure 4) capable of providing temperature of 1300°C with accuracy of ±5°C.

(III) **Machining:** The surfaces of the specimens were polished with fine emery paper. Density of the copper powder preforms was obtained simply by measuring specimen dimensions and weight. The relative density of the powder preform was obtained by the ratio of the powder preform density to the solid metal density (density of solid copper = 8.93g/cc).

### 9.3. Experimental Procedure and Measurements

The copper powder perform of known relative density was placed between a compound flow arrangements dies (Figure 1) and was compressed at room temperature. The compression was carried out in lubricated conditions. The sintered copper powder preforms of known initial relative densities (0.75, 0.80, 0.85 and 0.90) were deformed between flat dies. The load was gradually increased until cracks were observed. The % compression and the load value just at the time of the appearance of cracks were recorded. The deformed preforms are shown in Figures 6 and 7.

### 10. Results and Discussion

Figure 8, 9 and 10 shows the normal variation of $p/\sigma_0$ and $V_e$. In this normal case, from geometry of the Figure 1, it can be seen that only Case I is valid for the given set of input. In these cases, the direction of $V_e$ must be positive for Case I and II and negative for Case III. So, $V_e$ is one of the criteria for selecting...
the best situation for a particular case of sinter forming operation. Figure 9 shows how the velocity field changes from one case to another, Case II holds no more good as $h/r_2$ increases further more. So it is limit of using Case II in a particular situation. For higher value of $h/r_2$, Case III is going to apply for a particular situation (because $V_e$ is going down and it may attain $-ve$ value for higher value of $h/r_2$). Although the Case I is more general situation considered in a forming operation but for lower value of $h/r_2$ it gives the little high value of $p/\sigma_0$ (Figure 12). The suitability of different cases is also depending upon the various other geometry of the preform/die. For higher value of $r_0/r_2$ and $r_1/r_2$ the Case II holds good at a higher value of $h/r_2$ and Case III good for lower value of $h/r_2$. It means as $h/r_2$ increases Case III is transformed into Case II (Figure 12, 13 and 14). This will be another criterion for selecting the best suited velocity field for the particular situation. The selecting criteria of different cases are also depending upon the percentage reduction in height of the preform. Figure 15 and 16 shows the valid situation for all the cases for particular inputs at 20% reduction in height.

11. Conclusion

The deformation characteristics of the sintered metal powder preforms are much complex than wrought materials. It is sensitive to hydrostatic stress and instead of volume constancy; mass constancy is assumed during the deformation of the
metal powder preform. Again, when the geometry of the preform is not simple and/or some annular hole present in the die, then we should have the clear understanding of flow of material of sintered preform, different velocity fields are proposed for a particular flow condition of the metal powder.

1. Case II is best suited when $h/r_2$ is less.
2. Case III is best suited when $h/r_2$ is high.
3. Case I is universal but at lower value of $h/r_2$, gives higher value of $p/\sigma_0$.
4. Again the velocity field depends upon the geometry of the preform.
5. Velocity field depends upon the percentage reduction in height.
Figure 8: Variation of $p/\sigma_0$ and $V_a$ with % reduction in height

Figure 9: Variation of $p/\sigma_0$ and $V_a$ with % reduction in height

Figure 10: Variation of $p/\sigma_0$ and $V_a$ with % reduction in height

Figure 11: Variation of $V_a$ with respect to $h/r_2$

Figure 12: Variation of $p/\sigma_0$ and $V_a$ with $h/r_2$

Figure 13: Variation of $p/\sigma_0$ and $V_a$ with $h/r_2$
Figure 14: Variation of \( p/\sigma_0 \) and \( V_a \) with \( h/r_2 \)

Figure 15: Variation of \( p/\sigma_0 \) and \( V_a \) with \( h/r_2 \)

Figure 16: Variation of \( p/\sigma_0 \) and \( V_a \) with \( h/r_2 \)

Figure 17: Variation of \( p/\sigma_0 \) and \( V_a \) with \( h/r_2 \)

The experimental and theoretical results agrees satisfactorily. The \( P_{av}/\sigma_0 \) obtained theoretically agrees satisfactorily with the experimental results, it may be observed that as thickness of the preform increases the condition of flow changes from Case I to Case III through Case II. Also as \( r_1/r_2 \) increases, the conditions of flow changes from Case I to Case III. Selection criteria of the velocity field for the particular shape and size of the preform is very complex one, so it is expected that the result of this paper will help the designer/academician who are engaged and working in this field.

References


**A. Appendix**

**A.1. Velocity Fields — Case I**

In this case it is assumed that the metal of Zone 1 partly flows into the die hole. Assuming a parallel velocity field this suggests that at a radius $r_n$ the radial velocity is zero. For $r > r_n$, $U_r$ is positive and for $r < r_n$, $U_r$ is negative. Velocity components for three zones are obtained.

Zone 1: The $Z$-direction velocity at upper die is equal to $-1$ and the same at lower die is zero.

$$\frac{\partial U_r}{\partial r} + \frac{U_r}{r} + \left(1 - 2\eta\right)\frac{\partial U_z}{\partial z} = 0$$
\[ r \frac{\partial U_r}{\partial r} + U_r + \left( 1 - 2\eta \right) \frac{1}{1 + \eta} \frac{\partial U_z}{\partial z} r = 0; \quad U_Z = -\frac{z}{h} \quad (A1) \]

\[ rU_r = -\left( 1 - 2\eta \right) \frac{1}{1 + \eta} \frac{\partial U_z}{\partial z} \frac{r^2}{2} + f(z) \quad (A2) \]

Using boundary conditions \( U_r = 0 \) at \( r = r_n \) in equation (A2), we get

\[ 0 = -\left( 1 - 2\eta \right) \left( -\frac{1}{h} \right) \frac{r_n^2}{2} + f(z), \quad f(z) = -\left( 1 - 2\eta \right) \frac{r_n^2}{2h} \quad (A3) \]

By putting this in equation (A2),

\[ U_r = \frac{(1-2\eta)}{2(1+\eta)rh}(r^2 - r_n^2) \quad (A4) \]

Zone 3: Similar to Zone 1,

\[ U_z = -\frac{Z}{h} \quad (A5) \]

From (A2)

\[ -rU_r = -\left( 1 - 2\eta \right) \frac{1}{1 + \eta} \frac{\partial U_z}{\partial z} \frac{r^2}{2} + f(z) \quad (A6) \]

Using boundary conditions, \( U_r = 0 \) at \( r = 0 \), i.e. \( f(z) = 0 \).

Putting the value of \( f(z) = 0 \) in equation (A6),

we get

\[ -U_r = \frac{(1-2\eta)r}{2(1+\eta)h} \quad (A7) \]

Zone 2: Assuming linear distribution of

\[ U_z - U_z = \frac{Z}{h} V_e \quad (A8) \]

Using equation (A1) and (A2)

\[ rU_r = -\frac{(1-2\eta)V_e V_e r^2}{2(1+\eta)h} + f(z) \]

\( U_r \) at \( r = r_0 \) can be obtained from equations (A7) and (A4).

At \( r = r_0 \); \( U_r = \frac{(1-2\eta)r_0}{2(1+\eta)h} \) and at \( r = r_1 \);

\[ U_r = -\frac{(1-2\eta)r_n^2 - r_1^2}{2(1+\eta)hr_1} \quad (A9) \]
Using boundary conditions, at
\[ r = r_0 U_r = \frac{(1 - 2\eta) r_0^2}{2(1 + \eta) rh} - \frac{(1 - 2\eta) V_e (r^2 - r_0^2)}{2(1 + \eta) rh} \]  \hspace{1cm} (A10)

Applying mass constancy to Zone 2, \( V_e \) is obtained as:
\[ V_e = \rho r_0^2 + r_n^2 - r_1^2 r_1^2 - r_0^2 \]  \hspace{1cm} (A11)

(A10) suggest that at some radius \( r_e \) in Zone 2, \( U_r = 0 \). Using \( U_r = 0 \) in equation (A11)

\[ 0 = \frac{(1 - 2\eta) r_0^2}{2(1 + \eta) rh} - \frac{(1 - 2\eta) V_e (r^2 - r_0^2)}{2(1 + \eta) rh}; \]
\[ \Rightarrow r_e^2 = \frac{r_0^2 (1 + V_e)}{V_e} \]
\[ \Rightarrow r_e = r_0 \sqrt{\frac{(1 + V_e)}{V_e}} \]

**A.2. Velocity Fields — Case II**

It is assumed that the velocity \( V_e \) is positive, but no material from Zone 1 flows into the die hole. The material coming out of Zone 3 flows partly into the die hole and partly into Zone 1.

**Zone 3** : \( U_z = -\frac{z}{h}; \quad U_r = \frac{(1 - 2\eta)r}{2(1 + \eta)h} \)

**Zone 2** : \( U_z = -\frac{z}{h} V_e; \quad U_r = \frac{(1 - 2\eta)}{2(1 + \eta)hr} [r_0^2(1 + V_e) - V_e r^2] \) \hspace{1cm} (A12)

**Zone 1** : \( U_z = -\frac{z}{h}; \quad U_r = \frac{(1 - 2\eta)}{2(1 + \eta)hr} [r_0^2(1 + V_e) - r_1^2 r(1 + V_e) + r^2] \) \hspace{1cm} (A13)

**A.3. Velocity Fields — Case III**

It is assumed that \( V_e \) is negative. The material from Zone 3 and Zone 2 together flow to Zone 1. The velocity components for the different zones are as given below:

**Zone 3** : \( U_z = -\frac{z}{h}; \quad U_r = \frac{(1 - 2\eta)r}{2(1 + \eta)h} \)
Zone 2: \[ U_z = -\frac{z}{h} V_e; \quad U_r = \frac{(1 - 2\eta)}{2(1 + \eta)hr}[r_0^2(1 - V_e) - V_e r^2] \]

Zone 1: \[ U_z = -\frac{z}{h}; \quad U_r = \frac{(1 - 2\eta)}{2(1 + \eta)hr}[r_0^2(1 - V_e) - r_1^2(1 - V_e) + r^2] \quad (A14) \]

B. Nomenclature

\( \tau \) = Shear stress; \( \mu \) = Coefficient of friction;
\( \eta \) = Constant and a function of \( \rho \) only;
\( k \) = Constant equal to 2 in yield criterion;
\( r, \theta, z \) = Cylindrical co-ordinates;
\( n \) = A constant quantity much greater than 1;
\( V_e \) = Velocity of powder preform in annular hole;
\( \rho \) = Relative density of the perform;
\( h \) = Instantaneous thickness of perform; \( p \) = ram pressure;
\( \rho^*, \rho_r \) = Densities of apparent and real contact areas;
\( \varepsilon^*_{r}, \varepsilon^*_{\theta}, \varepsilon^*_{z} \) = Principal strain increment;
\( \sigma_0 \) = Yield stress of the non-work hardening matrix metal;
\( P \) = Die load;
\( J'_2 \) = Second invariant of deviatoric stress;
\( r_0, r_1, r_2, \dot{h} \) = Dimensions of specimen.