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SOME TYPES OF TYPE-2 TRIANGULAR FUZZY MATRICES

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Abstract: Type-2 fuzzy sets are fuzzy sets whose membership values are fuzzy sets on the interval [0, 1]. This concept was proposed by Zadeh, as an extension of fuzzy sets. Type-2 fuzzy sets possess a great expressive power and are conceptually quite appealing. Also fuzzy matrices play an important role in scientific developments. In this paper, some types of type-2 triangular fuzzy matrices (T2TFM) are defined. Also some basic properties of T2TFM are verified.

Key Words: Type-2 fuzzy set, Type-2 triangular fuzzy number, Type-2 triangular fuzzy matrices

1. Introduction

The concept of a type-2 fuzzy set, which is an extension of the concept of an ordinary fuzzy set, was introduced by Zadeh [11]. A type-2 fuzzy set is characterized by a membership function, i.e., the membership value for each element of this set is a fuzzy set in [0, 1], unlike an ordinary fuzzy set where the membership value is a crisp number in [0, 1]. Hisdal [1] discussed the IF THEN ELSE statement and interval-valued fuzzy sets of higher type. Jhon [2] studied an appraisal of theory and applications on type-2 fuzzy sets. Dinagar

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and Anbalagan [7] presented new ranking function and arithmetic operations on generalized type-2 trapezoidal fuzzy numbers.

The fuzzy matrices introduced first time by Thomason [9], and discussed about the convergence of powers of fuzzy matrix. Kim [3] presented some important results on determinant of square fuzzy matrices. Ragab et al. [5] presented some properties of the min–max composition of fuzzy matrices. Shyamal and Pal [6] first time introduced triangular fuzzy matrices. Recently Dinagar and Latha [8] introduced type-2 triangular fuzzy matrices.

The paper is organized as follows. Firstly in Section 2, some basic definitions are given. In Section 3 of this paper, we recall the definition of type-2 triangular fuzzy number and some operations on type-2 triangular fuzzy numbers. In Section 4, we review the definition of type-2 triangular fuzzy matrices (T2TFM) and some operations on T2TFMs. In Section 5, we define some types of T2TFMs. In Section 6, we present some properties of T2TFMs. Finally in Section 7, conclusion is also included.

2. Preliminaries

Definition 2.1. (Fuzzy set) A fuzzy set is characterized by a membership function mapping the elements of a domain, space or universe of discourse X to the unit interval [0,1].

A fuzzy set A in a universe of discourse X is defined as the following set of pairs:

$$A = \{(x, \mu_A(x)); x \in X\}.$$

Here $\mu_A : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

Definition 2.2. (Normal fuzzy set) A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exists at least one $x \in X$ such that $\mu_A(x) = 1$.

Definition 2.3. (Triangular fuzzy number) For a triangular fuzzy number $A(x)$, it can be represented by $A(a, b, c; 1)$ with membership function μ(x) given

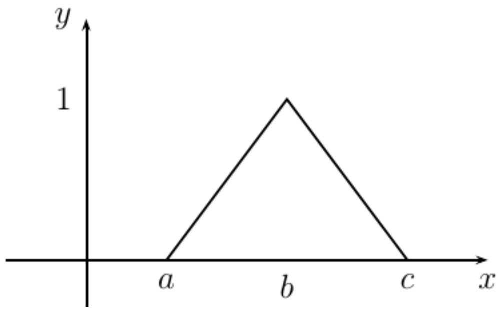

Figure 1: Graphical representation of triangular fuzzy number $(a, b, c; 1)$

by

$$\mu(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & x = b \\ \dfrac{c-x}{c-b}, & c \le x \le d \\ 0, & \text{otherwise.} \end{cases}$$

Satisfying the following conditions:

1. $\mu(x)$ is a continuous mapping from R to the closed interval [0, 1];
2. $\mu(x) = 0 \ \forall x \in (-\infty, a]$;
3. Strictly increasing and continuous on $[a, b]$;
4. $\mu(x) = 1$ at $x = b$;
5. Strictly decreasing and continuous on $[b, c]$;
6. $\mu(x) = 0 \ \forall x \in [c, \infty)$.

The graphical representation of a triangular fuzzy number is shown in Figure 1.

3. Type-2 Triangular Fuzzy Numbers

Definition 3.1. ((Zadeh): Type-2 fuzzy set) A type-2 fuzzy set is a fuzzy set whose membership values are fuzzy sets on [0, 1].

Definition 3.2. The type-2 fuzzy sets are defined by functions of the form $\mu_A : x \to \chi([0,1])$ where $\chi[0,1]$ denotes the set of all ordinary fuzzy sets that can be defined within the universal set [0, 1]. An example [4] of a membership function of this type is given in Figure 2.

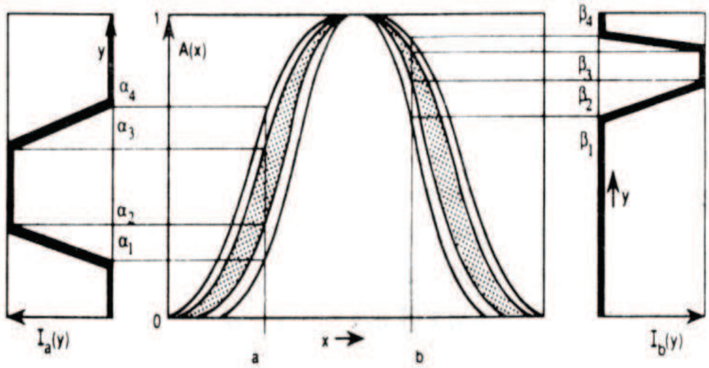

Figure 2: Illustration of the concept of a fuzzy set of type-2.

Definition 3.3. (Type-2 fuzzy number [7]) Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied:

1. $\tilde{\tilde{A}}$ is normal,
2. $\tilde{\tilde{A}}$ is a convex set,
3. The support of $\tilde{\tilde{A}}$ is closed and bounded, then $\tilde{\tilde{A}}$ is called a type-2 fuzzy number.

Definition 3.4. (Type-2 Triangular Fuzzy Number) A type-2 triangular fuzzy number $\tilde{\tilde{A}}$ on R is given by $\tilde{\tilde{A}} = \{(x, (\mu_A^1(x), \mu_A^2(x), \mu_A^3(x)); x \in R\}$ and $\mu_A^1(x) \le \mu_A^2(x) \le \mu_A^3(x)$, for all $x \in R$. Denote

$$\tilde{\tilde{A}} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3),$$

where $\tilde{A}_1 = (A_1^L, A_1^N, A_1^U)$, $\tilde{A}_2 = (A_2^L, A_2^N, A_2^U)$ and $\tilde{A}_3 = (A_3^L, A_3^N, A_3^U)$ are same type of fuzzy numbers.

3.1. Arithmetic Operations on Type-2 Triangular Fuzzy Numbers, see [8]

Let

$$\tilde{\tilde{a}} = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3) = ((a_1^L, a_1^N, a_1^U), (a_2^L, a_2^N, a_2^U), (a_3^L, a_3^N, a_3^U)) \text{ and}$$
$$\tilde{\tilde{b}} = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = ((b_1^L, b_1^N, b_1^U), (b_2^L, b_2^N, b_2^U), (b_3^L, b_3^N, b_3^U))$$

be two type-2 triangular fuzzy numbers. Then we define,

(i) Addition

$\tilde{\tilde{a}} + \tilde{\tilde{b}} = \big((a_1^L + b_1^L, a_1^N + b_1^N, a_1^U + b_1^U), (a_2^L + b_2^L, a_2^N + b_2^N, a_2^U + b_2^U), (a_3^L + b_3^L, a_3^N + b_3^N, a_3^U + b_3^U)\big)$.

(ii) Subtraction

$\tilde{\tilde{a}} - \tilde{\tilde{b}} = ((a_1^L - b_3^U, a_1^N - b_3^N, a_1^U - b_3^L), (a_2^L - b_2^U, a_2^N - b_2^N, a_2^U - b_2^L), (a_3^L - b_1^U, a_3^N - b_1^N, a_3^U - b_1^L))$.

(iii) Scalar Multiplication

If $k \geq 0$ and $k \in R$ then
$k\tilde{\tilde{a}} = ((ka_1^L, ka_1^N, ka_1^U), (ka_2^L, ka_2^N, ka_2^U), (ka_3^L, ka_3^N, ka_3^U))$ and if
$k < 0$ and $k \in R$ then
$k\tilde{\tilde{a}} = ((ka_3^U, ka_3^N, ka_3^L), (ka_2^U, ka_2^N, ka_2^L), (ka_1^U, ka_1^N, ka_1^L))$.

(iv) Multiplication

Define $\sigma b = b_1^L + b_1^N + b_1^U + b_2^L + b_2^N + b_2^U + b_3^L + b_3^N + b_3^U$.
If $\sigma b \geq 0$, then

$$\tilde{\tilde{a}} \times \tilde{\tilde{b}} = \left(\left(\frac{a_1^L \sigma b}{9}, \frac{a_1^N \sigma b}{9}, \frac{a_1^U \sigma b}{9} \right), \left(\frac{a_2^L \sigma b}{9}, \frac{a_2^N \sigma b}{9}, \frac{a_2^U \sigma b}{9} \right), \left(\frac{a_3^L \sigma b}{9}, \frac{a_3^N \sigma b}{9}, \frac{a_3^U \sigma b}{9} \right) \right).$$

If $\sigma b < 0$, then

$$\tilde{\tilde{a}} \times \tilde{\tilde{b}} = \left(\left(\frac{a_3^U \sigma b}{9}, \frac{a_3^N \sigma b}{9}, \frac{a_3^L \sigma b}{9} \right), \left(\frac{a_2^U \sigma b}{9}, \frac{a_2^N \sigma b}{9}, \frac{a_2^L \sigma b}{9} \right), \left(\frac{a_1^U \sigma b}{9}, \frac{a_1^N \sigma b}{9}, \frac{a_1^L \sigma b}{9} \right) \right).$$

(v) Division

Whenever $\sigma b \neq 0$ we define division as follows:

If $\sigma b > 0$, then

$$\frac{\tilde{\tilde{a}}}{\tilde{\tilde{b}}} = \left(\left(\frac{9a_1^L}{\sigma b}, \frac{9a_1^N}{\sigma b}, \frac{9a_1^U}{\sigma b} \right), \left(\frac{9a_2^L}{\sigma b}, \frac{9a_2^N}{\sigma b}, \frac{9a_2^U}{\sigma b} \right), \left(\frac{9a_3^L}{\sigma b}, \frac{9a_3^N}{\sigma b}, \frac{9a_3^U}{\sigma b} \right) \right).$$

If $\sigma b < 0$, then

$$\frac{\tilde{\tilde{a}}}{\tilde{\tilde{b}}} = \left(\left(\frac{9a_3^U}{\sigma b}, \frac{9a_3^N}{\sigma b}, \frac{9a_3^L}{\sigma b} \right), \left(\frac{9a_2^U}{\sigma b}, \frac{9a_2^N}{\sigma b}, \frac{9a_2^L}{\sigma b} \right), \left(\frac{9a_1^U}{\sigma b}, \frac{9a_1^N}{\sigma b}, \frac{9a_1^L}{\sigma b} \right) \right).$$

3.2. The Proposed Ranking Function, see [8]

Let $F(R)$ be the set of all type-2 normal triangular fuzzy numbers. One convenient approach for solving numerical valued problem is based on the concept of comparison of fuzzy numbers by use of ranking function. An effective approach for ordering the elements of $F(R)$ is to define a linear ranking function $\check{R} : F(R) \to R$ which maps each fuzzy number into R.

Suppose if
$\tilde{\tilde{A}} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3), = ((A_1^L, A_1^N, A_1^U), (A_2^L, A_2^N, A_2^U), (A_3^L, A_3^N, A_3^U))$
then we define
$\check{R}(\tilde{\tilde{A}}) = (A_1^L + A_1^N + A_1^U + A_2^L + A_2^N + A_2^U + A_3^L + A_3^N + A_3^U) \ / \ 9$.

Also we define orders on $F(R)$ by $\check{R}(\tilde{\tilde{A}}) \geq \check{R}(\tilde{\tilde{B}})$ if and only if $\tilde{\tilde{A}} \geq_{\check{R}} \tilde{\tilde{B}}$,

$\check{R}(\tilde{\tilde{A}}) \leq \check{R}(\tilde{\tilde{B}})$ if and only if $\tilde{\tilde{A}} \leq_{\check{R}} \tilde{\tilde{B}}$

and $\check{R}(\tilde{\tilde{A}}) = \check{R}(\tilde{\tilde{B}})$ if and only if $\tilde{\tilde{A}} =_{\check{R}} \tilde{\tilde{B}}$.

Definition 3.5. (Type-2 zero triangular fuzzy number)
If $\tilde{\tilde{A}} = \big((0,0,0),(0,0,0),(0,0,0)\big)$ then $\tilde{\tilde{A}}$ is said to be a type-2 zero triangular fuzzy number. It is denoted by 0.

Definition 3.6. (Type-2 zero-equivalent triangular fuzzy number)
A type-2 triangular fuzzy number $\tilde{\tilde{A}}$ is said to be a type-2 zero-equivalent triangular fuzzy number if $\check{R}(\tilde{\tilde{A}}) = 0$. It is denoted by $\tilde{\tilde{0}}$.

Definition 3.7. (Type-2 unit triangular fuzzy number)
If $\tilde{\tilde{A}} = \big((1,1,1),(1,1,1),(1,1,1)\big)$ then $\tilde{\tilde{A}}$ is said to be a type-2 unit triangular fuzzy number. It is denoted by 1.

Definition 3.8. (Type-2 unit-equivalent triangular fuzzy number) A type-2 triangular fuzzy number $\tilde{\tilde{A}}$ is said to be a type-2 unit-equivalent triangular fuzzy number if $\check{R}(\tilde{\tilde{A}}) = 1$. It is denoted by $\tilde{\tilde{1}}$.

Definition 3.9. (Inverse of type-2 triangular fuzzy number) If $\tilde{\tilde{a}}$ is a type-2 triangular fuzzy number and $\tilde{\tilde{a}} \neq \tilde{\tilde{0}}$ then we define $\tilde{\tilde{a}}^{-1} = \frac{\tilde{\tilde{1}}}{\tilde{\tilde{a}}}$.

4. Type-2 Triangular Fuzzy Matrices (T2TFMs), see [8]

Definition 4.1. A type-2 triangular fuzzy matrix (T2TFM) of order $m \times n$ is defined as $A = (\tilde{\tilde{a}}_{ij})_{m\times n}$ where the ijth element $\tilde{\tilde{a}}_{ij}$ of A is the type-2 triangular fuzzy number.

4.1. Operations on T2TFMs

As for classical matrices we define the following operations on T2TFMs. Let $A = (\tilde{\tilde{a}}_{ij})$ and $B = (\tilde{\tilde{b}}_{ij})$ be two T2TFMs of same order. Then we have the following:

1. $A + B = (\tilde{\tilde{a}}_{ij} + \tilde{\tilde{b}}_{ij})$.

2. $A - B = (\tilde{\tilde{a}}_{ij} - \tilde{\tilde{b}}_{ij})$.

3. For $A = (\tilde{\tilde{a}}_{ij})_{m\times n}$ and $B = (\tilde{\tilde{b}}_{ij})_{n\times k}$ then $AB = (\tilde{\tilde{c}}_{ij})_{m\times k}$ where $\tilde{\tilde{c}}_{ij} = \sum_{p=1}^{n} \tilde{\tilde{a}}_{ip} \cdot \tilde{\tilde{b}}_{pj}$, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, k$.

4. A^T or $A' = (\tilde{\tilde{a}}_{ji})$.

5. $kA = (k\tilde{\tilde{a}}_{ij})$, where k is a scalar.

5. Types of Type-2 Triangular Fuzzy Matrices

Definition 5.1. (Square T2TFM) A T2TFM $A = (\tilde{\tilde{a}}_{ij})_{m\times n}$ is said to be a square T2TFM if the number of rows is equal to the number of columns, i.e., $m = n$. Otherwise it is called non-square T2TFM.

Definition 5.2. (Symmetric T2TFM) A square T2TFM $A = (\tilde{\tilde{a}}_{ij})$ is said to be a symmetric T2TFM about the principal diagonal if $\tilde{\tilde{a}}_{ij} = \tilde{\tilde{a}}_{ji}$ for all $i, j = 1, 2, \ldots, n$, i.e., $A^T = A$.

Definition 5.3. (Skew-symmetric T2TFM) A square T2TFM $A = (\tilde{\tilde{a}}_{ij})$ is said to be skew-symmetric T2TFM if $\tilde{\tilde{a}}_{ij} = -\tilde{\tilde{a}}_{ji}$ for all $i, j = 1, 2, \ldots, n$ and $a_{ii} = 0$. i.e., $A^T = -A$.

Definition 5.4. (Skew-symmetric equivalent T2TFM)
A square T2TFM $A = (\tilde{\tilde{a}}_{ij})$ is said to be skew-symmetric equivalent T2TFM if $\tilde{\tilde{a}}_{ij} = -\tilde{\tilde{a}}_{ji}$ for all $i, j = 1, 2, \ldots, n$ and $a_{ii} = \tilde{\tilde{0}}$. i.e., $A^T = -A$.

Definition 5.5. (Diagonal T2TFM) A square T2TFM $A = (\tilde{\tilde{a}}_{ij})$ is said to be a diagonal T2TFM if all the elements outside the principal diagonal are 0.

Definition 5.6. (Diagonal-equivalent T2TFM) A square T2TFM $A = (\tilde{\tilde{a}}_{ij})$ is said to be a diagonal- equivalent T2TFM if all the elements outside the principal diagonal are $\tilde{\tilde{0}}$.

Definition 5.7. (Scalar T2TFM) A diagonal T2TFM $A = (\tilde{\tilde{a}}_{ij})$ is said to be a scalar T2TFM if every entry $\tilde{\tilde{a}}_{ii}$ in the principal diagonal are the same.

Definition 5.8. (Scalar–equivalent T2TFM) A diagonal-equivalent T2TFM $A = (\tilde{\tilde{a}}_{ij})$ is said to be a scalar-equivalent T2TFM if the value of $\check{R}(\tilde{\tilde{a}}_{ii})$ is the same for every entry $\tilde{\tilde{a}}_{ii}$ in the principal diagonal.

Definition 5.9. (Unit T2TFM or Identity T2TFM) A scalar T2TFM $A = (\tilde{\tilde{a}}_{ij})$ is said to be an unit T2TFM or identity T2TFM if $\tilde{\tilde{a}}_{ii} = 1$ for every entry $\tilde{\tilde{a}}_{ii}$ in the principal diagonal. It is denoted by I.

Definition 5.10. (Unit-equivalent T2TFM or Identity-equivalent T2TFM) A scalar-equivalent T2TFM $A = (\tilde{\tilde{a}}_{ij})$ is said to be an unit-equivalent T2TFM or identity-equivalent T2TFM if $\tilde{\tilde{a}}_{ii} = \tilde{\tilde{1}}$ for every entry $\tilde{\tilde{a}}_{ii}$ in the principal diagonal. It is denoted by $\hat{I}$.

Definition 5.11. (Null T2TFM or Zero T2TFM) The $m \times n$ T2TFM with each entry 0 is called the null T2TFM or zero T2TFM. It is denoted by O.

Definition 5.12. (Null-equivalent T2TFM or Zero-equivalent T2TFM) The $m \times n$ T2TFM with each entry $\tilde{\tilde{0}}$ is called the null-equivalent T2TFM or zero-equivalent T2TFM. It is denoted by $\hat{O}$.

Definition 5.13. (Upper triangular T2TFM) A square T2TFM $A = (\tilde{\tilde{a}}_{ij})$ is called an upper triangular T2TFM if all the entries below the principal diagonal are 0.

Definition 5.14. (Upper triangular-equivalent T2TFM)
A square T2TFM $A = (\tilde{\tilde{a}}_{ij})$ is called an upper triangular-equivalent T2TFM if all the entries below the principal diagonal are $\tilde{\tilde{0}}$.

Definition 5.15. (Lower triangular T2TFM) A square T2TFM $A = (\tilde{\tilde{a}}_{ij})$ is called a lower triangular T2TFM if all the entries above the principal diagonal are 0.

Definition 5.16. (Lower triangular-equivalent T2TFM)
A square T2TFM $A = (\tilde{\tilde{a}}_{ij})$ is called a lower triangular -equivalent T2TFM if all the entries above the principal diagonal are $\tilde{\tilde{0}}$.

Definition 5.17. (Row T2TFM) A $1 \times n$ T2TFM is called a row T2TFM.

Definition 5.18. (Column T2TFM) A $m \times 1$ T2TFM is called a column T2TFM.

6. Properties of Type-2 Triangular Fuzzy Matrices

Property 6.1. For any three T2TFMs A,B and C of order $m \times n$ we have:

1. $A + B = B + A$,
2. $A + (B + C) = (A + B) + C$,
3. $A + A = 2A$,
4. $A - A = \hat{O}$, i.e., a null-equivalent T2TFM.
5. $A + O = O + A = A$
6. $A - O = A$.

Property 6.2. Let A and B be two T2TFMs of the same order and p, q be two scalars, then:

1. $p(qA) = (pq)A$,
2. $p(A + B) = pA + pB$,
3. $p(A - B) = pA - pB$,
4. $(p + q)A = pA + qA$, if $p, q \geq 0$.

Property 6.3. If A and B are two T2TFMs and p, q are two scalars then:

1. $(A')' = A$,
2. $(A + B)' = A' + B'$,
3. $(pA)' = pA'$,
4. $(pA + qB)' = pA' + qB'$.

Property 6.4. The product of two lower triangular T2TFMs of order $n \times n$ is also a lower triangular T2TFM.

Proof. Let $A = (\tilde{\tilde{a}}_{ij})$ and $B = (\tilde{\tilde{b}}_{ij})$ be two lower triangular T2TFMs. Since A and B are lower triangular T2TFMs then $\tilde{\tilde{a}}_{ij} = 0$ and $\tilde{\tilde{b}}_{ij} = 0$ for all $i < j$; $i, j = 1, 2, \ldots, n$.

Let $AB = C = (\tilde{\tilde{c}}_{ij})$ where $\tilde{\tilde{c}}_{ij} = \sum_{k=1}^{n} \tilde{\tilde{a}}_{ik} \cdot \tilde{\tilde{b}}_{kj}$. Now we will show that $\tilde{\tilde{c}}_{ij} = 0$ for all $i < j$; $i, j = 1, 2, \ldots, n$.

For $i < j$ we have $\tilde{\tilde{a}}_{ik} = 0$ for $k = i + 1, i + 2, \ldots, n$, and similarly $\tilde{\tilde{b}}_{kj} = 0$ for $k = 1, 2, \ldots, i$.

Therefore

$$\tilde{\tilde{c}}_{ij} = \sum_{k=1}^{n} \tilde{\tilde{a}}_{ik} \cdot \tilde{\tilde{b}}_{kj} = \sum_{k=1}^{i} \tilde{\tilde{a}}_{ik}.\tilde{\tilde{b}}_{kj} + \sum_{k=i+1}^{n} \tilde{\tilde{a}}_{ik}.\tilde{\tilde{b}}_{kj} = 0$$

Now

$$\tilde{\tilde{c}}_{ii} = \sum_{k=1}^{n} \tilde{\tilde{a}}_{ik} \cdot \tilde{\tilde{b}}_{ki} = \sum_{k=1}^{i-1} \tilde{\tilde{a}}_{ik}.\tilde{\tilde{b}}_{ki} + \tilde{\tilde{a}}_{ii} \cdot \tilde{\tilde{b}}_{ii} + \sum_{k=i+1}^{n} \tilde{\tilde{a}}_{ik}.\tilde{\tilde{b}}_{ki} = \tilde{\tilde{a}}_{ii} \cdot \tilde{\tilde{b}}_{ii}$$

Hence the result follows. □

Property 6.5. The product of two upper triangular T2TFMs of order $n \times n$ is also an upper triangular T2TFM.

Proof. Let $A = (\tilde{\tilde{a}}_{ij})$ and $B = (\tilde{\tilde{b}}_{ij})$ be two upper triangular T2TFMs. Since A and B are upper triangular T2TFMs then $\tilde{\tilde{a}}_{ij} = 0$ and $\tilde{\tilde{b}}_{ij} = 0$ for all $i > j;\ i, j = 1, 2, \dots, n$.

Let $AB = C = (\tilde{\tilde{c}}_{ij})$ where $\tilde{\tilde{c}}_{ij} = \sum_{k=1}^{n} \tilde{\tilde{a}}_{ik} \cdot \tilde{\tilde{b}}_{kj}$. Now we will show that $\tilde{\tilde{c}}_{ij} = 0$ for all $i > j;\ i, j = 1, 2, \dots, n$.

For $i > j$ we have $\tilde{\tilde{a}}_{ik} = 0$ for $k = 1, 2, \dots, i-1$, and similarly $\tilde{\tilde{b}}_{kj} = 0$ for $k = i, i+1, \dots, n$.

$$\text{Therefore } \tilde{\tilde{c}}_{ij} = \sum_{k=1}^{n} \tilde{\tilde{a}}_{ik} \cdot \tilde{\tilde{b}}_{kj} = \sum_{k=1}^{i-1} \tilde{\tilde{a}}_{ik}.\tilde{\tilde{b}}_{kj} + \sum_{k=i}^{n} \tilde{\tilde{a}}_{ik}.\tilde{\tilde{b}}_{kj} = 0$$

$$\text{Now } \tilde{\tilde{c}}_{ii} = \sum_{k=1}^{n} \tilde{\tilde{a}}_{ik} \cdot \tilde{\tilde{b}}_{ki} = \sum_{k=1}^{i-1} \tilde{\tilde{a}}_{ik}.\tilde{\tilde{b}}_{ki} + \tilde{\tilde{a}}_{ii}.\tilde{\tilde{b}}_{ii} + \sum_{k=i+1}^{n} \tilde{\tilde{a}}_{ik}.\tilde{\tilde{b}}_{ki} = \tilde{\tilde{a}}_{ii}.\tilde{\tilde{b}}_{ii}$$

Hence the result follows. □

7. Conclusion

In this article some types of type-2 triangular fuzzy matrices are defined and also some properties of T2TFMs are verified. Using these types of T2TFMs, some important properties of T2TFMs, involving the notion like determinant of matrix, adjoint of matrix and inverse of matrix can be studied in future. Also the theories of the discussed T2TFMs may be utilized in further works.

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