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IMPLICIT MANN TYPE ITERATION METHOD INVOLVING THREE STRICTLY HEMICONTRACTIVE MAPPINGS

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Abstract: The purpose of this paper is to prove that the modified implicit Mann iteration process can be applied to approximate the common fixed point of three strictly hemicontractive mappings in smooth Banach spaces.

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1. Introduction

Let K be a nonempty subset of an arbitrary Banach space X and X^* be its dual space. The symbols $D(T)$ and $F(T)$ stand for the domain and the set of fixed points of T (for a single-valued mapping $T : X \to X$, $x \in X$ is called a *fixed point* of T iff $Tx = x$). We denote by J the normalized duality mapping from X to 2^{X^*} defined by

$$J(x) = \{f^* \in X^* : \langle x, f^* \rangle = \|x\|^2 = \|f^*\|^2\}, \quad x \in X,$$

where $\langle \cdot, \cdot \rangle$ denotes the duality pairing. In a smooth Banach space J is single-

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valued (we denoted by j).

Let $T : D(T) \subset X \to X$ be a mapping.

Definition 1.1. The mapping T is called *Lipshitzian* if there exists $L > 0$ such that

$$\|Tx - Ty\| \leq L \|x - y\|$$

for all $x, y \in D(T)$. If $L = 1$, then T is called *nonexpansive* and if $0 \leq L < 1$, then T is called *contractive.*

Definition 1.2. (see [2]) 1. The mapping T is said to be *pseudocontractive* if

$$\|x - y\| \leq \|x - y + r((I - T)x - (I - T)y\| \tag{1.1}$$

for all $x, y \in D(T)$ and $r > 0$.

2. The mapping T is said to be *strictly pseudocontractive* if there exists a $t > 1$ such that

$$\|x - y\| \leq \|(1 + r)(x - y) - rt(Tx - Ty)\| \tag{1.2}$$

for all $x, y \in D(T)$ and $r > 0$.

3. The mapping T is said to be *strictly hemicontractive* if $F(T) \neq \emptyset$ and if there exists a $t > 1$ such that

$$\|x - q\| \leq \|(1 + r)(x - q) - rt(Tx - q)\| \tag{1.4}$$

for all $x \in D(T)$, $q \in F(T)$ and $r > 0$.

Clearly, every strictly pseudocontractive mapping with a nonempty fixed point set is strictly hemicontractive, but the converse does not hold in general (see [2]).

Chidume [1] established that the Mann iteration sequence converges strongly to the unique fixed point of T in case T is a Lipschitzian strictly pseudocontractive mapping from a bounded closed convex subset of L_p (*or* l_p) into itself. Schu [10] generalized the result in [1] to both uniformly continuous strictly pseudocontractive mappings and real smooth Banach spaces. Park [8] extended the result in [1] to both strictly pseudocontractive mappings and certain smooth Banach spaces. Rhoades [9] proved that the Mann and Ishikawa iteration methods may exhibit different behaviors for different classes of nonlinear mappings. Afterwards, several generalizations have been made in various directions (see for example [2], [4], [5], [7], [8] and [11]).

In 2001, Xu and Ori [12] introduced the following implicit iteration process for a finite family of nonexpansive mappings $\{T_i : i \in I\}$ (here $I = \{1, 2, \ldots, N\}$), with $\{\alpha_n\}$ a real sequence in $(0, 1)$, and for an initial point $x_0 \in K$,

$$\begin{aligned} x_1 &= (1-\alpha_1)x_0 + \alpha_1 T_1 x_1, \\ x_2 &= (1-\alpha_2)x_1 + \alpha_2 T_2 x_2, \\ &\vdots \\ x_N &= (1-\alpha_N)x_{N-1} + \alpha_N T_N x_N, \\ x_{N+1} &= (1-\alpha_{N+1})x_N + \alpha_{N+1} T_{N+1} x_{N+1}, \\ &\vdots \end{aligned}$$

which can be written in the following compact form:

$$x_n = (1-\alpha_n)x_{n-1} + \alpha_n T_n x_n, \quad n \geq 1, \qquad (XO)$$

where $T_n = T_{n\,(mod\,N)}$ (here the $mod\,N$ function takes values in I). Xu and Ori [12] proved the weak convergence of this process to a common fixed point of the finite family defined in a Hilbert space. They further remarked that it is yet unclear what assumptions on the mappings and/or the parameters $\{\alpha_n\}$ are sufficient to guarantee the strong convergence of the sequence $\{x_n\}$.

Let K be a nonempty closed bounded convex subset of an arbitrary smooth Banach space X and $T, S, H : K \to K$ be three continuous strictly hemicontractive mappings. We proved that the implicit Mann type iteration method converges strongly to the common fixed point of T, S and H.

The results presented in this paper extend and improve the corresponding results particularly in [1], [5] and [6].

2. Preliminaries

We need the following results.

Lemma 2.1. (see [8]) *Let X be a smooth Banach space. Suppose that one of the following holds:*

(a) *J is uniformly continuous on any bounded subsets of X,*

(b) *$\langle x-y, j(x)-j(y)\rangle \leq \|x-y\|^2$ for all $x, y \in X$,*

(c) *for any bounded subset D of X, there is a function $c : [0,\infty) \to [0,\infty)$ such that*

$$Re\,\langle x-y,\ j(x)-j(y)\rangle \leq c(\|x-y\|)$$

for all $x, y \in D$*, where* c *satisfies* $\lim_{t\to 0^+} \frac{c(t)}{t} = 0$.
Then for any $\epsilon > 0$ *and any bounded subset* K*, there exists* $\delta > 0$ *such that*

$$\|sx + (1-s)y\|^2 \leq (1-2s)\|y\|^2 + 2sRe\langle x, j(y)\rangle + 2s\epsilon \tag{2.1}$$

for all $x, y \in K$ *and* $s \in [0, \delta]$.

Remark 2.2. 1. If X is uniformly smooth, then (a) in Lemma 2.1 holds.
2. If X is a Hilbert space, then (b) in Lemma 2.1 holds.

Lemma 2.3. (see [2]) *Let* $T : D(T) \subset X \to X$ *be a mapping with* $F(T) \neq \emptyset$*. Then* T *is strictly hemicontractive if and only if there exists* $t > 1$ *such that for all* $x \in D(T)$ *and* $q \in F(T)$*, there exists* $j(x-q) \in J(x-q)$ *satisfying*

$$Re\langle x - Tx, j(x-q)\rangle \geq \left(1 - t^{-1}\right)\|x-q\|^2. \tag{2.2}$$

Lemma 2.4. (see [5]) *Let* X *be an arbitrary normed linear space and* $T : D(T) \subset X \to X$ *be a mapping. If* T *is strictly hemicontractive, then* $F(T)$*is a singleton.*

Lemma 2.5. (see [5]) *Let* $\{\xi_n\}$*,* $\{\theta_n\}$ *and* $\{\omega_n\}$ *be nonnegative real sequences and* $\epsilon' > 0$ *be a constant satisfying*

$$\xi_{n+1} \leq (1-\theta_n)\xi_n + \epsilon'\theta_n + \omega_n, \quad n \geq 1,$$

where $\sum_{n=1}^{\infty} \theta_n = \infty$, $\theta_n \leq 1$ *for all* $n \geq 1$ *and* $\sum_{n=1}^{\infty} \omega_n < \infty$*. Then* $\lim_{n\to\infty}\sup \xi_n \leq \epsilon'$.

3. Main Results

We now prove our main results.

Lemma 3.1. *Let* X *be a smooth Banach space. Suppose that one of the following holds:*
(a) *J is uniformly continuous on any bounded subsets of* X,
(b) $\langle x - y, j(x) - j(y)\rangle \leq \|x-y\|^2$ *for all* $x, y \in X$,

(*c*) *for any bounded subset* D *of* X, *there is a function* $c : [0,\infty) \to [0,\infty)$ *such that*

$$Re\langle x-y, j(x)-j(y)\rangle \le c(\|x-y\|)$$

for all $x, y \in D$, *where* c *satisfies* $\lim\limits_{t\to 0^+} \frac{c(t)}{t} = 0$.
Then for any $\epsilon > 0$ *and any bounded subset* K, *there exists* $\delta > 0$ *such that*

$$\begin{aligned} &\|\alpha_1 x + \alpha_2 y + \alpha_3 z + \alpha_4 w\|^2 \\ &\le (1-2\alpha_1)\|x\|^2 + 2\frac{\alpha_1\alpha_2}{1-\alpha_1} Re\langle y, j(x)\rangle \\ &\quad + 2\frac{\alpha_1\alpha_3}{1-\alpha_1} Re\langle z, j(x)\rangle + 2\frac{\alpha_1\alpha_4}{1-\alpha_1} Re\langle w, j(x)\rangle + 2\epsilon\alpha_1 \end{aligned} \tag{3.1}$$

for all $x,\ y, z, w \in K$ *and* $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in [0,\delta]$ *with* $\alpha_1+\alpha_2+\alpha_3+\alpha_4 = 1$.

Proof. For $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in [0,\delta]$ with $\alpha_1+\alpha_2+\alpha_3+\alpha_4 = 1$, by using (2.1), consider

$$\begin{aligned} &\|\alpha_1 x + \alpha_2 y + \alpha_3 z + \alpha_4 w\|^2 \\ &= \left\|\alpha_1 x + (1-\alpha_1)\left(\frac{\alpha_2}{1-\alpha_1}y + \frac{\alpha_3}{1-\alpha_1}z + \frac{\alpha_4}{1-\alpha_1}w\right)\right\|^2 \\ &\le (1-2\alpha_1)\|x\|^2 + 2\epsilon\alpha_1 \\ &\quad + 2\alpha_1 \mathrm{Re}\left\langle \frac{\alpha_2}{1-\alpha_1}y + \frac{\alpha_3}{1-\alpha_1}z + \frac{\alpha_4}{1-\alpha_1}w, j(x)\right\rangle \\ &= (1-2\alpha_1)\|x\|^2 + 2\epsilon\alpha_1 + 2\frac{\alpha_1\alpha_2}{1-\alpha_1}\mathrm{Re}\langle y, j(x)\rangle \\ &\quad + 2\frac{\alpha_1\alpha_3}{1-\alpha_1}\mathrm{Re}\langle z, j(x)\rangle + 2\frac{\alpha_1\alpha_4}{1-\alpha_1}\mathrm{Re}\langle w, j(x)\rangle. \end{aligned}$$

This completes the proof. □

Theorem 3.2. *Let* X *be a smooth Banach space satisfying one of the Axioms (a)-(c) of Lemma 3.1. Let* K *be a nonempty closed bounded convex subset of* X *and* $T, S, H : K \to K$ *be three continuous strictly hemicontractive mappings. Let* $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ *and* $\{\delta_n\}$ *be real sequences in* $[0,1]$ *satisfying conditions*

(i) $\alpha_n + \beta_n + \gamma_n + \delta_n = 1$;

(ii) $\sum_{n=1}^{\infty} \alpha_n = \infty$;

(iii) $\lim\limits_{n\to\infty} \beta_n = \lim\limits_{n\to\infty} \gamma_n = \lim\limits_{n\to\infty} \delta_n = 0$.

For arbitrary $x_0 \in K$, let $\{x_n\}$ be a sequence generated by

$$x_n = \alpha_n x_{n-1} + \beta_n T x_n + \gamma_n S x_n + \delta_n H x_n, \quad n \geq 1. \tag{3.2}$$

Then the sequence $\{x_n\}$ converges strongly to the common fixed point q of T, S and H.

Proof. By [3, Corollary 1], T, S and H have the unique fixed point q in K. It follows from Lemma 2.4 that $F(T) \cap F(S) \cap F(H)$ is a singleton. That is, $F(T) \cap F(S) \cap F(H) = \{q\}$ for some $q \in K$.

Set $M = 1 + diam\, K$. It is easy to verify that

$$\begin{aligned} M = &\sup_{n\geq 1} \|x_n - q\| + \sup_{n\geq 1} \|Tx_n - q\| + \sup_{n\geq 1} \|Sx_n - q\| \\ &+ \sup_{n\geq 1} \|Hx_n - q\| . \end{aligned} \tag{3.3}$$

Consider

$$\begin{aligned} \|x_n - q\|^2 &= \|\alpha_n x_{n-1} + \beta_n T x_n + \gamma_n S x_n + \delta_n H x_n - q\|^2 \\ &= \|\alpha_n (x_{n-1} - q) + \beta_n (T x_n - q) \\ &\quad + \gamma_n (S x_n - q) + \delta_n (H x_n - q)\|^2 \\ &\leq \alpha_n \|x_{n-1} - q\|^2 + \beta_n \|T x_n - q\|^2 \\ &\quad + \gamma_n \|S x_n - q\|^2 + \delta_n \|H x_n - q\|^2 \\ &\leq \|x_{n-1} - q\|^2 + M^2 (\beta_n + \gamma_n + \delta_n), \end{aligned} \tag{3.4}$$

where the first inequality holds by the convexity of $\|\cdot\|^2$.

Now we put $k = \frac{1}{t}$, where t satisfies (2.2). Using (3.2) and Lemma 3.1, we infer that

$$\begin{aligned} &\|x_n - q\|^2 \\ &= \|\alpha_n x_{n-1} + \beta_n T x_n + \gamma_n S x_n + \delta_n H x_n - q\|^2 \\ &= \|\alpha_n (x_{n-1} - q) + \beta_n (T x_n - q) + \gamma_n (S x_n - q) + \delta_n (H x_n - q)\|^2 \\ &\leq (1 - 2\alpha_n) \|x_{n-1} - q\|^2 + 2\frac{\alpha_n \beta_n}{1 - \alpha_n} \mathrm{Re} \langle T x_n - q, j(x_{n-1} - q)\rangle \\ &\quad + 2\frac{\alpha_n \gamma_n}{1 - \alpha_n} \mathrm{Re} \langle S x_n - q, j(x_{n-1} - q)\rangle \\ &\quad + 2\frac{\alpha_n \delta_n}{1 - \alpha_n} \mathrm{Re} \langle H x_n - q, j(x_{n-1} - q)\rangle + 2\epsilon\alpha_n \end{aligned}$$

$$
\begin{aligned}
&= (1-2\alpha_n)\left\|x_{n-1}-q\right\|^2 + 2\frac{\alpha_n\beta_n}{1-\alpha_n}\operatorname{Re}\left\langle Tx_n-q, j(x_n-q)\right\rangle \\
&\quad + 2\frac{\alpha_n\beta_n}{1-\alpha_n}\operatorname{Re}\left\langle Tx_n-q, j(x_{n-1}-q)-j(x_n-q)\right\rangle \\
&\quad + 2\frac{\alpha_n\gamma_n}{1-\alpha_n}\operatorname{Re}\left\langle Sx_n-q, j(x_n-q)\right\rangle \\
&\quad + 2\frac{\alpha_n\gamma_n}{1-\alpha_n}\operatorname{Re}\left\langle Sx_n-q, j(x_{n-1}-q)-j(x_n-q)\right\rangle \\
&\quad + 2\frac{\alpha_n\delta_n}{1-\alpha_n}\operatorname{Re}\left\langle Hx_n-q, j(x_n-q)\right\rangle \\
&\quad + 2\frac{\alpha_n\delta_n}{1-\alpha_n}\operatorname{Re}\left\langle Hx_n-q, j(x_{n-1}-q)-j(x_n-q)\right\rangle + 2\epsilon\alpha_n \\
&\le (1-2\alpha_n)\left\|x_{n-1}-q\right\|^2 + 2\frac{\alpha_n\beta_n}{1-\alpha_n}k\left\|x_n-q\right\|^2 \\
&\quad + 2\frac{\alpha_n\beta_n}{1-\alpha_n}\left\|Tx_n-q\right\|\left\|j(x_{n-1}-q)-j(x_n-q)\right\| \\
&\quad + 2\frac{\alpha_n\gamma_n}{1-\alpha_n}k\left\|x_n-q\right\|^2 \\
&\quad + 2\frac{\alpha_n\gamma_n}{1-\alpha_n}\left\|Sx_n-q\right\|\left\|j(x_{n-1}-q)-j(x_n-q)\right\| \\
&\quad + 2\frac{\alpha_n\delta_n}{1-\alpha_n}k\left\|x_n-q\right\|^2 \\
&\quad + 2\frac{\alpha_n\delta_n}{1-\alpha_n}\left\|Hx_n-q\right\|\left\|j(x_{n-1}-q)-j(x_n-q)\right\| + 2\epsilon\alpha_n \\
&\le (1-2\alpha_n)\left\|x_{n-1}-q\right\|^2 + 2k\alpha_n\left\|x_n-q\right\|^2 \\
&\quad + 2M\alpha_n\varepsilon_n + 2\epsilon\alpha_n, \qquad (3.5)
\end{aligned}
$$

where

$$\varepsilon_n = \left\|j(x_{n-1}-q)-j(x_n-q)\right\|. \qquad (3.6)$$

Since J is uniformly continuous on any bounded subsets of X, we have

$$
\begin{aligned}
&\left\|x_{n-1}-x_n\right\| \\
&= \left\|x_{n-1}-\alpha_n x_{n-1}-\beta_n Tx_n-\gamma_n Sx_n-\delta_n Hx_n\right\| \\
&= \left\|\beta_n(x_{n-1}-Tx_n)+\gamma_n(x_{n-1}-Sx_n)+\delta_n(x_{n-1}-Hx_n)\right\| \\
&\le \beta_n\left\|x_{n-1}-Tx_n\right\| + \gamma_n\left\|x_{n-1}-Sx_n\right\| + \delta_n\left\|x_{n-1}-Hx_n\right\| \\
&\le 2M(\beta_n+\gamma_n+\delta_n) \\
&\to 0
\end{aligned}
$$

as $n \to \infty$, which implies that

$$\varepsilon_n \to 0 \quad \text{as } n \to \infty. \tag{3.7}$$

For given any $\epsilon > 0$ and the bounded subset K, there exists a $\delta > 0$ satisfying (2.1). Note that (iii) and (3.7) ensure that there exists an N such that

$$\beta_n, \gamma_n, \delta_n < \min\left\{\delta, \frac{\epsilon}{12M^2k}\right\}, \quad \varepsilon_n \leq \frac{\epsilon}{4M}, \quad n \geq N. \tag{3.8}$$

Now substituting (3.4) in (3.5) to obtain

$$\begin{aligned} \|x_n - q\|^2 &\leq (1 - 2(1-k)\alpha_n)\|x_{n-1} - q\|^2 \\ &\quad + 2M^2k\alpha_n(\beta_n + \gamma_n + \delta_n) + 2M\alpha_n\varepsilon_n + 2\epsilon\alpha_n \\ &\leq (1 - 2(1-k)\alpha_n)\|x_{n-1} - q\|^2 + 3\epsilon\alpha_n, \quad n \geq N. \end{aligned} \tag{3.9}$$

Putting

$$\begin{aligned} \xi_n &= \|x_{n-1} - q\|, \quad \theta_n = 2(1-k)\alpha_n, \\ \epsilon' &= \frac{3\epsilon}{2(1-k)}, \quad \omega_n = 0, \end{aligned}$$

we have from (3.9)

$$\xi_{n+1} \leq (1-\theta_n)\xi_n + \epsilon'\theta_n + \omega_n, \quad n \geq 1.$$

Set $\delta = \frac{1}{2(1-k)}$ for $k \leq \frac{1}{2}$. Because $\alpha_n \leq \delta$ we imply $2(1-k)\alpha_n \leq 1$. Observe that $\sum_{n=1}^{\infty} \theta_n = \infty$ and $\theta_n \leq 1$ for all $n \geq 1$. It follows from Lemma 2.5 that

$$\lim_{n\to\infty} \sup \|x_n - q\|^2 \leq \epsilon'.$$

Letting $\epsilon' \to 0^+$, we obtain that $\lim_{n\to\infty} \sup \|x_n - q\|^2 = 0$, which implies that $x_n \to q$ as $n \to \infty$. This completes the proof. □

Corollary 3.3. *Let X be a smooth Banach space satisfying one of the Axioms (a)-(c) of Lemma 3.1. Let K be a nonempty closed bounded convex subset of X and $T, S, H : K \to K$ be three Lipschitzian strictly hemicontractive mappings. Let $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ and $\{\delta_n\}$ be real sequences in $[0,1]$ satisfying the conditions* (i)-(iii)

For arbitrary $x_0 \in K$, let $\{x_n\}$ be the sequence defined by the implicit iteration process (3.2)*. Then the sequence $\{x_n\}$ converges strongly to the common fixed point q of T, S and H.*

Corollary 3.4. *Let X be a smooth Banach space satisfying one of the Axioms (a)-(c) of Lemma 3.1. Let K be a nonempty closed bounded convex subset of X and $T : K \to K$ be a continuous strictly hemicontractive mapping. Let $\{\alpha_n\}$ be a real sequence in $[0, 1]$ satisfying conditions*

(iv) $\lim_{n\to\infty} \alpha_n = 0$;

(v) $\sum_{n=1}^{\infty}(1 - \alpha_n) = \infty$.

For arbitrary $x_0 \in K$, let $\{x_n\}$ be the sequence defined by the implicit iteration process (XO) with $T_i = T$ $(i = 1, \dots, N)$. Then the sequence $\{x_n\}$ converges strongly to a unique fixed point q of T.

Corollary 3.5. *Let X be a smooth Banach space satisfying one of the Axioms (a)-(c) of Lemma 3.1. Let K be a nonempty closed bounded convex subset of X and $T : K \to K$ be a Lipschitzian strictly hemicontractive mapping. Let $\{\alpha_n\}$ be a real sequence in $[0, 1]$ satisfying the conditions* (iv) *and* (v).

For arbitrary $x_0 \in K$, let $\{x_n\}$ be the sequence defined by the implicit iteration process (XO) with $T_i = T$ $(i = 1, \dots, N)$. Then the sequence $\{x_n\}$ converges strongly to a unique fixed point q of T.

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