CATCHING A GANG – A MATHEMATICAL MODEL OF
THE SPREAD OF GANGS IN A POPULATION
TREATED AS AN INFECTIOUS DISEASE

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Abstract: In this study, criminal gang membership is treated as an infection that spreads through a community by interactions among gang members and the population. A mathematical model consisting of a system of coupled, nonlinear ordinary differential equations is used to describe this spread and to suggest control mechanisms to minimize this infection. The analysis shows the existence of three equilibrium states - two of which contain no gang members. When parameters such as recruitment, conviction and recidivism rates and longer jail sentences are varied, the greatest reduction occurs by changing the parameters in combination. A bifurcation analysis shows transcritical bifurcations and no hopf bifurcations.

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1. Introduction

Gangs and their associated violence are becoming an increasingly worldwide phenomenon [8]. Research into this growing phenomenon is vital in the development of public policies and social movements which may decrease the gang

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population [8]. Policies are generally based on linear systems [1] which aim to describe a system and make predictions about general trends. Linear systems have identifiable cause-and-effect relationships, repeatability and proportionality between inputs and outputs [1]. These properties make linear systems particularly useful for prediction and manipulation [9] - hence their popularity in modelling. However, some have postulated that gangs as described as organised crime in [2] are better described by a nonlinear and dynamical system. Hence, we have applied an infectious disease model to criminal gangs and our goal is to give a general description of how the gang population responds to policy changes.

Association with delinquent peers is one of the strongest risk factors for gang membership [14]. This association therefore lends itself to an ‘infection hypothesis’ of growth in the gang subpopulation. We therefore treat gang membership as an infection that multiplies due to interaction or peer contagion whereby delinquent youth convert vulnerable youth through verbal and non-verbal communication. Prior research has applied the infection disease to other social phenomena [11], [13], [7], but only one group [10] and [4] has attempted to model violent crime using the Infectious Disease model but this group did not consider gang-related crimes.

In this paper, the population is divided into four disjoint groups who differ in their potential to become a gang member. A system of four differential equations is used to model the flows between these groups over time. The strength of these flows is based upon factors identified in the literature related to the formation and desistance from gangs. The model examines the impact of various crime fighting strategies to determine their influence on gang membership. This may be used to assist policy-makers in the development of effective gang control policies. The behavior of the model is investigated through stability analysis and numerical simulation.

The paper is organized as follows: Section 1 contains a description of the model and its inherent assumptions. The different equilibria are and their stability are analysed in Section 2. Sections 3 and 4 present the numerical simulations and discussion.

2. The Model

Figure 1 is an adaptation of the basic SIR models (S for susceptible, I for infectious and R for recovered) used in mathematical epidemiology [3]. The population T is divided into four disjoint compartments based on infection

Figure 1: The Gang Model

status and risk factors with respect to gang membership. These are denoted by the letters N , S , G , and R and are based on the model for gang hierarchy in [15]. N represents those who are not susceptible to a life in the gang. S denotes potential gang members or susceptibles - these correspond to the fringe and wannabes. The fringe gang member is not yet committed to the gang lifestyle and may drift in and out of the gang. The entry level or wannabes are potential gang members. The compartment G contains committed core gang members – the gang leaders, hard core members and associates. The removed compartment R represents the gang members who are removed from the gang by placing them in jail. To ensure continuity, gang members must actively recruit and retain its members [8]. Each gang member acts as a potential ‘carrier’. Since we divide the gang hierarchy from [15] into two compartments, the model contains two standard incidence contact or recruitment terms.

Non-susceptibles (N) may by contact rate β_1 with susceptibles (S) and gang members (G) become susceptible (S) at a rate $\beta_1 (S + G) \left(\frac{N}{T}\right)$. However it is possible to intervene at this stage – with the intervention/interference parameter α – and sway the youth back into the Non-susceptible class (N). The susceptibles (S) by contact rate β_2 with gang members (G) may be recruited into the gang (G) at a rate $\beta_2 \frac{SG}{T}$. These gang members (G) may then go to jail with an imprisonment rate of Φ . Some individuals on serving their jail sentence may relapse to G or may have been rehabilitated in jail and recover to join the Non-Susceptible (N) class. ρ^{-1} gives the average length of the jail sentence. The proportion of prisoners moving back to G after release is defined as $(1 - f)$, hence $(1 - f)$ denotes the recidivism rate while f denotes the rehabilitation rate. Figure 1 shows the structure of flows and parameters used within the model, which is given by following non-linear system:

$$\frac{dN}{dt} = -\beta_1(S + G)\frac{N}{T} + \alpha S + f\rho R, \quad (1)$$

$$\frac{dS}{dt} = \beta_1(S + G)\frac{N}{T} - \beta_2\frac{SG}{T} - \alpha S, \quad (2)$$

$$\frac{dG}{dt} = \beta_2\frac{SG}{T} + (1 - f)\rho R - \Phi G, \quad (3)$$

$$\frac{dR}{dt} = \Phi G - \rho R, \quad (4)$$

$$N + S + G + R = T, \quad (5)$$

where

$$N(0) \geq 0, \quad S(0) \geq 0, \quad G(0) \geq 0, \quad R(0) \geq 0.$$

Since the total population T is constant, we may consider three equations. Re-scaling, so that we will be working with proportion of subsets of the population where

$$s = \frac{S}{T}, \quad g = \frac{G}{T}, \quad r = \frac{R}{T}, \quad n = \frac{N}{T}$$

gives the following system:

$$\frac{dn}{dt} = -\beta_1 n(s + g) + \alpha s + f\rho r, \quad (6)$$

$$\frac{ds}{dt} = \beta_1 n(s + g) - \beta_2 sg - \alpha s, \quad (7)$$

$$\frac{dg}{dt} = \beta_2 sg + (1 - f)\rho r - \Phi g, \quad (8)$$

$$\frac{dr}{dt} = \Phi g - \rho r, \quad (9)$$

$$n + s + g + r = 1. \quad (10)$$

2.1. Assumptions

1. We make the basic assumption that belonging to a gang is contagious i.e. crime breeds more crime. In this model, gang membership will be considered as an infection.
2. Social contact plays an important role in the recruitment of gang members.
3. All gang members and susceptibles play an equal part in the recruitment process.

4. We ignore the details of infection within an individual, considering him to be in one of a small number of discrete states, such as infected or susceptible. We consider an individual who is infective as infectious.
5. It is easier to convince someone to consider joining a gang than it is to persuade someone who is a potential member to take the final step and join the gang $\beta_1 \geq \beta_2$.
6. This model does not take into account births and deaths of the population. Hence T is assumed to be constant.

3. Mathematical Analysis

3.1. Calculation of R_0

The issue of whether or not a disease can invade a host population and persist or remain endemic involves the introduction of a threshold - the basic reproductive number R_0 - the average number of secondary cases caused by an infected individual. In this case, R_0 represents the average number of secondary gang members recruited by a gang member. R_0 can be computed in various ways. Here, we use the next generation operator method [5]. In applying this method, s and g are considered the infective compartments, n represents the non-infective compartments and r represents individuals who are infected but do not transmit the disease.

We obtain $R_0 = \frac{\beta_1}{\alpha}$. The condition $R_0 < 1$ is, at least, a necessary condition for a globally asymptotically stable disease free state. On the other hand, $R_0 > 1$ allows for the possibility of multiple stable endemic states. For $R_0 < 1$, a new epidemic cannot be started and an endemic disease will fade out. In this model, the transmission may be considered a collective phenomenon – societal and peer pressure play important roles – so R_0 may be considered as indicative of the suitability of the environment to harbouring gangs.

3.2. Disease Dynamics: Equilibria

There are three possible equilibrium states - the Criminal-free equilibrium, the Core-free equilibrium and the Coexistence equilibrium.

3.2.1. Case 1 and 2: $g^* = 0$: Criminal-free equilibrium and Core-free equilibrium

Case 1: $s = 0$, then from (6) to (10) $g = 0$, $r = 0$, and $n = 1$. This is the criminal-free equilibrium.

Case 2: $n = \frac{\alpha}{\beta_1}$, then from (6) to (10) $g = 0$, $r = 0$, $s = \frac{\beta_1 - \alpha}{\beta_1}$. This means that $\beta_1 > \alpha$ ($R_0 > 1$). This is the core-free equilibrium.

3.2.2. Case 3: $s^* = \frac{f\Phi}{\beta_2}$ where $f\Phi < \beta_2$: Coexistence equilibria

Another possible end state for the model is one in which all the subpopulations coexist and gangs exist in an endemic state within the population. Substituting for s and n in (6) using (9) and (10), will result in a quadratic in g of the form $b_1 g^2 + b_2 g + b_3 = 0$, that can be solved for g where

$$b_1 = \beta_1 \left(1 + \frac{\Phi}{\rho}\right) > 0, \quad (11)$$

$$b_2 = \frac{2\beta_1 f\Phi}{\beta_2} + \frac{\beta_1 f\Phi^2}{\beta_2 \rho} - \beta_1 + f\Phi, \quad (12)$$

$$b_3 = \frac{\beta_1 f^2 \Phi^2}{\beta_2^2} - \frac{f\Phi}{\beta_2} (\beta_1 - \alpha). \quad (13)$$

Since $b_1 > 0$, there may be zero, one or two coexistence solutions for g depending on the values of b_2 and b_3 .

3.3. Stability of Equilibria

To check the stability of the equilibrium points, we linearise the system by taking a small perturbation about the equilibrium points (n, s, g) by substituting

$$n = n^* + u; \quad s = s^* + v; \quad g = g^* + w \quad (14)$$

where u, v, w are small perturbations. We expand all terms about the equilibria using Taylor's theorem and neglect higher order terms in u, v, w . The linearised form of the equations can be written as

$$\frac{dV}{dt} = JV, \quad (15)$$

where

$$J = \begin{pmatrix} -B - f\rho & -G + \alpha & -G \\ B & \beta_1 n - \beta_2 g - \alpha & \beta_1 n - \beta_2 s \\ -F & \beta_2 g - F & \beta_2 s - F - \Phi \end{pmatrix},$$

with

$$B = \beta_1(s + g); \quad F = (1 - f)\rho; \quad G = \beta_1 n + f\rho.$$

Also

$$V = \begin{pmatrix} u \\ v \\ w \end{pmatrix}.$$

The associated characteristic equation is obtained from

$$|J - \lambda I| = 0$$

and is of the form

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$$

where

$$a_1 = B + F + \Phi + \alpha + f\rho + \beta_2 g - \beta_1 n - \beta_2 s,$$

$$a_2 = B(G - \alpha) + \phi_1\phi_2 + (B + f\rho)\phi_3 - FG + \phi_4,$$

$$a_3 = \phi_5\phi_1 - \phi_6\phi_7 - (F - \beta_2 g)\phi_8 + F\phi_9,$$

with

$$\phi_1 = F + \Phi - \beta_2 s,$$

$$\phi_2 = B + \alpha + f\rho + \beta_2 g - \beta_1 n,$$

$$\phi_3 = \alpha + \beta_2 g - \beta_1 n,$$

$$\phi_4 = (F - \beta_2 g)(\beta_1 n - \beta_2 s),$$

$$\phi_5 = (B(G - \alpha) + (B + f\rho)(\alpha + \beta_2 g - \beta_1 n)),$$

$$\phi_6 = (FG - (F - \beta_2 g)(\beta_1 n - \beta_2 s)),$$

$$\phi_7 = (B + \alpha + f\rho + \beta_2 g - \beta_1 n),$$

$$\phi_8 = ((\beta_1 n - \beta_2 s)(\alpha + \beta_2 g - \beta_1 n) + BG),$$

$$\phi_9 = (G(B + f\rho) - (\beta_1 n - \beta_2 s)(G - \alpha)).$$

The Routh-Hurwitz criteria

$$a_1 > 0, \quad a_3 > 0, \quad a_1 a_2 - a_3 > 0 \tag{16}$$

are used to determine the stability of each case.

3.3.1. Case 1:

At the criminal-free equilibrium

$$(n, s, g) = (1, 0, 0)$$

the condition for stability using (16) is:

$$\alpha > \beta_1. \quad (17)$$

This condition on the stability of the criminal-free equilibrium is the same as the condition that the basic reproduction number $R_0 < 1$ so that an epidemic does not occur. This means that once potential recruits can be dissuaded from joining a gang faster than they can be recruited, no gangs will arise in the population.

3.3.2. Case 2: $n^* = \frac{\alpha}{\beta_1}$, $s^* = \frac{\beta_1 - \alpha}{\beta_1}$, $g^* = 0$, $r^* = 0$, $\beta_1 > \alpha$

In this section it is assumed that the disease has become established, that is, $R_0 > 1$. The conditions for stability using (16) are:

$$\beta_1 > \beta_2 ; f\Phi > \left(1 - \frac{\alpha}{\beta_1}\right) \beta_2. \quad (18)$$

3.3.3. Case 3: $s^* = \frac{f\Phi}{\beta_2}$, $f\Phi < \beta_2$, $n^* = 1 - \frac{f\Phi}{\beta_2} - g^*(1 - \frac{\Phi}{\rho})$, g^*

The condition for stability using (16) is

$$\eta_1 (\eta_2 - \Phi (1 - f) (Q\beta_1 - \alpha)) > \eta_3$$

where

$$\begin{aligned} \eta_1 &= \Phi (1 - f) + \rho + \alpha + \beta_2 g - \beta_1 Q, \\ \eta_2 &= \beta_2 g - \Phi - \beta_1 Q \beta_2 g + \rho (\alpha + \beta_2 g - \beta_1 Q), \\ \eta_3 &= \left(\Phi \beta_1 \left(\frac{f\Phi}{\beta_2} + g \right) + \rho f \Phi - \beta_1 \rho Q \right) \beta_2 g, \\ Q &= 1 - \frac{2f\Phi}{\beta_2} - g \left(2 - \frac{\Phi}{\rho} \right). \end{aligned}$$

4. Bifurcation Analysis

The parameters used in the numerical simulations were estimated using local data [10] as given in Table 4.

Parameter	Value
β_1	0.71
β_2	0.21
α	0.126 5
Φ	0.115
ρ	0.2
f	0.44

MATCONT, a graphical MATLAB package for the interactive numerical study of ODE dynamical systems is used for the numerical analysis. An ODE 45 differential equation solver based on the Runge-Kutta method with a variable time step was used to numerically integrate the equations using initial conditions to locate all the equilibrium points over a range of values for the parameters. This value was then used to draw the equilibrium curve by varying one parameter and keeping the others fixed. MATCONT indicates ‘BP’ or branch point at the extinction point transcritical bifurcation, while ‘LP’ represents a limit point or saddle-node bifurcation.

4.1. Bifurcation with respect to α

We do a bifurcation analysis using MATCONT taking α as the bifurcation parameter. Figure 2 and Figure 3 represent bifurcation diagrams for $0 < \alpha < 1$. These figures show the existence of two branch points and one limit point at $\alpha = 0.538924, 0.71, 0.583374$ respectively.

The branch points may be calculated from (17) and (18), where for stability:

$$\alpha > 0.71 \text{ for Case 1 and } \alpha > 0.538924 \text{ for Case 2.}$$

The limit point may be calculated from the condition for one coexistence equilibrium value using (11) - (13) where:

$$b_1^2 = 4b_2b_3,$$

On substituting parameter values we get $\alpha = 0.583374$.

Figure 4 shows that for $0 < \alpha < 0.538924$, the system tends to the Case 3 coexistence equilibrium and that for $0.538924 < \alpha < 0.583374$, there are three

Figure 2: Bifurcation Diagram for α – g vs α Figure 3: Bifurcation Diagram for α – n vs α

possible equilibria- a stable Case 3 coexistence equilibrium, an unstable Case 3

Figure 4: Bifurcation Diagram for α showing dependence on the initial conditions

Figure 5: $\alpha = 0.1265$: Case 3 stable

Figure 6: $\alpha = 0.75$: Case 3 unstable and tending to stable Case 1

coexistence equilibrium and the Case 2 core-free equilibrium. For $0.583374 < \alpha < 0.71$, the system tends to the Case 2 core-free equilibrium and for $\alpha > 0.71$, the system tends to the Case 1 criminal-free equilibrium.

We illustrate the stability for $\alpha = 0.1265$ and $\alpha = 0.75$ using (14) with a small perturbation of 0.0001. Figure 5 shows the stability of the Case 3 coexistence equilibrium:

$$n = 0.1107, s = 0.2410, g = 0.4116, r = 0.2367,$$

when $\alpha = 0.1265$. When α is increased past the bifurcation point of $\alpha = 0.71$ to $\alpha = 0.75$ in Figure 6 with no change to the other parameters, the Case 3 equilibrium tends to the stable Case 1 criminal-free equilibrium, resulting in equilibrium values:

$$n = 1, s = 0, g = 0, r = 0.$$

4.2. Bifurcations with respect to β_1 , β_2 and Φ

We also do bifurcation analyses using MATCONT taking β_1 , β_2 and Φ as the bifurcation parameters. Figures 7-10 show the bifurcation diagrams for β_1 , β_2 and Φ . The branch points may be calculated from (18) where for stability of

Figure 7: Bifurcation Diagram for $\beta_1 - g$ vs β_1 Figure 8: Bifurcation Diagram for $\beta_2 - g$ vs β_2

the Case 2 equilibrium:

$$\beta_1 < 0.1667, \beta_2 < 0.0615 \text{ and } \Phi > 0.39997. \quad (19)$$

Figure 9: Bifurcation Diagram for $\beta_2 - s$ vs β_2 Figure 10: Bifurcation Diagram for $\phi - g$ vs ϕ

There are two branch points with respect to β_1 in Figure 7. $\beta_1 = 0.1265$ may be derived from the condition for the criminal-free Case 1 equilibrium (17) and

the second branch point $\beta_1 = 0.1667$ from (19).

From Figure 7, when $\beta_1 < 0.1265$, the system tends to the criminal-free Case 1 equilibrium. For $0.1265 < \beta_1 < 0.1659$, the system tends to the Case 2 core-free equilibrium. The limit point for β_1 , $\beta_1 = 0.1659$ may be calculated from the condition for the existence of one coexistence equilibrium solution using (11) - (13). For $0.1659 < \beta_1 < 0.1667$, there are three possible equilibria- a stable Case 3 coexistence equilibrium, an unstable Case 3 coexistence equilibrium and the Case 2 core-free equilibrium. When $\beta_1 > 0.1667$, the system tends to the Case 3 coexistence equilibrium.

Figures 8 and 9 show the bifurcation diagram for $0 < \beta_2 < 1$. Using (19) when $\beta_2 < 0.0615$, the system tends to the Case 2 core-free equilibrium, otherwise it tends to the Case 3 coexistence equilibrium.

Figure 10 shows the bifurcation diagram for $0 < \Phi < 1$, where for $\Phi > 0.39997$, the system tends to the Case 2 core-free equilibrium, otherwise it tends to the Case 3 coexistence equilibrium.

We illustrate the stability for $\beta_1 = 0.14$ and $\Phi = 0.3$ in Figures 11 and 12 using (14) with a small perturbation of 0.0001. From Figure 7, when $\beta_1 = 0.14$, the system tends to the Case 2 core-free equilibrium, where

$$n = 0.9036, \quad s = 0.0964, \quad g = 0, \quad r = 0,$$

as illustrated in Figure 11. Similarly if we increase Φ to 0.3, past the bifurcation point with no change to the other parameters, the case 2 equilibrium

$$n = 0.1782, \quad s = 0.8218, \quad g = 0, \quad r = 0,$$

becomes unstable - Figure 12.

5. Discussion

The model can be used to predict the effects of different gang reduction strategies by varying the parameters in the model. The approach to reducing gangs is usually one involving combinations of prevention, intervention and suppression strategies [12]. In this model, the prevention and intervention programs are related to α , β_1 and β_2 . Suppression strategies will be inherent in Φ , ρ and f .

The deterministic approach used in this model is useful in detecting bifurcation points. At these points, small changes in certain parameters of a nonlinear system can cause equilibria to appear or disappear, or to change from attracting to repelling and vice versa, leading to large and sudden changes in the behavior

Figure 11: $\beta_1 = 0.14$: Case 2 stable Figure 12: $\Phi = 0.3$: Case 2 unstable

of the system. Epidemics have bifurcation points at which an ordinary and stable phenomenon can turn into a public health crisis. Though mathematical

models differ from the real world, the possibility of bifurcation points resulting in a sudden change in the behavior of the system is important in designing policies.

The bifurcation analysis showed bifurcations with respect to α , Φ , β_1 and β_2 . This means that by changing these values past the bifurcation values we can shift the system into different equilibrium states and potentially reduce the number of gang members to zero.

When the intervention/interference parameter α is such that $0.583374 < \alpha < 0.583374$, the system will tend to either the Case 2 core-free equilibrium value or the Case 3 coexistence equilibrium. This behavior is different from typical SIR models where there is no dependence on the initial population values. For values of $\alpha > 0.71 (= \beta_1)$, the system tends to the Case 1 criminal-free equilibrium where $s = 0$, $g = 0$, $r = 0$ and $n = 1$, which may be desirable since there are no gang members in this state. However, even if more resources are utilised to increase α above this value, the number of gang members will still be zero and from this point of view will be an unnecessary expenditure of resources.

If we consider reducing β_1 , the bifurcation points occur at values less than $\beta_2 = 0.21$, which violates our assumption that $\beta_1 > \beta_2$. It is possible to reduce Φ and β_2 to obtain the Case 2 core-free equilibrium state, but not to the Case 1 criminal-free equilibrium. From the point of view of eliminating gangs, this means that it is not possible to eliminate all the gang members from the population by varying β_1 , β_2 or Φ .

6. Conclusion

Terms like crime ‘epidemic’, ‘crime waves’ or crime ‘spreading’ from one locale to another are reminiscent of terms used in epidemiology. This research applies existing techniques from modelling infectious diseases to model gangs. Both international and local research are used to determine the key features of the behavior of gang members that should be included in a crime model. The model gives an insight into the effectiveness of responses aimed at reducing gangs. However, the model results are not predictions, and analysis of the model is used to yield insights into dealing with the gang problem. The model shows that nipping the problem at the S creation stage by varying α , β_1 and β_2 as well as the imprisonment rate Φ will have the biggest impact on decreasing gang membership.

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