International Journal of Pure and Applied Mathematics
Volume 82 No. 1 2013, 87-94
ISSN: 1311-8080 (printed version)
url: http://www.ijpam.eu

ijpam.eu

AXIALLY SYMMETRIC SPACE-TIME WITH STRANGE QUARK MATTER ATTACHED TO STRING CLOUD IN BIMETRIC THEORY

P.K. Sahoo[1 §], B. Mishra[2]
[1,2]Department of Mathematics
Birla Institute of Technology and Science-Pilani
Hyderabad Campus, Hyderabad, 500078, INDIA

Abstract: In this paper, we study axially symmetric space-time with strange quark matter attached to string cloud in Rosen's (1973) bimetric theory. It is shown that there is no contribution from strange quark matter, hence vacuum model is presented and studied.

AMS Subject Classification: 83D05
Key Words: axially symmetric space-time, quark matter, cosmic strings, bimetric theory

1. Introduction

The bimetric theory of gravitation was proposed by Rosen (1973) to modify the Einstein's general theory of relativity, by assuming two metric tensors viz. a Riemannian metric tensor g_{ij} and the background metric tensor γ_{ij}. The metric tensor g_{ij} determines the Riemannian geometry of the curved space time which plays the same role as given in Einstein's general relativity and interacts with matter. The background metric tensor γ_{ij} refers to the geometry of the empty (free from matter and radiation) universe and describes the inertial forces. The metric tensor γ_{ij} has no direct physical significance but appears in the field equations. Moreover, the bimetric theory also satisfied the covariance

Received: September 25, 2012

§Correspondence author

and equivalence principles: the formation of general relativity. The theory agrees with the present observational facts pertaining to general relativity. Thus at every point of space-time, there are two metrics

$$ds^2 = g_{ij}dx^i dx^j \tag{1}$$

and

$$d\sigma^2 = \gamma_{ij}dx^i dx^j \tag{2}$$

where ds is the interval between two neighboring events as measured by a clock and a measuring rod. The interval $d\sigma$ is an abstract or a geometrical quantity which is not directly measurable. One can regard it as describing the geometry that exists if no matter were present.

The field equations of Rosen's bimetric theory of gravitation derived from variational principles are

$$N^i_j - \frac{1}{2}N\delta^i_j = -8\pi\kappa T^i_j \tag{3}$$

where

$$N^i_j = \frac{1}{2}\gamma^a_b(g^{hi}g_{hj|a})_{|b}$$

and

$$N = N^i_i,\ g = \det(g_{ij}),\ \gamma = \det(\gamma_{ij}),\ \kappa = \sqrt{(\frac{g}{\gamma})}$$

A vertical bar ($|$) denotes the covariant differentiation with respect to γ_{ij} and T^i_j is the energy momentum tensor of the matter field.

Several aspects of bimetric theory of gravitation have been studied by Rosen (1974), Karade (1980), Israelit (1981), Reddy and Venkateswarlu (1989), Reddy and Venkateswara Rao (1998), Adhav et al. (2002, 2005). In particular Mohanty et al. (2002), Mohanty and Sahoo (2002) have shown the non-existence of perfect fluid or mesonic perfect fluid models in bimetric theory. Sahoo et al. (2011) have studied Bianchi type cosmic string models coupled with Maxwell fields in this theory.

In quantum field theories, broken symmetries are restored at high temperatures. One of the interesting consequences of the first order phase transition from quark phase to hadron phase in the early universe is the formation of strange quark matter and it has been attracting much interest (Witten (1984), Fahri and Jaffe (1984)).

In this study, we will attach strange quark matter to the string cloud. It is plausible to attach strange quark matter to the string cloud. Because, one of such transitions during the phase transitions of the universe could be Quark

Glucon Plasma (QGP) harden gas (called quark-hadron phase transition) when cosmic temperature was $T \approx 200Mev$. Experiments at CERN's Super Proton Synchrotron (SPS) first tried to create the QGP in the 1980s and 1990s: the results led CERN to announce indirect evidence for a "new state of matter" in 2000. Current experiments at Brookhaven National Laboratory's Relativistic Heavy Ion Collider (RHIC) are continuing this effort. Itoh (1970), Bodmar (1971) and Witten (1984) have formed two ways for creation of strange quark matter. One is the quark hadron phase transition in the early universe and another is the conversion of neutron stars into strange ones at ultrahigh density. In strong interaction theories, it is supposed that breaking of physical vacuum takes place inside hadrons to form quark bag model. As a result, vacuum energy densities inside and outside a hadron become essentially different, and the vacuum pressure on the bag wall equilibrates the pressure of quarks, thus stabilizing the system. Alcock et al. (1986) and Haensel et al. (1986) examined that if the hypothesis of the quark matter is true, then some of neutrons stars could actually be strange stars built entirely of strange matter. Cheng et al. (1998) have studied strange star properties. Yavuz et al. (2005) and Yilmaz (2005, 2006) have studied 5-D Kaluza-Klein cosmological models with quark matter attached to the string cloud and domain walls. Burau et al. (2005) have studied anisotropic flow of charged and identified hadrons in the quark-gluon string model for Au + Au collisions at 200GeV. Arakelyan et al. (2006) have studied quark-gluon string model description of baryon production. Filho and Brage (2006) have examined duality between static strings and quark anti-quark configuration in the Randall-Sendrum Scenarios.

Strange quark matter is modeled with an equation of state based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume. In this model, quarks are through as degenerate Fermi gas, which exists only in a region of space endowed with a vacuum energy density Bc (called as the bag constant). In the framework of this model, the quark matter is composed of massless u and d quarks, massive s quarks and electrons. In the simplified version of the bag model, it is assumed that quarks are massless and non-interacting.

Therefore, we have quark pressure

$$p_q = \frac{\rho_q}{3} \tag{4}$$

where ρ_q is the quark energy density.

The total energy density is

$$\rho = \rho_q + B_C \tag{5}$$

But the total pressure is

$$p = p_q - B_C \tag{6}$$

Mak and Harko (2004) have studied charged strange quark matter in the spherically symmetric space-time admitting conformal motion. The study of strange quark matter attached to the string cloud in the spherical symmetric space-time admitting conformal motion has been done by Yavuz et al. (2005). Letelier (1983) and Stachel (1980) have proposed the study of general relativistic treatment of strings. Dey et al. (1998) have obtained new sets of EOSs for strange matter based on a model of inter quark potential which has the following features: (a) asymptotic freedom, (b) confinement at zero baryon density and deconfinement at high baryon density, (c) chiral symmetry restoration and (d) gives stable uncharged β-stable matter. Adhav et al. (2008, 2009) have discussed string cloud and domain walls with quark matter in n-dimensional Kaluza-Klein cosmological model in general relativity and strange quark matter attached to string cloud in Bianchi type-III space time in general relativity. Khadekar et al. (2009) have confirmed their work to the quark matters which attached to the topological defects in general relativity. Katore (2012) have obtained cosmological model with strange quark matter attached to cosmic string for axially symmetric space-time in general relativity. Recently Jain et al. (2012) have studied axially symmetric space-time with wet dark fluid in bimetric theory.

In this paper, we will study the field equations for axially symmetric space time with strange quark matter attached to the string cloud in Rosen's bimetric theory.

2. Metric and Field Equations

We consider the axially symmetric space-time as

$$ds^2 = dt^2 - A^2[d\chi^2 + f^2(\chi)d\phi^2] - B^2dz^2 \tag{7}$$

with the convention $x^1 = \chi, x^2 = \phi, x^3 = z, x^4 = t$.

Here A and B are functions of t alone while f is a function of the coordinate χ alone.

The flat space-time corresponding to the metric (7) is

$$d\sigma^2 = dt^2 - d\chi^2 - d\phi^2 - dz^2 \tag{8}$$

The energy momentum tensor for string cloud [Letelier (1983)] is given by

$$T^i_j = \rho U^i U_j - \rho_s X^i X_j \tag{9}$$

here ρ is the rest energy density for the cloud of strings with particles attached to them and ρ_s is the string tension density. They are related by

$$\rho = \rho_p + \rho_s \tag{10}$$

where ρ_p is the particle energy density. We know that string is free to vibrate. The different vibrational models of the string represent different types of particles because these different models are seen as different masses or spins. Therefore, here we will take quarks instead of particles in the string cloud. Hence we consider quark matter energy density instead of particle energy density in the string cloud.

In this case from (10), we get

$$\rho = \rho_p + \rho_s + B_C \tag{11}$$

From (10) and (11), we have energy momentum tensor for strange quark matter attached to the string cloud as (Yavuz et al. 2005)

$$T^i_j = (\rho_p + \rho_s + B_C)U^iU_j - \rho_s X^i X_j \tag{12}$$

where U^i is the four velocity of the particles and X^i is the unit space like vector representing the direction of string.

We have U^i and X^i with satisfying conditions:

$$U^iU_i = 1, \quad U^iX_i = 0, \quad X^iX_i = -1 \tag{13}$$

We have taken the direction of string along z-axes. Then the components of energy momentum tensor are

$$T^1_1 = 0 = T^2_2, T^3_3 = \rho_s, T^4_4 = \rho \tag{14}$$

The Rosen's field equations (3) for the metric (7) and (8) can be written as

$$\left(\frac{f_1}{f}\right)_1 - \left(\frac{B_4}{B}\right)_4 = 0 \tag{15}$$

$$-\left(\frac{f_1}{f}\right)_1 - \left(\frac{B_4}{B}\right)_4 = 0 \tag{16}$$

$$\left(\frac{f_1}{f}\right)_1 - 2\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 = 16\pi\kappa\rho_s \tag{17}$$

$$\left(\frac{f_1}{f}\right)_1 - 2\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 = -16\pi\kappa\rho \tag{18}$$

where the suffixes 1 and 4 following an unknown function denotes ordinary differentiation with respect to χ and t respectively.

The functional dependence of the metric together with (15) and (16) imply

$$\left(\frac{f_1}{f}\right)_1 = 0 \tag{19}$$

which immediately yields $f(\chi) = exp(k\chi)$ for some constant k.

Without loss of generality, by taking $k = 1$, we have $f(\chi) = exp(\chi)$.

Now the field equations (15)to (18) reduces to

$$\left(\frac{B_4}{B}\right)_4 = 0 \tag{20}$$

$$-2\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 = 16\pi\kappa\rho_s \tag{21}$$

$$2\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 = 16\pi\kappa\rho \tag{22}$$

From (20), we get

$$B = exp(c_1 t + c_2) \tag{23}$$

Using equations (20)-(22) we have

$$\rho + \rho_s = 0 \tag{24}$$

For reality conditions to hold we need $\rho > 0$ and $\rho_s > 0$.

Equation (24) gives us

$$\rho = 0 = \rho_s \tag{25}$$

Equation (25) shows that there is no contribution from strange quark matter to axially symmetric model in bimetric relativity.

Hence axially symmetric vacuum model in bimetric relativity can be expressed, after a proper choice of coordinates and constants, in the form

$$ds^2 = dt^2 - exp(c_1 t)\left[d\chi^2 + exp(2\chi)d\phi^2) + dz^2\right] \tag{26}$$

It is interesting to note that the vacuum model (26) is isotropic and has no singularity.

3. Conclusion

Here we have shown that axially symmetric model do not exist in Rosen's (1973) bimetric theory of gravitation in presence of strange quark matter attached to cloud of strings. We obtained a vacuum model which is free from singularity and reduces to a flat space-time when $t = 0$.

Acknowledgments

PKS acknowledges UGC, New Delhi, India for financial assistance to carry out the Minor Research Project [No. F. 41-1385/2012(SR)].

References

[1] Adhav, K. S. et al, Bulletin of Pure and Applied Science, **E21**(2002), 531.

[2] K.S. Adhav, S.D. Katore, S.V. Thakare, *Astrophysics and Space Science*, **299** (2005), 233.

[3] K.S. Adhav, A.S. Nimkar, M.V. Dawande, *Int. J. Theor. Phys.*, **47** (2008), 2002.

[4] K.S. Adhav, A.S. Nimkar, V.B. Raut, R.S. Thakare, *Astrophysics and Space Sci.*, **319** (2009), 81.

[5] C. Alcock, E. Farhi, A. Olinto, *Astrophys. J.*, **310** (1986), 261.

[6] G.H. Arakelyen, C. Merino, Y.M. Shabelski, *ArXiv,* hep-ph/0604103 (2006).

[7] A.R. Bodmar, *Phys. Rev.*, **D4** (1971), 1601.

[8] H. Boschi-Filho, N.R.F. Brage, *Phys. Rev.*, **D73** (2006), 106006.

[9] G. Burau, et al. *Phys. Rev.*, **C71** (2005), 54905.

[10] K.S. Cheng, Z.G. Dai, T. Lu, *Int. J. Mod. Phys.*, **D7** (1998), 139.

[11] M. Dey, I. Bombaci, J. Dey, S. Ray, B.C. Samanta, *Phys. Lett. B*, **438** (1998), 123.

[12] E. Farhi, R.L. Jaffe, *Phys. Rev. D*, **30** (1984), 2379.

[13] P. Haensel, J.L. Zdunik, R. Sehaeffer, *Astron. Astrophys.*, **160** (1986), 121.

[14] N. Itoh, *Prog. Theor. Phys.*, **44** (1970), 291.

[15] M. Israelit, *Gen. Relativ. Gravit.*, **13** (1981), 681.

[16] P. Jain, P.K. Sahoo, B. Mishra, *Int. J. Theor. Phys.*, **51** (2012), 2546.

[17] T.M. Karade, *Ind. J. Pure Appl. Math.*, **11** (1980), 1202.

[18] S.D. Katore, A.Y. Shaikh, *Int. J. Theor. Phys.*, **51** (2012), 1881.

[19] G.S. Khadekar, R. Wanjari, C. Ozel, *Int. J. Theor. Phys.*, **48** (2009), 2250.

[20] P.S. Latelier, *Phys. Rev. D*, **28** (1983), 2414.

[21] M.K. Mak, T. Harko, *Int. J. Mod. Phys. D*, **13** (2004), 149.

[22] G. Mohanty, P.K. Sahoo, *Czech. J. Phys.*, **52** (2002), 1041.

[23] G. Mohanty, P.K. Sahoo, B. Mishra, *Astrophys. Space Sci.*, **281** (2002), 609.

[24] D.R.K. Reddy, R. Venkateswarlu, *Astrophys. Space Sci.*, **158** (1989), 169.

[25] D.R.K. Reddy, Venkateswara N. Rao, *Astrophys. Space Sci.*, **257** (1998), 293.

[26] N. Rosen, *Gen. Relativ. Gravit.*, **4** (1973), 435.

[27] N. Rosen, *Annls. of Phy.*, **84** (1974), 455.

[28] J. Stachel, *Phys. Rev. D*, **21** (1980), 2171.

[29] P.K. Sahoo, B. Mishra, A. Ramu, *Int. J. Theor. Phys.*, **50** (2011), 349.

[30] E. Witten, *Phys. Rev. D*, **30** (1984), 272.

[31] I. Yavuz, I. Yilmaz, H. Baysal, *Int. J. Mod. Phys. D*, **14** (2005), 1365.

[32] I. Yilmaz, *Phys. Rev. D*, **71** (2005), 103503.

[33] I. Yilmaz, *Gen. Rel. Grav.*, **38** (2006), 1397.